ISSN 2063-5346

MULTI-CRITERIA DECISION MAKING PROBLEM USING ENERGY OF INVERSE INTUITIONISTIC FUZZY GRAPH

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Article History: Received: 02.07.2023 Revised: 15.07.2023 Accepted: 23.07.2023

Abstract

In this Paper, we study the TOPSIS method on the inverse intuitionistic fuzzy graph (IIFG). The inverse intuitionistic fuzzy relations were defined from the inverse intuitionistic fuzzy matrix (IIFM). An entropy measure defined is used for calculating the weights. Also, the positive and negative ideal solution for inverse intuitionistic fuzzy matrix is formed. Moreover, the relative closeness coefficient is calculated and based on these values, the alternatives are ranked.

Keywords: Inverse intuitionistic fuzzy matrix, energy, entropy function, ideal solution, TOPSIS method.

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DOI:10.48047/ecb/2023.12.9.206

1 Introduction

The concept of fuzzy set was introduced by Zadeh [1]. Atanasssov [2, 3] introduced the concept of fuzzy sets as a generalization of fuzzy sets and also he developed the concepts of intuitionistic fuzzy relations and intuitionistic fuzzy graphs. Alfuraidan et al. [4] proposed the inverse graph with finite groups. Gutman [5] defined the energy of a graph and Anjali et al. [6] proposed the energy of a fuzzy graph. John Stephen et al. [7] investigated the idea of an inverse domination set and inverse domination number of an IFG. Akram et al. [9] defined some operations on IFG structures. Certain concepts of IVIFG with its application was discussed by Talebi et al. [10] and Peng et al. [11].

TOPSIS technique is the one of the popular MCDM model, it has been commonly used to solve decision making problem, which was first discovered by Hwang and Yoon in 1981. Boran et al. [12] proposed a multi criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Kumar et al. [13] and Wang et al. [14] developing a fuzzy TOPSIS approach based on subjective weights and objective weights. The application of IF set TOPSIS method in employee performance has been proposed by Yinghui [15]. Zulqarnain et al. [16] investigated the intuitionistic fuzzy TOPSIS method for selection of best alternative of an automotive company.

Motivated by these theories, Inverse intuitionistic fuzzy graph matrix has been proposed and developed by using TOPSIS method. Further, a numerical example is given to the proposed method.

2 Preliminaries

In this section, we give some definitions for the further reference.

Definition 2.1. [6] The adjacency matrix A of a fuzzy graph $G = (\upsilon, \sigma, \mu)$ is an $n \times n$ matrix where $n = |\upsilon|$ defined as $A = [a_{ij}]$ where $a_{ij} = \mu(\upsilon_{i,}\upsilon_j)$.

Definition 2.2. [6] Let $G = (\upsilon, \sigma, \mu)$ be a fuzzy graph and A be its adjacency matrix. The eigenvalues of A are called eigenvalues of G. The spectrum of A is called spectrum of G. It is denoted by $Spec\ G$.

Definition 2.3. [6] Let $G = (\upsilon, \sigma, \mu)$ be a fuzzy graph and A be its adjacency matrix. Energy of G is defined as the sum of the absolute values of the eigenvalues of A.

Definition 2.4. Let $(G,*)$ be a finite group and let $S = \{u \in G | u^{-1} \notin G\}$ be a non-empty finite subset of non self-invertible elements of G such that u and v are adjacent if and only if $u * v \in S$ or $* u \in S$, then the *inverse intuitionistic fuzzy graph* $I_G = (\tau, \rho)$ is an intuitionistic fuzzy graph with vertex set $\tau = G$ and let $\Omega = \{\mu_\Omega, \upsilon_\Omega\}$ be an intuitionistic fuzzy subset of τ, where μ_Ω and υ_Ω are the membership and non-membership functions of IIFG satisfies the condition $\mu_\Omega + \upsilon_\Omega \leq 1$. The inverse intuitionistic fuzzy edge relation $\rho(x, y)$ on τ is defined by

$$\rho(x, y) = \{\mu_\Omega(x * y), \upsilon_\Omega(x * y) |\ x, y \in \tau\ and\ (x * y) \in S\}$$

3. Inverse Intuitionistic Fuzzy Graph on Z_4

In this section, we study the inverse intuitionistic fuzzy graphs on group Z4.

The inverse intuitionistic fuzzy vertex relation of Z_4 is defined by

$$\mu_{\Omega ij} = \begin{cases} \dfrac{\sqrt[4]{n}}{4} & , \quad when \quad n = 4 \\ \dfrac{n}{4} & , \quad when \quad n = 1, 2, 3 \end{cases}$$

where $I_1 = 0, I_2 = 1, I_3 = 2, I_4 = 3$ are the vertex set of Z_4 for all n = 4 and $\nu_{\Omega ij} = (1 - \mu_{\Omega ij})$.

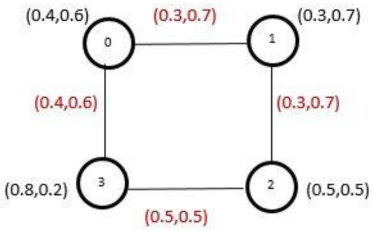

Figure 1: Inverse Intuitionistic Fuzzy Graph Z_4

Definition 3.1. The adjacency matrix of IIFG is defined using inverse intuitionistic fuzzy relation and is denoted by $IIFM_{ij}$.

$$IIFM_{ij} = \begin{matrix} \\ I_1 \\ I_2 \\ \vdots \\ I_n \end{matrix} \begin{matrix} I_1 & I_2 & & I_r \\ \left(\begin{matrix} (\mu_{\Omega_{11}}, \upsilon_{\Omega_{11}}) & (\mu_{\Omega_{12}}, \upsilon_{\Omega_{12}}) & \cdots & (\mu_{\Omega_{1r}}, \upsilon_{\Omega_{1r}}) \\ (\mu_{\Omega_{21}}, \upsilon_{\Omega_{21}}) & (\mu_{\Omega_{22}}, \upsilon_{\Omega_{22}}) & \cdots & (\mu_{\Omega_{2r}}, \upsilon_{\Omega_{2r}}) \\ \cdots & \cdots & \cdots & \cdots \\ (\mu_{\Omega_{n1}}, \upsilon_{\Omega_{n1}}) & (\mu_{\Omega_{n2}}, \upsilon_{\Omega_{n2}}) & \cdots & (\mu_{\Omega_{nr}}, \upsilon_{\Omega_{nr}}) \end{matrix}\right) \end{matrix}$$

In short, $\mu_{\Omega ij} = \mu_{ij}$ and $\upsilon_{\Omega ij} = \upsilon_{ij}$, $i = 1,2,...,r$ and $j = 1,2,...,n.$.

Definition 3.2. The energy $E(I_G)$ of a IIFG is the sum of the absolute values of the eigenvalues of $IIFM_{ij}$

$E(I_G) = \sum_{i=1}^{n} A_i$, where A_1 , A_2 , ..., A_n are the eigen values of the adjacency matrix of IIFG.

Definition 3.3. The inverse intuitionistic fuzzy normalized decision matrix is denoted by $IIFNDM_{ij}$ and is defined by

$$IIFNDM_{ij} = \begin{matrix} \\ I_1 \\ I_2 \\ \vdots \\ I_n \end{matrix} \begin{matrix} I_1 & I_2 & & I_r \\ \left(\begin{matrix} (\alpha_{11}, \beta_{11}) & (\alpha_{12}, \beta_{12}) & \cdots & (\alpha_{1r}, \beta_{1r}) \\ (\alpha_{21}, \beta_{21}) & (\alpha_{22}, \beta_{22}) & \cdots & (\alpha_{2r}, \beta_{2r}) \\ \cdots & \cdots & \cdots & \cdots \\ (\alpha_{n1}, \beta_{n1}) & (\alpha_{n2}, \beta_{n2}) & \cdots & (\alpha_{nr}, \beta_{nr}) \end{matrix}\right) \end{matrix}$$

where $\alpha_{ij} = \dfrac{\mu_{ij}}{\sqrt{\sum_{i=1}^{r} \left(E(I_G) - \mu_{ij}\right)^2}}, \beta_{ij} = \dfrac{\nu_{ij}}{\sqrt{\sum_{i=1}^{r} \left(E(I_G) - \nu_{ij}\right)^2}}$, $j = 1,2,3,4.$,

where E is the energy $E(I_G)$ of $IIFM_{ij}$.

Definition 3.4. Given an $IIFM_{ij}$, Let $d_{ij} = 1 - |\mu_{ij} - \nu_{ij}|$ is the fuzzy degree. The IIF entropy of evaluation index is defined as, $E_j = \dfrac{1}{n}\sum_{i=1}^{n} d_{ij}, j = 1,2,...,r.$

The weight of evaluation index of $IIFM_{ij}$ is defined as, $w_i = \dfrac{E_j}{\sum_{j=1}^{n}(E_j)}, i = 1,2,...,r.$

Thus the weight vector $w_i = (w_1, w_2, ..., w_r)$ which satisfies $\sum_{i=1}^{r} w_i = 1$.

Definition 3.5. The IIF weighted normalized decision matrix is defined as,

$W_{\mu_{ij}} = w_i \alpha_{ij}, W_{\nu_{ij}} = w_i \beta_{ij}$.

$$IIFWNDM_{ij} = \begin{matrix} & I_1 & I_2 & & I_r \\ I_1 & (W_{\mu_{11}}, W_{\nu_{11}}) & (W_{\mu_{12}}, W_{\nu_{12}}) & \cdots & (W_{\mu_{1r}}, W_{\nu_{1r}}) \\ I_2 & (W_{\mu_{21}}, W_{\nu_{21}}) & (W_{\mu_{22}}, W_{\nu_{22}}) & \cdots & (W_{\mu_{2r}}, W_{\nu_{2r}}) \\ \vdots & \cdots & \cdots & \cdots & \cdots \\ I_n & (W_{\mu_{n1}}, W_{\nu_{n1}}) & (W_{\mu_{n2}}, W_{\nu_{n2}}) & \cdots & (W_{\mu_{nr}}, W_{\nu_{nr}}) \end{matrix}$$

Definition 3.6. The IIF positive ideal solution ($IIFPIS$) and IIF negative ideal solution ($IIFNIS$), with respect to the criteria are defined as,

$IIFPIS_j^+ = \left(\max W_{\mu_{ij}}, \min W_{\nu_{ij}}\right)$ and $IIFNIS_j^+ = \left(\min W_{\mu_{ij}}, \max W_{\nu_{ij}}\right)$.

Definition 3.7. The distance between PIS and NIS of $IIFM_{ij}$ is defined by $IIFD_i^+$ and $IIFD_i^-$ is,

$$IIFD_i^+ = \sqrt{\sum_{j=1}^{s} \left(\left(W_{\mu_{ij}} - IIFPIS_{\mu_{ij}}^+\right)^2 + \left(W_{\nu_{ij}} - IIFPIS_{\nu_{ij}}^+\right)^2 \right)}$$

$$IIFD_i^- = \sqrt{\sum_{j=1}^{s} \left(\left(W_{\mu_{ij}} - IIFNIS_{\mu_{ij}}^-\right)^2 + \left(W_{\nu_{ij}} - IIFNIS_{\nu_{ij}}^-\right)^2 \right)}, \ i = 1,2,...,r.$$

Definition 3.8. The relative closeness co-efficient denoted by $IIFR_i$ is defined as

$$IIFR_i = \frac{IIFD_i^-}{\left(IIFD_i^+ + IIFD_i^-\right)}, \ i = 1,2,...,r.$$

3.1 Procedure

The procedure to solve the decision making problem using TOPSIS method is given below.

Step:1 Construct the IIF decision matrix $IIFM_{ij}$ from the available data.

Step:2 Estimate the energy value of $IIFM_{ij}$.

Step:3 Construct the IIF normalized decision matrix $IIFNDM_{ij}$ using *Definition 3.3*.

Step:4 Compute the weights w_i corresponding to the parameters e_i, by using *Definition 3.4.*

Step:5 Calculate the IIF weighted normalized decision matrix using *Definition 3.5.*

Step:6 Determine the inverse intuitionistic fuzzy PIS $IIFPIS_j^+$ and NIS $IIFNIS_j^-$ by using *Definition 3.6.*

Step:7 Calculate the distance $IIFD_i^+$ and $IIFD_i^-$ using *Definition 3.7.*

Step:8 Compute the relative closeness coefficient $IIFR_i$ by using *Definition 3.8.*

Step:9 According to the relative closeness coefficient, rank the order of IIF. The alternative with the highest ranking score is chosen as the best alternative.

4 Application

The problem of finding the best choice of power transmission is considered. Alternatives $S = \{I_1, I_2\ I_3, I_4\}$ denote four transmission tower locations. Let the vertices of IIFG represent the transmission tower locations. The membership and non-membership value of vertices represents the transmission strength and transmission range. The edge between two vertices denotes spectrum analyzer, in which the membership value of edge represents the electrical power and non-membership value denotes electromagnetic interferences. The signal strength at each level is compared and energy level is found. Using TOPSIS method, the alternatives are ranked and the best tower location is found.

Step 1. IIF decision matrix $IIFM_{ij}$ is as follows:

$$\begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{array} \begin{array}{c} \begin{array}{cccc} I_1 & I_2 & I_3 & I_4 \end{array} \\ \begin{pmatrix} (0,0) & (0.3,0.7) & (0,0) & (0.4,0.6) \\ (0.3,0.7) & (0,0) & (0.3,0.7) & (0,0) \\ (0,0) & (0.3,0.7) & (0,0) & (0.5,0.5) \\ (0.4,0.6) & (0,0) & (0.5,0.5) & (0,0) \end{pmatrix} \end{array}$$

Step 2. The energy of $IIFM_{ij}$ is $E(I_G) = (1.9644, 2.6311)$.

Step 3. IIF normalized decision matrix $IIFNDM_{ij}$ is:

$$\begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{array} \begin{array}{c} \begin{array}{cccc} I_1 & I_2 & I_3 & I_4 \end{array} \\ \begin{pmatrix} (0,0) & (0.13,0.25) & (0,0) & (0.18,0.21) \\ (0.13,0.26) & (0,0) & (0.13,0.26) & (0,0) \\ (0,0) & (0.14,0.24) & (0,0) & (0.23,0.17) \\ (0.19,0.2) & (0,0) & (0.23,0.17) & (0,0) \end{pmatrix} \end{array}$$

Step 4. The weight function values w_i, corresponding to the parameters e_j are:

$w_1 = 0.25,\ w_2 = 0.23,\ w_1 = 0.25,\ w_2 = 0.27.$

Step 5. IIF weighted normalized decision matrix is as follows:

$$\begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{array} \begin{array}{c} \begin{array}{cccc} I_1 & I_2 & I_3 & I_4 \end{array} \\ \begin{pmatrix} (0,0) & (0.03,0.06) & (0,0) & (0.05,0.05) \\ (0.03,0.06) & (0,0) & (0.03,0.06) & (0,0) \\ (0,0) & (0.04,0.06) & (0,0) & (0.06,0.04) \\ (0.05,0.05) & (0,0) & (0.06,0.05) & (0,0) \end{pmatrix} \end{array}$$

Step 6. IIF PIS, IIF NIS.

For the parameter I_1:
$IIFPIS_1^+ = (0.05.0)$
$IIFNIS_1^- = (0, 0.06)$

For the parameter I_2:
$IIFPIS_2^+ = (0.04, 0)$
$IIFNIS_2^- = (0, 0.06)$

For the parameter I_3:
$IIFPIS_3^+ = (0.06, 0)$
$IIFNIS_3^- = (0, 0.05)$

For the parameter I_4:
$IIFPIS_4^- = (0.06, 0)$
$IIFNIS_4^- = (0, 0.05)$

Step 7. Distances $IIFD_i^+$, $IIFD_i^+$ are tabulated as follows:

S	$IIFD_i^+$	$IIFD_i^-$
I_1	0.11136	0.09747
I_2	0.10099	0.08944
I_3	0.10630	0.10677
I_4	0.11747	0.11091

Step 8. The relative closeness coefficient of IIF is defined below

$IIFR_i = \frac{IIFD_i^-}{\left(IIFD_i^+ + IIFD_i^-\right)}$
0.20883
0.19043
0.21307
0.22838

Step 9. Ranking the alternatives based on their relative closeness coefficient values $I_4 > I_3 > I_1 > I_2$. Hence I_4 is the best transmission tower location.

5 Conclusion:

In this article, TOPSIS method on IIFG of Z_4 matrix is developed. IIF normalized, weighted normalized, PIS and NIS on IIFG matrix are calculated. Distances between ideal solutions are calculated and alternatives are ranked. It is found that I_4 is the best alternative. i.e, Taking four transmission tower locations and rank the locations using TOPSIS method and found the best tower transmission location.

It can help a lot in finding a deep insight and to make decisions in complex situations as well. This approach can also be used for finite abelian group problems in decision making scenarios.

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