TRIBONACCI GRACEFUL DECOMPOSITION OF DIAMOND RELATED GRAPHS

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Abstract
Let G be a graph with p vertices and q edges.A decomposition of G is a collection $\psi_{tg} = \{H_1, H_2, ..., H_n\}$, such that H_i are edge disjoint and every edge in H_i belongs to G.If each H_i is a Tribonacci graceful graph, then ψ_{tg} is called a Tribonacci graceful decomposition of G.The minimum cardinality of a Triboonacci graceful decomposition of G is called the Tribonacci graceful decomposition number of G and is denoted by $\pi_{tg}(G)$.In this paper, we investigate the bounds of Tribonacci graceful decomposition of Diamond Snake graph Ds_n, Mongolian tent graph Mt_n and Triangular Diamond graph TD_n.
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1.Introduction

Graphs considered throughout this paper are finite, simple, undirected and nontrivial.Labeling of graph is the assignment of values to vertices or edges or both subject to certain conditions.The parameter π was introduced by Arumugam et al [2].The Graceful labeling of graphs was introduced by Rosa[5] in 1967.Tribonacci graceful Labeling was introduced by K.Sunitha and Sheriba.M in 2021[7].In this sequel, we introduced a new concept called Tribonacci graceful Decomposition of graphs.For standard terminology and notations, we follow D.B.West[9] and J.A.Gallian[3].

Definition 1.1[7]

Let G be a graph with p vertices and q edges. An injective function $\phi : V(G) \to \{0,1,2,...,T_q\}$, where T_q is the q^{th} Tribonacci number in the Tribonacci sequence is said to be Tribonacci graceful if the induced edge labeling $\phi^*(uv) = |\phi(u) - \phi(v)|$ is a bijection onto the set $\{T_1, T_2, ..., T_q\}$.If a graph G admits Tribonacci graceful labeling, then G is called a Tribonacci graceful graph.

Remark 1.1

The Tribonacci sequence is obtained as follows:

$$T_0 = 0,\ T_1 = T_2 = 1 \ and\ T_n = T_{n-1} + T_{n-2} + T_{n-3}\ \forall\ n \geq 3$$

$ie, \{0, 1, 1, 2, 4, 7, 13, 24, 44, 81, ...\}$ is the Tribonacci sequence.

Definition 1.2[2]

A decomposition π of a graph G is a collection of edge disjoint subgraphs $G_1, G_2, ..., G_n$ of G such that every edge of G belongs to exactly one $G_i, 1 \leq i \leq n$.

Definition 1.3

Let G be a graph with p vertices and q edges.A decompositon of G is a collection $\psi_{tg} = \{H_1, H_2, ..., H_n\}$, such that H_i are edge disjoint and every edge in H_i belongs to G.If each H_i is a Tribonacci graceful graph, then ψ_{tg} is called a Tribonacci graceful decomposition of G.The minimum cardinality of a Triboonacci graceful decomposition of G is called the Tribonacci graceful decomposition number of G and is denoted by $\pi_{tg}(G)$.

Definition 1.4[4]

A diamond snake graph DS_n is obtained by joining u_i and u_{i+1} to a new vertex u_i^1 and u_i^2 for $1 \leq i \leq n-1$.

Definition 1.5[1]

Mongolian tent graph MT_n is obtained from the ladder graph L_n by adding a new vertex u and joining each vertex $v_i, 1 \leq i \leq n$ with u.

Definition 1.6[1]

A Triangular diamond graph $TD_n, n \geq 3$ is obtained by joining a single vertex w to all vertices $u_i, 1 \leq i \leq n$ of Triangular ladder graph TL_n.

2.Main Result

Theorem 2.2 The bounds of Tribonacci graceful decomposition of the Diamond Snake graph DS_n is $2 \leq \pi_{tg}(DS_n) \leq 4n-4,\ n \geq 3$.

Proof

Let DS_n be Diamond Snake graph whose vertex set

$V(DS_n) = \{\{u_i / 1 \leq i \leq n\} \bigcup \{u_i^1 / 1 \leq i \leq n-1\}$

$\bigcup \{u_i^2 / 1 \leq i \leq n-1\}\}$ and edge set

$E(DS_n) = \{u_i u_i^1 / 1 \leq i \leq n-1\} \bigcup \{u_i u_i^2 / 1 \leq i \leq n-1\} \bigcup \{u_i^1 u$

such that

$|V(DS_n)| = 3n-2\ and\ |E(DS_n)]| = 4n-4.$

Claim: $\psi_{tg}(DS_n) = \{P_{2n-1}, P_{2n-1}\}$ is a Tribonacci graceful decomposition of the Diamond Snake graph DS_n

Case 1 Let $\psi_{tg}(DS_n)] = P_{2n-1}$ and let $\{u_2, u_3, ..., u_n, u_1^1, u_2^1, ..., u_{n-1}^1\}$ be the vertices of P_{2n-1}.

Define $\phi : V(P_{2n-1}) \to \{0, 1, ..., T_{2n-2}\}$ by

$\phi(u_1) = T_1, \phi(u_1^1) = T_0$

$\phi(u_i) = \phi(u_{i-1}^1) + (-1)^i T_{2n-2-(2i-4)}, \quad 2 \leq i \leq n$

$\phi(u_i^1) = \phi(u_i) - T_{2n-2-(2i-3)}, 2 \leq i \leq n-1$

Thus ϕ admits Tribonacci graceful labeling.

Hence $P_{2n-1}, n \geq 3$ is a Tribonacci graceful graph.

Case 2 Let $\psi_{tg}(DS_n) = P_{2n-1}$

Let $\{u_1, u_2, ..., u_n, u_1^2, u_2^2, ..., u_{n-1}^2\}$ be the vertices of P_{2n-1}.

In this case, ϕ admits Tribonacci graceful labeling.

Hence $P_{2n-1}, n \geq 3$ is a Tribonacci graceful graph.

Therefore $\psi_{tg}(DS_n) = \{P_{2n-1}, P_{2n-1}\}$ is a Tribonacci graceful decomposition of the Diamond Snake graph DS_n. Clearly $\psi_S(DS_n) \supseteq S(P_{2n-1}) \bigcup S(P_{2n-1})$

Therefore $\pi_{tg}(DS_n) \geq 2$. The edge set is $E = S(P_{2n-1}) \bigcup S(P_{2n-1})$

Note that P_2 is a Tribonacci graceful graph. Number of P_2 in DS_n is $S(P_{2n-1}) + S(P_{2n-1})$

$$|\psi_S(DS_n)| \leq |S(P_{2n-1})| + |S(P_{2n-1})| = 2n-2+2n-2 = 4n$$

Hence $\pi_{tg}(DS_n) \leq 4n-4$

Therefore the bounds of Tribonacci graceful decomposition of the Diamond Snake graph DS_n is $2 \leq \pi_{tg}(DS_n) \leq 4n-4, n \geq 3$.

Example 2.1 The Tribonacci graceful decomposition of Diamond Snake graph DS_5 is in Figure 2.2

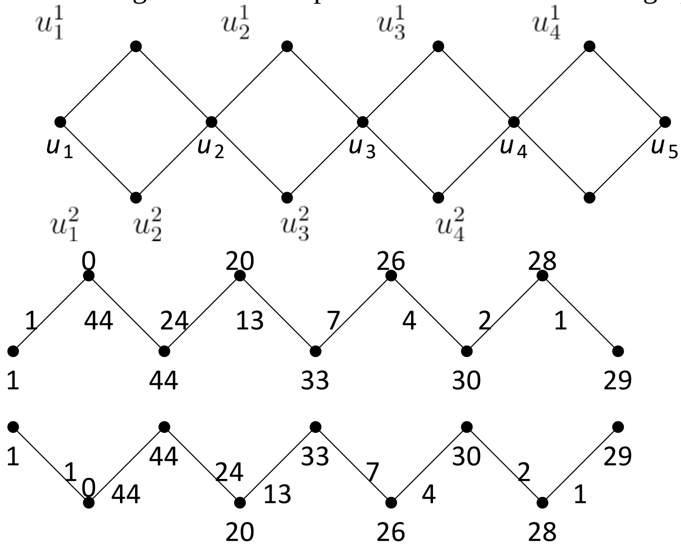

Figure 2.1

Theorem 2.2 The bounds of Tribonacci graceful decomposition of the Mongolian tent graph MT_n is $3 \leq \pi_{tg}(MT_n) \leq 4n-2, n \geq 3$.

Proof

Let L_n be the Ladder graph. Join each vertices $v_i, 1 \leq i \leq n$ to a new vertex u. The resultant graph is MT_n whose vertex set $V(MT_n) = \{\{u_i / 1 \leq i \leq n\} \bigcup \{v_i \ / 1 \leq i \leq n\} \bigcup \{u\}\}$ and edge set $E(MT_n) = \{\{u_i u_{i+1} / 1 \leq i \leq n-1\} \bigcup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \bigcup \{u_i v_i / 1 \leq i \leq n\} \bigcup \{u v_i / 1 \leq i \leq n\}\}$ such that $|V(MT_n)| = 2n+1 \ and \ |E(MT_n)]| = 4n-2$.

Claim: $\psi_{tg}(MT_n) = \{K_{1,n}, P_n, P_n \Theta K_1\}$ is the Tribonacci graceful decomposition of the Mongolian tent graph MT_n

Case 1 Let $\psi_{tg}(MT_n)] = K_{1,n}$

Let $\{u, v_1, v_2, ..., v_n\}$ be the vertices of $K_{1,n}$.

Define $\phi : V(K_{1,n}) \to \{0, 1, ..., T_n\}$ by

$\phi(u) = T_0, \phi(v_1) = T_1, \phi(v_i) = T_i, 2 \leq i \leq n$

Thus ϕ admits Tribonacci graceful labeling.

Hence $K_{1,n}, n \geq 3$ is a Tribonacci graceful graph.

Case 2 Let $\psi_{tg}(MT_n) = P_n$

Let $\{v_1, v_2, ..., v_n\}$ be the vertices of P_n.

In this case, $P_n, n \geq 3$ is a Tribonacci graceful graph.

Case 3 Let $\psi_{tg}(MT_n)] = P_n \Theta K_1$

Let $\{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n\}$ be the vertices of $P_n \Theta K_1$.

Define $\phi : V(P_n \Theta K_1) \to \{0, 1, ..., T_{2n-1}\}$ by

$\phi(u_1) = T_0, \phi(u_i) = \phi(u_{i-1}) + T_i, 2 \leq i \leq n$,

$\phi(v_n) = \phi(u_n) + T_1, \phi(u_i) + T_{2n-1-(i-1)}, 1 \leq i \leq n-1$

Thus ϕ admits Tribonacci graceful labeling.

Hence $P_n \Theta K_1, n \geq 3$ is a Tribonacci graceful graph.

Therefore $\psi_{tg}(MT_n)=\{K_{1,n}, P_n, P_n\Theta K_1\}$ is a Tribonacci graceful decomposition of the Mongolian tent graph MT_n.

Clearly

$\psi_S(MT_n)\supseteq S(K_{1,n})\bigcup S(P_n)\bigcup S(P_n\Theta K_1)$

Therefore $\pi_{tg}(MT_n)\ge 3$. The edge set is

$E=S(K_{1,n})\bigcup S(P_n)\bigcup S(P_n\Theta K_1)$

Note that P_2 is a Tribonacci graceful graph.

Number of P_2 in MT_n is $|S(K_{1,n})|+|S(P_n)|+|S(P_n\Theta K_1)|$

$|\psi_S(MT_n)|\le |S(K_{1,n})|+|S(P_n)|+|P_n\Theta K_1|$

$$=n+n-1+2n-1=4n-2$$

Hence $\pi_{tg}(MT_n)\le 4n-2$

Therefore the bounds of Tribonacci graceful decomposition of the Mongolian tent graph MT_n is $3\le \pi_{tg}(MT_n)\le 4n-2, n\ge 3.$

Example 2.1 The Tribonacci graceful decomposition of Mongolian tent graph Mt5 is in Figure 2.2

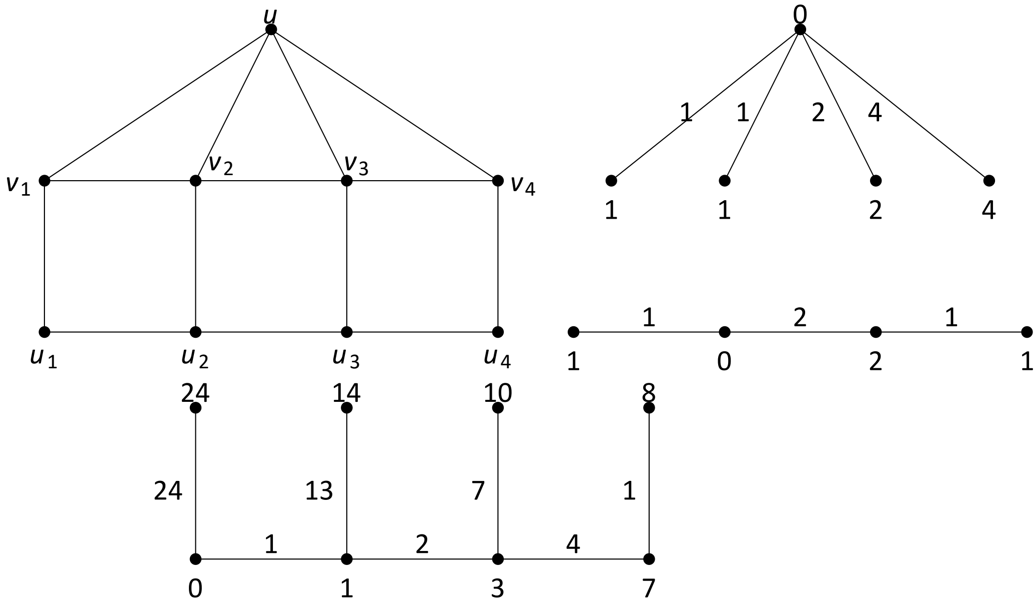

Figure 2.2

Theorem 2.3 The bounds of Tribonacci graceful decomposition of the Triangular diamond graph TD_n is $4\le \pi_{tg}(TD_n)\le 5n-5, n\ge 3.$

Proof

Let TL_n be a Triangular ladder graph. Join a new vertex w to each vertices $u_i, 1\le i\le n$. The resultant graph is TD_n whose vertex set $V(TD_n)=\{\{u_i/1\le i\le n\}\bigcup\{v_i/1\le i\le n\}\bigcup\{w\}\}$ and edge set $E(TD_n)=\{\{u_iu_{i+1}/1\le i\le n-1\}\bigcup\{v_iv_{i+1}/1\le i\le n-2\}\bigcup\{u_iv_i/1\le i\le n-1\}\bigcup\{v_iu_{i+1}, 1\le i\le n-1\}$ $\bigcup\{wu_i/1\le i\le n\}\}$ such that $|V(TD_n)|=2n \ and |E(TD_n)]|=5n-5.$

Claim: $\psi_{tg}(TD_n)=\{P_{n-1}, P_n, P_{2n-1}, K_{1,n}\}$ is a Tribonacci graceful decomposition of the Triangular diamond graph TD_n

Case 1 Let $\psi_{tg}(TD_n)]=P_{n-1}$

Let $\{v_1, v_2, ..., v_{n-1}\}$ be the vertices of P_{n-1}.

Define $\phi: V(P_{n-1})\to\{0,1,...,T_{n-2}\}$ by

$\phi(u_1)=T_1, \phi(u_2)=T_0,$

$\phi(u_i)=\phi(u_{i-1})+(-1)^{i+1}T_{n-i+1}, \quad 3\le i\le n-1$

Thus ϕ admits Tribonacci graceful labeling.

Hence $P_{n-1}, n\ge 3$ is a Tribonacci graceful graph.

Case 2 Let $\psi_{tg}(TD_n)=P_n$

Let $\{u_1, u_2, ..., u_n\}$ be the vertices of P_n.

Define $\phi: V(P_n)\to\{0,1,...,T_n\}$ by

$\phi(u_1)=T_1, \phi(u_2)=T_0,$

$\phi(u_i)=\phi(u_{i-1})+(-1)^{i+1}T_{n-i+1}, \quad 3\le i\le n$

Thus ϕ admits Tribonacci graceful labeling.

Hence $P_n, n\ge 3$ is a Tribonacci graceful graph.

Case 3 Let $\psi_{tg}(TD_n)=P_{2n-1}$

Let $\{u_1, u_2, ..., u_n, v_1, v_2, ..., v_{n-1}\}$ be the vertices of P_{2n-1}.

Define $\phi: V(P_{2n-1})\to\{0,1,...,T_{2n-1}\}$ by

$\phi(u_1)=T_1, \phi(v_1)=T_0$,

$\phi(u_i)=\phi(v_{i-1})+(-1)^{i+1}T_{2(n-i+1)}, 2\le i\le n$

$\phi(v_i)=\phi(u_i)-T_{2n-2i+1}, 2\le i\le n-1$

Thus ϕ admits Tribonacci graceful labeling.

Hence $P_{2n-1}, n\ge 3$ is a Tribonacci graceful graph.

Case 4 Let $\psi_{tg}(TD_n)]=K_{1,n}$

Let $\{w, u_1, u_2, ..., u_n\}$ be the vertices of $K_{1,n}$.

Define $\phi: V(K_{1,n})\to\{0,1,...,T_n\}$ by

$\phi(w)=T_0, \phi(u_1)=T_1, \phi(u_i)=T_i, 2\le i\le n$

Thus ϕ admits Tribonacci graceful labeling.

Hence $K_{1,n}, n\ge 3$ is a Tribonacci graceful graph.

Therfore $\psi_{tg}(TD_n)=\{P_{n-1}, P_n, P_{2n-1}.K_{1,n}\}$ is a Tribonacci graceful decomposition of the Triangular diamond graph TD_n

Clearly

$\psi_S(TD_n)\supseteq S(P_{n-1})\cup S(P_n)\cup S(P_{2n-1})\cup S(K_{1,n})$

Therefore $\pi_{tg}(TD_n)\ge 4$. The edge set is

$E=S(P_{n-1})\cup S(P_n)\cup S(P_{2n-1})\cup S(K_{1,n})$

Note that P_2 is a Tribonacci graceful graph.

Number of P_2 in TD_n is

$|S(P_{n-1})|+|S(P_n)|+|S(P_{2n-1})|+|S(K_{1,n})|$

$$|\psi_S(TD_n)|\le|S(P_{n-1})|+|S(P_n)|+|S(P_{2n-1})|+|S(K_{1,n})|$$
$$=n-2+n-1+2n-2+n=5n-5$$

Hence $\pi_{tg}(TD_n)\le 5n-5$

Therefore the bounds of Tribonacci graceful decomposition of the Triangular Diamond graph TD_n is $4\le\pi_{tg}(TD_n)\le 5n-5, n\ge 3.$

Example 2.3 The Tribonacci graceful decomposition of the Triangular diamond graph TD_4 is in Figure 2.3

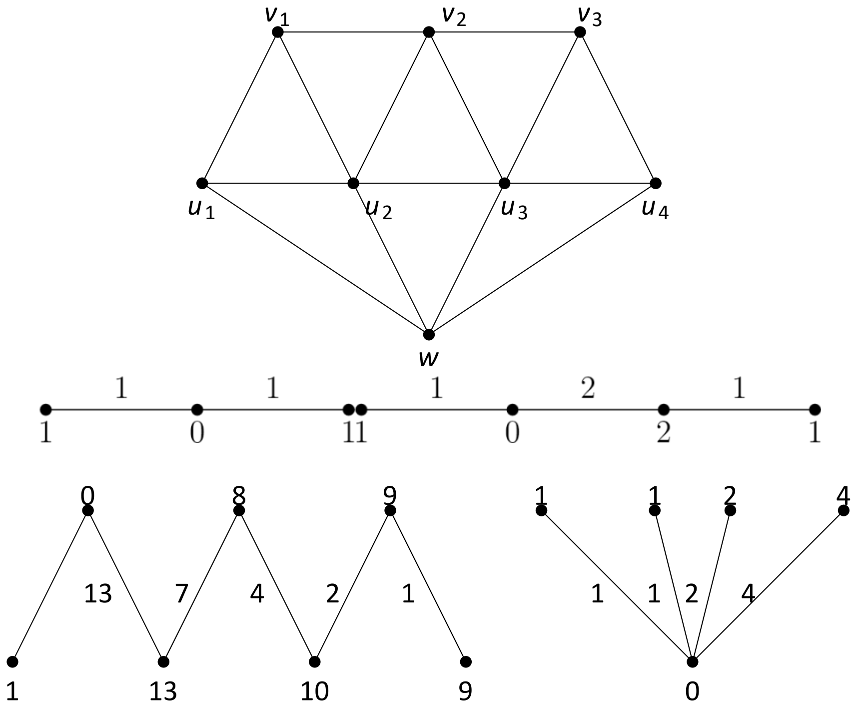

Figure 2.3

Conclusion

In this paper,we investigate the bounds of Tribonacci graceful decomposition of Diamond snake graph DS_n, Mongolian tent graph MT_n and Triangular diamond graph TD_n.

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