BINARY S_α CONTINUOUS,IRRESOLUTE ANDSTRONGLY $_bS_\alpha$ CONTINUOUS FUNCTION INBINARY TOPOLOGICAL SPACE.

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Abstract: In this paper we introduce the new class of functions called binary semi α ($_bS_\alpha$) continuous, binary semi α ($_bS_\alpha$) irresolute function and strongly binary semi α ($_bS_\alpha$) continuous functions. Further its properties are studied using suitable examples.

Keywords and phrases Binary S_α ($_bS_\alpha$) continuous function,Binary S_α ($_bS_\alpha$) open mapping,Binary S_α ($_bS_\alpha$) closed mapping

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1 Introduction

In 2000, G.B.Navalagi[4], proposed the idea of new set called semi-α open sets in topological spaces. S.Nithyanantha Jothi and P.Thangavelu[5] in 2011 introduced topology between two sets which was named as binary topological space in which they investigate some of the basic properties, where a binary topology from X to

Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology and In 2022, J.Elekiah and G.Sindhu[2] introduced binary semi α open sets in binary topological spaces and studied its relationship with other existing sets. And the concept of continuous function in a Binary topological space was studied by S.Nithyanantha Jothi and P.Thangavelu[] in 2011.

In this paper the main purpose is to study the concept of binary S_α Continuous and Irresolute function in a binary topological space and also its properties are discussed.

2 Preliminaries

Definition 2.1. *A subset* (A, B) *of a binary topological space* (X, Y, M) *is called*

(i) binary α open [5]if $(A, B) \subseteq b\text{-}int(b\text{-}cl(b\text{-}int(A, B)))$.

(ii) binary semi open set [4] if $(A, B) \subseteq b\text{-}int(b\text{-}cl(A, B))$.

Definition 2.2 (3)**.** *In a topological space* (X, τ)*, the subset A of X is said to be semi-α-open if there exists a α-open set U in X such that* $U \subseteq A \subseteq cl(U)$*. The family of all semi-α-open sets of X is denoted by* $S_\alpha(X)$.

Definition 2.3 (1)**.** *Let* (X, Y, M) *be a binary topological space and* $(A, B) \subseteq (X, Y)$*. The subset (A,B) is said to be binary semi α -open (*$_bS_\alpha$*)if there exists an binary α-open set (U,V) in X such that* $(U, V) \subseteq (A, B) \subseteq cl(U, V)$.

Definition 2.4 (6)**.** *Let* (Z, τ) *be a topological space and* (X, Y, M) *be a binary topological space. then the map* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *is called binary semi continuous if* $f^{-1}(A, B)$ *is semi open in Z for every binary open set* (A, B) *in* (X, Y, M)

Definition 2.5 (7)**.** *Let* (Z, τ) *be a topological space and* (X, Y, M) *be a binary topological space. then the map* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *then is called binary continuous if* $f^{-1}(A, B)$ *is open in* (Z, τ) *for every binary open set* (A, B) *in* (X, Y, M)

Definition 2.6 (2). *let* (X, Y, M) *be a binary topological space and let* (Z, τ) *be a topological space and let* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *be a function,then* f *is said to be binary* α ($_b\alpha$) *continuous function if* $f^{-1}(A, B)$ *is a* α *open set in* (Z, τ) *for every binary open set* (A, B) *in* (X, Y, M)

Proposition 2.7. *(i) binary* α *open [5]if* $(A, B) \subseteq b\text{-}int(b\text{-}cl(b\text{-}int(A, B)))$.

(ii) binary semi open set [4] if $(A, B) \subseteq b\text{-}int(b\text{-}cl(A, B))$.

3 Binary S_α continuous function

Definition 3.1. *let* (X, Y, M) *be a binary topological space and let* (Z, τ) *be a topological space and let* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *be a function,then* f *is said to be binary* S_α ($_bS_\alpha$) *continuous function if* $f^{-1}(A, B)$ *is a* S_α *open set in* (Z, τ) *for every binary open set* (A, B) *in* (X, Y, M)

Definition 3.2. *A mapping* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *is said to be a binary* S_α ($_bS_\alpha$) *open mapping if the image of each open set in* (Z, τ) *is a binary* S_α *open set in* (X, Y, M)

Definition 3.3. *A mapping* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *is said to be a binary* S_α ($_bS_\alpha$) *closed mapping if the image of each closed set in* (Z, τ) *is a binary* S_α *closed set in*(X, Y, M)

Example 3.4. $Z = \{a, b, c\}$; $\tau = \{\phi, Z, \{a, b\}, \{b, c\}, \{b\}\}$; $X = \{x_1, x_2\}$; $Y = \{y_1, y_2\}$; $\mathrm{M} =$
$\{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_2\}), (\{x_2\}, \{y_1\})\}$ *we define a function* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *by* $f(a) = (x_1, \emptyset), f(b) = (x_2, y_2), f(c) = (\emptyset, y_1)$ *then* f *is a binary* S_α *continuous mapping*

Example 3.5. $Z = \{a, b, c, d\}; \tau = \{\phi, Z, \{a, b\}, \{b, c\}, \{b\}, \{c, d\}, \{c\}, \{a, b, c\}\}; X =$ $\{x_1, x_2, x_3\}; Y = \{y_1, y_2, y_3\}; \mathrm{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1, x_2\}, \{y_1\}), (\{x_3\}, \{y_2\})\}$ *we define a function* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *by* $f(a) = (x_1, y_3), f(b) = (x_2, \emptyset), f(c) = (x_3, y_1), f(d) = (\emptyset, y_2)$ *then* f *is a binary* S_α *continuous mapping*

Theorem 3.6. *let* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *be a mapping then the following statements are equivalent*

- *(i)* f *is binary* S_α *continuous*
- *(ii)* *for each* $x \in Z$ *and each open set* $(A, B) \subseteq (X, Y, \mathrm{M})$ *containing* $f(x)$*, there exists* $W \in \alpha(X)$ *such that* $x \in W$, $f(W) \subseteq (A, B)$
- *(iii)* *the inverse image of each binary closed set in* (X, Y, M) *is* S_α *closed in* (Z, τ)

Proof. $(i) \Longrightarrow (ii)$
let $f : (Z, \tau) \to (X, Y, \mathrm{M})$ be a binary S_α continuous mapping then for every binary open set (A, B) in (X, Y, M) there exist a S_α open set W in (Z, τ) such that $f^{-1}(A, B) = x \Longrightarrow x \in W \Longrightarrow f(x) \subseteq f(W) \subseteq (A, B)$ where $W \in \alpha(X)$

$(ii) \Longrightarrow (iii)$
let $f : (Z, \tau) \to (X, Y, \mathrm{M})$ be a function. let $x \in Z$ and $(A, B) \subseteq (X, Y, \mathrm{M})$ containing $f(x)$ and $W \in \alpha(X)$ such that $x \in W, f(W) \subseteq (A, B)$.

$\Longrightarrow f(x) \subseteq (A, B)$

$\Longrightarrow x \in f^{-1}(A, B)$,where $x \in W$ and W is S_α open set

$\Longrightarrow$ *f is a* S_α continuous map.

since f is S_α continuous map, the inverse image of each binary closed set in (X, Y, M) is S_α closed in (Z, τ)

$(iii) \Longrightarrow (i)$
let the inverse image of each binary closed set in (X, Y, M) is S_α closed in (Z, τ)then it is obvious that f is binary S_α continuous function. □

Theorem 3.7. *Every* $_bS_\alpha$ *continuous mapping* $f : (Z, \tau) \to (X, Y, M)$ *is a binary semi continuous mapping.*

Proof. Let $f : (Z, \tau) \to (X, Y, M)$ be a binary S_α continuous mapping then by definition every inverse image of binary open set (A, B) in (X, Y, M) is a S_α openset in (Z, τ). since every S_α open set is a semi open set. it implies that every inverse image of binary open set is a semi open set in (Z, τ), which implies that the function f is a binary semi continuous mapping. □

The following example shows that the converse of the above theorem need not be true

Example 3.8. $Z = \{1, 2, 3, 4\}; \tau = \{\phi, Z, \{1\}, \{2, 3\}, \{1, , 2, 3\}\}; X = \{x_1, x_2, x_3\}; Y = \{y_1, y_2, y_3\}; M = \{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_2, y_3\}), (\{x_2\}, \{y_1\}), (\{x_1, x_2\}, Y)\}$ *we define a function* $f : (Z, \tau) \to (X, Y, M)$ *by* $f(1) = (x_3, y_3), f(2) = (x_1, \emptyset), f(3) = (\emptyset, y_2), f(4) = (x_2, y_1)$ *then* f *is a binary Semi continuous but not* $_bS_\alpha$ *continuous.*

Theorem 3.9. *Every* $_b\alpha$ *continuous mapping* $f : (Z, \tau) \to (X, Y, M)$ *is a binary* S_α *continuous mapping*

Proof. Let $f : (Z, \tau) \to (X, Y, M)$ be a binary α continuous mapping then by definition every inverse image of binary open set (A, B) in (X, Y, M) is a α open set in (Z, τ). since every α open set is a S_α open set. it implies that every inverse image of binary open set is a S_α open set in (Z, τ), which implies that the function f is a binary S_α continuous mapping. □

The following example shows that the converse of the above theorem need not be true.

Example 3.10. $Z = \{1, 2, 3, 4\}; \tau = \{\phi, Z, \{1\}, \{2, 3\}, \{1, , 2, 3\}\}; X = \{x_1, x_2, x_3\}; Y = \{y_1, y_2, y_3\}; M = \{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_2, y_3\}), (\{x_2\}, \{y_1\}), (\{x_1, x_2\}, Y),$

$(\{x_2\}, \{y_1, y_2\}), (\{x_1, x_2\}, \{y_1\}), (\{x_1, x_2\}, \{y_1, y_2\}), (\emptyset, \{y_2\}), (\{x_1\}, \emptyset), (\{x_1\}, \{y_2\})\}$ *we define a function* $f : (Z, \tau) \rightarrow (X, Y, \mathrm{M})$ *by* $f(1) = (x_3, y_3), f(2) = (x_1, \emptyset), f(3) = (\emptyset, y_2), f(4) = (x_2, y_1)$ *then* f *is a* ${}_bS_\alpha$ *continuous but not* ${}_b\alpha$ *continuous mapping.*

Theorem 3.11. *Let* $f : (Z, \tau) \rightarrow (X, Y, \mathrm{M})$ *be a mapping where* (Z, τ) *is a general topological space and* (X, Y, M) *is a binary topological space. Then* f *is said to be* ${}_bS_\alpha$ *continuous if and only if for* $f(w) \in (A, B)$*, there exist an* $Q \in S_\alpha O$ *in* (Z, τ)*such that* $w \in Q$ *and* $f(Q) \subseteq (A, B)$*.*

Proof. Necessity:

Let $f(w) \in (A, B)$ then $w \in f^{-1}(A, B)$ since f is a ${}_bS_\alpha$ continuous map, $w \in f^{-1}(A, B) \in SO$. let $Q = f^{-1}(A, B)$. then $w \in Q$ and $f(Q) \subseteq (A, B)$.

sufficiency:

Let (A, B) be a binary open set in (X, Y, M) and $f(w) \in (A, B)$. where Q is a $S_\alpha O$ in (Z, τ) such that $w \in Q$

$\Longrightarrow$ $w \in f^{-1}(A, B), w \in Q, Q$ is a $S_\alpha O$ set

$\Longrightarrow$ $f^{-1}(A, B) \subseteq Q$ this implies that f is a ${}_bS_\alpha$ continuous map. □

Theorem 3.12. *Let* $f : (Z, \tau) \rightarrow (X, Y, \mathrm{M})$ *be a* ${}_bS_\alpha$ *continuous mapping and let* $q_1 : (Z, \tau) \rightarrow (X, \sigma)$ *and* $q_2 : (Z, \tau) \rightarrow (Y, \sigma)$ *where* $f(W) = (A, B)$ *and* $q_1(w) = A$ *and* $q_2(w) = B$ *then both* q_1, q_2 *are* S_α *continuous mapping.*

Proof. We shall show only that $q_1 : (Z, \tau) \rightarrow (X, \sigma)$ is S_α continuous mapping. let A be a open set in (X, σ). then (A, B) is a binary open set in (X, Y, M). since f is a ${}_bS_\alpha$ continuous mapping $f^{-1}(A, B)$ is a S_α open set. but $q_1^{-1}(A) = f^{-1}(A, B)$ hence $q_1^{-1}(A)$ is a $S_\alpha O$ set.

hence q_1 is a S_α continuous mapping. similarly q_2 is also a S_α continuous mapping.

□

Theorem 3.13. *The two functions* $_bS_\alpha$ *continuous mapping and* $_bS_\alpha$ *open mapping are independent of each other.*

Proof. Let $f : (Z, \tau) \to (X, Y, \mathrm{M})$ be a $_bS_\alpha$ continuous mapping,then the inverse image of a binary open set is a S_α open set in (Z, τ). w.k.t every open set is a S_α open set but every S_α open set need not be a open set. hence the function doesn't satisfy the definition of a $_bS_\alpha$ open mapping,similarly if a function is $_bS_\alpha$ open mapping then it can't be a $_bS_\alpha$ continuous mapping. □

Theorem 3.14. *A mapping* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *is* $_bS_\alpha$ *continuous if and only if the inverse image of every binary closed set in* (X, Y, M) *is* S_α *closed set in* (Z, τ).

Proof. Assume that f is $_bS_\alpha$ continuous and let (A, B) be any binary closed set in (X, Y, M) then $(A, B)^c$ is a binary open set then by definition $f^{-1}\{(A, B)^c\}$ is a S_α open set. But $f^{-1}\{(A, B)^c\} = Z - f^{-1}(A, B)$. then $f^{-1}(A, B)$ is a S_α closed set in (Z, τ).

Conversely,

Assume that the inverse image of every binary closed set is S_α closed set in (Z, τ).Let (A, B) be any binary open set in (X, Y, M) then $(A, B)^c$ is a binary closed set. and by our assumption $f^{-1}\{(A, B)^c\}$ is a S_α closed set.

$\Longrightarrow X - f^{-1}(A, B)$ is a S_α closed set

$\Longrightarrow f^{-1}(A, B)$ is a S_α open set

$\Longrightarrow f$ is a $_bS_\alpha$ Continuous mapping. □

4 Binary S_α Irresolute function

Definition 4.1. *let* (X, Y, M) *be a binary topological space and let* (Z, τ) *be a topo-logical space and let* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *be a function,then* f *is said to be binary* S_α ($_bS_\alpha$) *Irresolute function if* $f^{-1}(A, B)$ *is a* S_α *open set in* (Z, τ) *for every binary* S_α *open set* (A, B) *in* (X, Y, M)

Example 4.2. $Z = \{a, b, c, d\}; \tau = \{\phi, Z, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}; X = \{x_1, x_2, x_3\}; Y = \{y_1, y_2\}; \mathrm{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1, x_2\}, \{y_2\}),$
$(\{x_3\}, \{y_1\}), (\{x_2\}, \{y_1\}), (\{x_1, x_2\}, \{Y\}), (\{X\}, \{y_2\}), (\{x_3\}, \emptyset), (\{x_2\}, \emptyset)\}$ *we define*
a function $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *by* $f(a) = (x_1, \emptyset), f(b) = (x_2, y_2), f(c) = (\emptyset, y_1), f(d) = (x_3, y_2)$ *then f is a binary* S_α *Irresolute mapping*

Theorem 4.3. *Every* ${}_bS_\alpha$ *continuous mapping is* ${}_bS_\alpha$ *Irresolute mapping.*

Proof. Let $f : (Z, \tau) \to (X, Y, \mathrm{M})$ be a ${}_bS_\alpha$ continuous mapping,then by definition $f^{-1}(A, B)$ is a S_α open set in (Z, τ) for every binary open set (A, B) in (X, Y, M).Since every binary open set is a ${}_bS_\alpha$ open set, every (A, B) is a ${}_bS_\alpha$ open set $\Longrightarrow$ f is a ${}_bS_\alpha$ Irresolute mapping. □

The following example shows that the converse of the above theorem need not be true.

Example 4.4. $Z = \{a, b, c, d\}; \tau = \{\phi, Z, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}; X = \{x_1, x_2, x_3\}; Y = \{y_1, y_2\}; \mathrm{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1, x_2\}, \{y_2\}),$
$(\{x_3\}, \{y_1\}), (\{x_2\}, \{y_1\}), (\{x_1, x_2\}, \{Y\}), (\{X\}, \{y_2\}), (\{x_3\}, \emptyset), (\{x_2\}, \emptyset)\}$ *we define*
a function $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *by* $f(a) = (x_1, \emptyset), f(b) = (x_2, y_2), f(c) = (\emptyset, y_1), f(d) = (x_3, y_2)$ *this f is a binary* S_α *Irresolute bot not a binary* S_α *continuous function.*

Theorem 4.5. *A map* $f : (Z, \tau) \to (X, Y, \mathrm{M})$ *is* ${}_bS_\alpha$ *Irresolute if and only if theinverse image of every* ${}_bS_\alpha$ *closed in* (X, Y, M) *is* S_α *closed in* (Z, τ).

Proof. Assume that f is ${}_bS_\alpha$ irresolute and let (A, B) be any ${}_bS_\alpha$ closed set in (X, Y, M) then $(A, B)^c$ is a ${}_bS_\alpha$ open set then by definition $f^{-1}\{(A, B)^c\}$ is a S_α open set. But $f^{-1}\{(A, B)^c\} = Z - f^{-1}(A, B)$. then $f^{-1}(A, B)$ is a S_α closed set in (Z, τ).

Conversely,

Assume that the inverse image of every ${}_bS_\alpha$ closed set is S_α closed set in (Z, τ).Let (A, B) be any ${}_bS_\alpha$ open set in (X, Y, M) then $(A, B)^c$ is a ${}_bS_\alpha$ closed set. and by our assumption $f^{-1}\{(A, B)^c\}$ is a S_α closed set.
$\Longrightarrow X - f^{-1}(A, B)$ is a S_α closed set

$\Longrightarrow f^{-1}(A, B)$ is a S_α open set
$\Longrightarrow f$ is a ${}_bS_\alpha$ Irresolute mapping. □

5 Strongly ${}_bS_\alpha$ Continuous.

Definition 5.1. *A mapping $f : (Z, \tau) \to (X, Y, \mathrm{M})$ be a Strongly ${}_bS_\alpha$ continuous mapping if $f^{-1}(A, B)$ is a open set in (Z, τ) for every ${}_bS_\alpha$ open set (A, B) in(X, Y, M)*

Theorem 5.2. *If a map $f : (Z, \tau) \to (X, Y, \mathrm{M})$ is a Strongly ${}_bS_\alpha$ continuous map then it is binary continuous.*

Proof. Assume that $f : (Z, \tau) \to (X, Y, \mathrm{M})$ is Strongly ${}_bS_\alpha$ continuous map. Let (A, B) be any binary open set in (X, Y, M). Since every binary open set is a ${}_bS_\alpha$ open set, (A, B) is a ${}_bS_\alpha$ open set and since f is Strongly ${}_bS_\alpha$ continuous. $f^{-1}(A, B)$ is a open set in (Z, τ)
$\Longrightarrow f$ is a binary continuous map. □

The following example shows that the converse of the above theorem is not true.

Example 5.3. $Z = \{a, b, c\}; \tau = \{\phi, Z, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}; X = \{x_1, x_2, x_3\}; Y =$
$\{y_1, y_2, y_3\}; \mathrm{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1, x_3\}, \{y_1, y_2\}), (\{x_1\}, \{y_3\}), (\{x_1, x_3\}, Y),$
$(\{x_1\}, \emptyset)\}$ *we define a function $f : (Z, \tau) \to (X, Y, \mathrm{M})$ by $f(a) = x_1 = y_3, f(b) = x_2 = y_2$, $f(c) = x_3 = y_1$ this f is a binary continuous bot not a Strongly ${}_bS_\alpha$ continuous*

Theorem 5.4. *If a mapping $f : (Z, \tau) \to (X, Y, \mathrm{M})$ is Strongly binary continuous then it is Strongly ${}_bS_\alpha$ continuous.*

Proof. Assume that f is strongly binary continuous. Let (C, D) be any binary S_α open set in (X, Y, M). since f is strongly binary continuous, $f^{-1}(C, D)$ is both open and closed set in (Z, τ). by the definition of strongly ${}_bS_\alpha$ continuous f is strongly ${}_bS_\alpha$ continuous. □

the following example shows that the converse of the above theorem is not true.

Example 5.5. $Z = \{a, b, c, d\}; \tau = \{\phi, Z, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}; X =$
$\{x_1, x_2, x_3\}; Y = \{y_1, y_2, y_3\}; M = \{(\emptyset, \emptyset), (X, Y), (\{x_1, x_3\}, \{y_1, y_2\}), (\{x_1\}, \{y_3\}),$ $(\{x_1, x_3\}, Y), (\{x_1\}, \emptyset)\}$ *we define a function* $f : (Z, \tau) \to (X, Y, M)$ *by* $f(a) = x_1 = y_1, f(b) = x_3 = y_2, f(c) = \emptyset = \emptyset, f(d) = x_2 = y_3$ *this* f *is a Strongly* ${}_bS_\alpha$ *continuous bot not strongly binary continuous.*

Theorem 5.6. *A mapping* $f : (Z, \tau) \to (X, Y, M)$ *is Strongly binary* ${}_bS_\alpha$ *continuous iff* $f^{-1}(A_1, B_1)$ *is a closed set in* (Z, τ) *for every* ${}_bS_\alpha$ *closed set* (A_1, B_1) *in* (X, Y, M)

Proof. Assume that f is a strongly ${}_bS_\alpha$ continuous mapping. Let (A_1, B_1) be a closed set in (X, Y, M). then $(A_1, B_1)^c$ is a open set. which implies $f^{-1}(A_1, B_1)^c$ isa open set in (Z, τ).
$\Longrightarrow f^{-1}(A_1, B_1)^c = X - f^{-1}(A_1, B_1)$
$\Longrightarrow f^{-1}(A_1, B_1)$ is a closed set in $(Z, \tau$
).conversely,
Assume that $f^{-1}(A_1, B_1)$ is a closed set for every ${}_bS_\alpha$ closed set (A_1, B_1) in (X, Y, M).Let

(A, B) be a binary open set in (X, Y, M), since every ${}_bOS$ is ${}_bS_\alpha$ open set, (A, B) is ${}_bS_\alpha OS$
$\Longrightarrow (A, B)^c$ is a binary closed set in (X, Y, M)
$\Longrightarrow f^{-1}(A, B)^c$ is a closed set in (Z, τ)
$\Longrightarrow f^{-1}(A, B)$ is a open set in (Z, τ)
$\Longrightarrow f$ is strongly ${}_bS_\alpha$ continuous □

Strongly Binary Continuous — Strongly ${}_bS_\alpha$ Continuous — Binary Continuous

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