BIANCHI TYPE IX INFLATIONARY UNIVERSE IN GENERAL RELATIVITY

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Abstract:
Having looked into Inflationary Bianchi Type IX Space-time with Flat Potential in General Relativity. We use the condition that the metric potentials are related by $b = a^n$ where n is constant, in order to produce a determinate mode. Also, the various geometrical and physical features of the model viz proper volume, expansion coefficient, shear scalar, Hubble parameter are explored.

Keywords: General relativity, Bianchi Type IX, Inflationary universes

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DOI: 10.48047/ecb/2023.12.si10.00486

1. Introduction

Analysing the Bianchi type IX universe is of great interest to many relativist for the reason that well-known solutions like the Robertson-Walker [1] universe, the de-Sitter [2] universe, the Taub-NUT[3] solutions, etc. are of Bianchi type IX space-times. Closed FRW models are included in the Bianchi type IX cosmological models. These models are generally anisotropic and permit rotation, shear, and expansion in addition to expansion. The FRW cosmological models, commonly used to describe the universe, are a specific instance of Bianchi types I, V, and IX space-times. These models assume a homogeneous and isotropic universe where the curvature of the physical three-space at t=0 is constant, either negative, positive or zero. The high radiation entropy and degree of isotropy of the cosmic background radiation are explained by neutrino viscosity in these models.

The concept of an initial inflationary period was keep putting up by Guth [4] under the framework of grand unification theories. Inflationary universes can help to partially explain a number of unresolved cosmological problems, such are the flatness, homogeneity, and isotropy of the observed universe As studied by Linde[5], Abbott and Wisethere[6],Abrecht and Steinhardt[7],Mataresse and Luechinare [8]various inflationary scenarios. With the use of the Higgs field model and potential V(φ), it can be shown that inflation will occur if V(φ) has a flat region and the field changes slowly, while the universe grows exponentially because of the energy in the vacuum field [9]. For the universe to achieve homogeneity and isotropy on a scale comparable to the size of the observable horizon, it is postulated that the scalar field must undergo a significant duration of traversal through the flat region.

Bali and Dave [10] investigated a cosmological model in General Relativity, specifically focusing on the Bianchi type IX string model. Tyagi and Sharma [11] examined Bianchi IX string cosmological model for perfect fluid distribution in General Relativity. Bali and Kumawat [12] investigated a cosmological model characterized by a tilted stiff fluid in Bianchi type IX geometry, taking into account bulk viscosity. Bali and Poonia [13] and Poonia and Sharma [14] and Sharma et.al [15] examined Inflationary Scenario in Bianchi Type IX in different context.

In this current study, we have examined homogeneous anisotropic Bianchi type IX space-time cosmological model for inflationary scenario. We use the condition $b = a^n$ between metric potentials, where n is constant, in order to produce a determinate mode Also, the some geometrical & physical features of the model are explored.

2. Field and Metric Equation

The metric for Bianchi type IX space time is taken as

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + (b^2sin^2y + a^2cos^2y)dz^2 - 2a^2 \cos y \, dxdz \quad (1)$$

Where a & b depend on cosmic time t

The effective potential V ($\boldsymbol{\phi}$) is used, the Lagrangian is that of gravity that is minimum coupled to is given by Stein-Schabes [16]

$$L = \int \sqrt{-g} \left(R - V(\varphi) - \frac{1}{2} g_{ij} \partial_i \varphi \partial^j \varphi \right) d^4x \quad (2)$$

We can get Einstein Field equations from the variation of L concerning the massless scalar fields, as

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij} \quad (3)$$

We can assume $8\pi G = C = 1$

The energy-momentum tensors, which are given as

$$T_{ij} = \partial_i \varphi \partial_j \varphi - \left[\frac{1}{2} \partial_p \varphi \partial^p \varphi + V(\varphi) \right] g_{ij} \quad (4)$$

$$\frac{1}{\sqrt{-g}} \left[\sqrt{-g} \varphi_{,i} \right]_{,i} = -\frac{dV(\varphi)}{d\varphi} \quad (5)$$

Where v_i (0, 0, 0, 1) is the flow vector and g_{ij} metric tensor and

$$\varphi_{,i} = \frac{\partial \varphi}{\partial x^i} \text{ And } \partial^p \varphi = g^{pl} \frac{\partial \varphi}{\partial x^l} \quad (6)$$

Now using equations (4) with metric (1), Einstein field equations (3) reduced to

$$\frac{b_4^2}{b^2}+2\frac{b_{44}}{b}+\frac{1}{b^2}-\frac{3a^2}{4b^4}=-\frac{1}{2}\varphi_4^2+V(\varphi) \quad (7)$$

$$\frac{a_{44}}{a}+\frac{b_{44}}{b}+\frac{a_4b_4}{ab}+\frac{a^2}{4b^2}=-\frac{1}{2}\varphi_4^2+V(\varphi) \quad (8)$$

$$\frac{b_4^2}{b^2}+2\frac{a_4b_4}{ab}+\frac{1}{b^2}-\frac{a^2}{4b^4}=\frac{1}{2}\varphi_4^2+V(\varphi) \quad (9)$$

From equation (5)

$$\varphi_{44}+\left[\frac{a_4}{a}+2\frac{b_4}{b}\right]\varphi_4=-\frac{dV}{d\varphi} \quad (10)$$

The physical terms, proper volume (V) is derived by

$$R^3=V=ab^2 \quad (11)$$

Expansion coefficient (θ) is derived by

$$\theta=\frac{a_4}{a}+2\frac{b_4}{b} \quad (12)$$

Shear scalar (σ) is derived by

$$\sigma^2=\frac{1}{3}\left[\frac{a_4}{a}-\frac{b_4}{b}\right] \quad (13)$$

Hubble factor (H) is derived by

$$H=\frac{1}{3}\left[\frac{a_4}{a}+2\frac{b_4}{b}\right] \quad (14)$$

3.The solution of Field Equations

For the purpose of achieving relevant outcomes, we assume V (φ) Is Constant
As a result, the conservation relation

$$\varphi_{44}+\left(\frac{a_4}{a}+2\frac{b_4}{b}\right)\varphi_4=0 \quad (15)$$

$$\varphi_4(ab^2)=L \quad (16)$$

L is a constant
By resolving non-linear differential equations, we are looking for an inflationary solution, from equation (7)-(8), we get

$$\frac{b_{44}}{b}-\frac{a_{44}}{a}+\frac{b_4^2}{b^2}-\frac{a_4b_4}{ab}+\frac{1}{b^2}-\frac{a^2}{b^4}=0 \quad (17)$$

Assuming that the metric coefficients have the following relationship

$b=a^n$ Where n≠1 (18)

Where n is a constant. Hence, using (18), one can get the differential equation for b from Eq. (17) as

$$\frac{2a_{44}}{a}+4n\frac{a_{44}^2}{a^2}=\frac{2}{(1-n)a^{2n}}-\frac{2a^{-4n+2}}{(1-n)} \quad (19)$$

$$2a_{44}+4n\frac{a_{44}^2}{a}=\frac{2}{1-n}\left[\frac{1}{a^{2n-1}}-\frac{1}{a^{4n-3}}\right] \quad (20)$$

Put $a=f(a)$ which gives $a_{44}=ff'$ where $f'=\frac{df}{dt}$ equation (21) gives

$$\frac{df^2}{dt}+4n\frac{f^2}{a}=\frac{2}{(1-n)}\left[\frac{1}{a^{2n-1}}-\frac{1}{a^{4n-3}}\right] \quad (21)$$

On solving, it gives

$$f^2=\frac{a^{2(1-n)}}{(1-n^2)}+\frac{a^{4(1-n)}}{2(1-n)}+Da^{-4n} \quad (22)$$

D is a constant

This gives to

$$\int \left[\frac{a^{2(1-n)}}{(1-n^2)} + \frac{a^{4(1-n)}}{2(n-1)} + Da^{-4n}\right]^{\frac{1}{2}} da = t + t_0 \qquad (23)$$

t_0 Is a constant and $\frac{1}{(1-n^2)} = k$ and $\frac{1}{2(1-n)} = k_1$

Using these values equation (26) gives to

$$\int \left[ka^{2(1-n)} + k_1 a^{4(1-n)} + Da^{-4n}\right]^{\frac{1}{2}} da = t + t_0 \quad (24)$$

The space-time metric (1) becomes to

$$ds^2 = -\left(kT^{2(1-n)} + k_1 T^{4(1-n)} + DT^{-4n}\right)^{\frac{1}{2}} dT^2 + T^2 dX^2 + T^{2n} dY^2 + (T^{2n} sin^2 y + T^2 cos^2 y) dZ^2 - 2T^2 \cos Y \, dX dZ \qquad (25)$$

Where transformation is $a = T, x = X,\ y = Y,\ z = Z$ (26)

4. PHYSICAL FEATURES OF THE MODEL

The Higg's Field is

$$\varphi_4 = \frac{L}{ab^2} = \frac{L}{T^{2n+1}} \quad (27)$$

$$\varphi = \int \frac{L}{T^{2n+1}} dT + c \qquad (28)$$

The spatial volume is provided by

$$V = ab^2 = T^{2n+1} \qquad (29)$$

The scalar of expansion (θ) is provided by

$$\theta = (2n+1)\left[\frac{k}{T^{2n}} + \frac{k_1}{T^{2(2n-1)}} + \frac{D}{T^{2(2n+1)}}\right]^{\frac{1}{2}} \qquad (30)$$

The shear scalar (σ) is given by

$$\sigma = \left(\frac{n-1}{\sqrt{2}}\right)\left[\frac{k}{T^{2n}} + \frac{k_1}{T^{2(2n-1)}} + \frac{D}{T^{2(2n+1)}}\right]^{\frac{1}{2}} \qquad (31)$$

$$\frac{\sigma}{\theta} = constant \quad (32)$$

The Hubble factor is given by

$$H = \left(\frac{2n+1}{3}\right)\left[\frac{k}{T^{2n}} + \frac{k_1}{T^{2(2n-1)}} + \frac{D}{T^{2(2n+1)}}\right]^{\frac{1}{2}} \qquad (33)$$

5. Conclusion

In this paper, we investigated the expansion of the universe on a cosmic scale while considering the influence of a massless scalar field with a flat potential. Due to the vacuum field energy, the Higgs field changes slowly over time as the cosmos expands. The proper volume rises as time increases is provided inflationary scenario. The shear scalar σ to expansion θ ratio tends to finite value i.e. $\frac{\sigma}{\theta}$ = constant. The Hubble parameter (H) is Large at initial stage but become finite at late T As a result, the model does not tend towards isotropy when T reaches large values. When T = 0, the big bang initiates the model's expansion.

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