NEIGHBORHOOD PRIME DECOMPOSITION OF GRAPHS

K. Sunitha[1], T.Revathi[2*]

Abstract
A decomposition of a graph is a list of subgraphs $\psi_{NP} = \{H_1, H_2, \ldots, H_r\}$ such that each edge appears in exactly one subgraph H_i. If each H_i is a neighborhood prime graphs then ψ_{NP} is called a neighborhood prime decomposition (NPD) of G. The minimum cardinality of NPD is called a NPD number of G and it is denoted by $\pi_{NP}(G)$. In this paper, we investigate NPD of the cycle book graph $B[(C_4, m), 2]$, jelly fish graph $J(m,n)$, pineapple graph K_n^m and windmill graph W_n^m.

Key words: Jelly fish, Pineapple, Windmill, Neighborhood and Decomposition.
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[1]Assistant Professor, Department of Mathematics Scott Christian College (Autonomous), Nagercoil, India - 629001. E-mail: ksunithasam@gmail.com
[2]*Research Scholar, Scott Christian College (Autonomous), Nagercoil, India- 629001. Affiliated to Manonmaniam Sundaranar University,Tirunelveli- 627012. E-mail: revathinathan77@gmail.com

***Corresponding Author:** T.Revathi
*Research Scholar, Scott Christian College (Autonomous), Nagercoil, India- 629001. Affiliated to Manonmaniam Sundaranar University,Tirunelveli- 627012. E-mail: revathinathan77@gmail.com

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1. Introduction

In this paper, all graphs considered are finite, simple and undirected. Prime labeling was introduced by Tout at al [4]. Patel and Shrimali have introduced neighborhood prime labeling of graphs [6]. Rajeev Gandhi has introduced prime decomposition of graphs [5]. In this sequel we introduced the neighborhood prime decomposition of graphs and investigate neighborhood prime decomposition of the cycle book $B[(C_4, m), 2]$, jelly fish $J(m,n)$, pineapple K_n^m and windmill graph W_n^m.

Definition1.1 Let G be a simple graph. A bijective function $\phi^+ : V(G) \to \{1, 2, 3, ..., n\}$ is said to be neighborhood prime labeling, if for every vertex $\lambda \in V(G)$ with $\deg(\lambda) > 1$, gcd $\{\phi^+(\beta) : \beta \in N(\lambda)\} = 1$. A graph which admits neighborhood prime labeling is called a neighborhood prime (NP) graph.

Definition 1.2 A decomposition of a graph is a list of subgraphs $\psi_p = \{H_1, H_2, \ldots, H_r\}$ such that each edge appears in exactly one H_i. If each H_i is a prime graph, then ψ_p is called a prime decomposition of G. The minimum cardinality of a prime decomposition of G is called the prime decomposition number of G and is denoted by $\pi_p(G)$.

Definition 1.4 A cycle book graph $B[(C_4, m), 2]$ consists of m cycles C_4 with a common path P_2.

Definition 1.5[3] The jelly fish graph $J(m,n)$ is obtained by joining a cycle C_4 whose vertices $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ with vertices λ_1 and λ_3 defined by an edge and appending m pendent edges to λ_2 and n pendent edges to λ_4.

Definition 1.6 A pineapple graph K_n^m is a graph obtained by appending m pendent edges to a vertex of a complete graph K_n.

Definition 1.7 A windmill graph W_n^m is a graph obtained by combining m copies of the complete graph K_n with a common vertex. [For n=3, windmill graph is a generalized friendship graph $F_{3,m}$. So the windmill graph is started with $n \geq 4$ *and* $m \geq 2$.]

2. Main Results

Theorem 2.1 The decomposition of the cycle book $B[(C_4, m), 2]$, $m \geq 2$ is neighborhood prime (NP) graph.

Proof. Let $B[(C_4, m), 2]$ be a cycle book graph with

$V[B[(C_4,m),2]] = \{u_i, v_i\} \cup \{u_i, v_i / 1 \leq i \leq m\} \cup \{u_j^{|}, v_j^{|} / 1 \leq j \leq m\}$

and

$E[B[(C_4,m),2]] = \{uv\} \cup \{uu_i / 1 \leq i \leq m\} \cup \{u_i v_i / 1 \leq i \leq m\} \cup \{vv_i / 1 \leq i \leq m\} \cup \{uu_j^{'} / 1 \leq j \leq m\} \cup \{vv_j^{'} / 1 \leq j \leq m\} \cup \{u_j^{'} v_j^{'} / 1 \leq j \leq m\}$

Clearly, $|V[B((C_4,m),2)]| = 2(2m+1)$ and $|E[B((C_4,m),2)]| = 2m+1$

Let $\psi_{NP} = \{SP(1^1, 2^{2m}), K_{1,2m}\}$ be a decomposition of $B[(C_4, m), 2]$.

Let n be the positive integer and d be the decomposition number.

Let

$$\psi_{NP} = \begin{cases} (m-d)\ SP(1^1, 2^{2m})\ \&\ K_{1,2m} & if\ m \equiv 0\ (\mathrm{mod}\ 2)\ \&\ d = 1,3,5,... \\ (m-d)\ SP(1^1, 2^{2m})\ \&\ K_{1,2m} & if\ m \equiv 1\ (\mathrm{mod}\ 2)\ \&\ d = 2,4,6,... \end{cases}$$

The decomposition of the cycle book graph $B[(C_4, m), 2]$ contains a spider $SP(1^1, 2^{2m})$ and a star $K_{1,2m}$.

This implies that $\psi_{NP} \supseteq \{SP(1^1, 2^{2m}), K_{1,2m}\}$

That is $|\psi_{NP}| \geq |SP(1^1, 2^{2m})| + |K_{1,2m}|$

Hence $\pi_{NP}[B((C_4, m), 2)] \geq 2$.

We claim that ψ_{NP} is a NPD of $B[(C_4, m), 2]$.

Let 'λ' be any vertex of $SP(1^1, 2^{2m})$ and $K_{1,2m}$.

Case (i): Let $H_1 = SP(1^1, 2^{2m}), m \geq 2$

Define a function $\phi^+ : V(H_1) \to \{1, 2, 3, \cdots, 4m+2\}$ by

$\phi^+(u_0) = 1$

$\phi^+(v_0) = 2$

$\phi^+(v_i) = i+2, \quad 1 \leq i \leq 4m$

Let $\lambda = u_0$ with $\deg(\lambda) \geq 3$.

Then $\gcd\{\phi^+(w) / w \in N_V(\lambda)\} = 1$

Let $\lambda = \{v_{2i-1} / 1 \leq i \leq 2m\}$ with $\deg(\lambda) = 2$.

Then $\gcd\{\phi^+(w) / w \in N_V(\lambda)\} = 1$

Case (ii): Let $H_2 = K_{1,2m}, m \geq 2$

Define a function $\phi^+ : V(H_2) \to \{1, 2, 3, \cdots 2m+1\}$ by

$\phi^+(u_i) = i, \quad 1 \leq i \leq 2m+1$

Let $\lambda = u_1$ with $\deg(\lambda) \geq 2$.

Then $\gcd\{\phi^+(w) / w \in N_V(\lambda)\} = 1$

Hence the decomposition of the cycle book $B[(C_4, m), 2]$ is NP graph.

Theorem 2.3 The decomposition of the jelly fish $J(m,n), m,n \geq 2$ is neighborhood prime graph.

Proof. Let $J(m,n)$ be the jelly fish graph with

$V[J(m,n)] = \{v_i / 1 \leq \mathrm{i} \leq 4\} \cup \{u_i / 1 \leq \mathrm{i} \leq \mathrm{m}\} \cup \{w_i / 1 \leq \mathrm{i} \leq \mathrm{n}\}$

and

$E[J(m,n)] = \{v_i v_{i+1} / 1 \leq i \leq 3\} \cup \{v_1 v_4\} \cup \{v_1 v_3\} \cup \{v_2 u_i / 1 \leq i \leq m\} \cup \{v_4 w_i / 1 \leq i \leq n\}$

Clearly, $|V[J(m,n)]| = m+n+4$ and $|E[J(m,n)]| = m+n+5$

Let $\psi_{NP} = \{SP(1^{m+1}, 2^1), K_{1, n+2}\}$ b e a decomposition of $J(m,n)$.

Let n be the positive integer and d be the decomposition number.

Then

$$\psi_{NP} = \begin{cases} (n-d)\, SP(1^{m+1}, 2^1)\ \&\ K_{1, n+2} & if\ m \equiv 0 \pmod 2, n = 2,3,\ldots \&\ d = 1,2,3,\ldots \\ (n-d)\, SP(1^{m+1}, 2^1)\ \&\ K_{1, n+2} & if\ m \equiv 1 \pmod 2, n = 2,3,\ldots \&\ d = 1,2,3,\ldots \end{cases}$$

The decomposition of the jellyfish graph $J(m,n)$ contains a spider $SP(1^{m+1}, 2^1)$ and a star $K_{1, n+2}$.

This implies that $\psi_{NP} \supseteq \{SP(1^{m+1}, 2^1), K_{1, n+2}\}$

That is $|\psi_{NP}| \geq |SP(1^{m+1}, 2^1)| + |K_{1, n+2}|$

Hence $\pi_{NP}(J(m,n) \geq 3$.

We claim that ψ_{NP} is a NPD of $J(m,n)$.

Let 'λ' be any vertex of $SP(1^{m+1}, 2^1)$ and $K_{1, n+2}$.

Case (i): Let $H_1 = SP(1^{m+1}, 2^1), m \geq 2$

Define a function $\phi^+ : V(H_1) \to \{1, 2, 3, \cdots, m+4\}$ by

$\phi^+(u_0) = 1$

$\phi^+(v_i) = i+1, \quad 1 \le i \le m+3$

Let $\lambda = u_0$ with $\deg(\lambda) \ge 4.$

Then $\gcd\{\, \phi^+(w)/\, w \in N_V(\lambda)\,\} = 1$

Let $\lambda = \{v_{m+2}, m \ge 2\}$ with $\deg(\lambda) = 2.$

Then $\gcd\{\, \phi^+(w)/\, w \in N_V(\lambda)\,\} = 1$

Case (ii): Let $H_2 = K_{1,\, n+2}, n \ge 2$

Define a function $\phi^+ : V(H_2) \to \{1, 2, 3, \cdots, n+3\}$ by

$\phi^+(u_i) = i\,, \quad 1 \le i \le n+3$

Let $\lambda = u_1$ with $\deg(\lambda) \ge 4.$

Then $\gcd\{\, \phi^+(w)/\, w \in N_V(\lambda)\,\} = 1$

Hence the decomposition of the jelly fish $J(m,n)$ is NP graph.

Theorem 2.5 The decomposition of the pineapple $K_n^m, n \ge 3, m \ge 2$ is neighborhood prime graph.

Proof. Let K_n^m be the pineapple graph with

$V[\, K_n^m \,] = \{u_i \,/\, 1 \le \text{i} \le \text{n}\,\} \cup \{v_i \,/\, 1 \le \text{i} \le \text{m}\}$

and

$E[\, K_n^m \,] = \{u_i u_j,\, i \ne j \,/\, 1 \le \text{i} \le \text{n}, 1 \le \text{j} \le \text{n}\,\} \cup \{u_n v_i \,/\, 1 \le \text{i} \le \text{m}\,\}$

Clearly, $\left| V[K_n^m\,] \right| = n+m$ and $\left| E\,[K_n^m\,] \right| = \dfrac{n(n-1)}{2} + m$

Let $\psi_{NP} = \{K_n,\ K_{1,\,m}\}$ be a decomposition of K_n^m.

Let m, n be the positive integers and d be the decomposition number.

Then

$$\psi_{NP} = \begin{cases} (m-d)\, K_n \,\&\, K_{1,\,m} & if\ m \equiv 0 \,(\text{mod}\, 2), n = 3,4,5,\ldots \&\, d = 1,2,3,\cdots \\ (m-d)\, K_n \,\&\, K_{1,\,m} & if\ m \equiv 1 \,(\text{mod}\, 2), n = 3,4,5,\ldots \&\, d = 1,2,3,\cdots \end{cases}$$

The decomposition of the pineapple graph K_n^m contains a complete graph K_n. and a star $K_{1,\,m}$.

This implies that $\psi_{NP} \supseteq \{K_n, K_{1,\,m}\}$

That is $|\psi_{NP}| \ge |\, K_n \,| + |\, K_{1,\,m}\,|$

Hence $\pi_{NP}(K_n^m) \ge 2.$

We claim that ψ_{NP} *is a* NPD of K_n^m.

Let 'λ' be any vertex of K_n and $K_{1,\,m}$

Case (i): Let $H_1 = K_n, n \ge 3$

Define a function $\phi^+ : V(H_1) \to \{1, 2, 3, \cdots, n\}$ by

$\phi^+(u_i) = i, \quad 1 \le i \le n$

Le $\lambda = \{u_i \,/\, 1 \le i \le n\}$ with $\deg(\lambda) \ge 2.$

Then $\gcd\{\, \phi^+(w)/\, w \in N_V(\lambda)\,\} = 1$

Case (ii): Let $H_2 = K_{1,\,m}, m \ge 2$

Define a function $\phi^+ : V(H_2) \to \{1, 2, 3, \ldots, m+1\}$ by

$\phi^{+}(v_i) = i\,,\quad 1 \le i \le m+1$

Let $\lambda = v_1$ with $\deg(\lambda) \ge 2$.

Then gcd$\{\ \phi^{+}(w) / w \in N_V(\lambda)\ \} = 1$

Hence the decomposition of the pineapple K_n^m is a NP graph.

Theorem 2.7 The decomposition of the windmill $W_n^m, n \ge 4, m \ge 2$ is a NP graph.

Proof. Let W_n^m be the windmill graph with $V[W_n^m] = \{v_0\} \cup \{v_i^j / 1 \le i \le n-1, 1 \le j \le m\}$ and

$E[W_n^m] = \{v_0 v_i^j / 1 \le i \le n-1, 1 \le j \le m\} \cup \{v_i^j v_k^j / i \ne k, 1 \le i \le n, 1 \le j \le m, 1 \le k \le n-1\}$

Clearly, $\left| V[W_n^m] \right| = m\,(n-1)+1$ and $\left| E[\,W_n^m] \right| = \dfrac{mn\,(n-1)}{2}$

Let $\psi_{NP} = \left\{ K_n, K_n, \ldots, K_n\ (m\,times) \right\}$ be a decomposition of W_n^m.

Let m, n be the positive integers and d be the decomposition number.

Then $\psi_{NP} = \begin{cases} (2m-d-1)\,K_n & if\ m \equiv 0\ (\mathrm{mod}\ 2), n = 4,5,6,\ldots \&\ d = 1,2,3,\cdots \\ (2m-d-1)\,K_n & if\ m \equiv 1\ (\mathrm{mod}\ 2), n = 4,5,6,\ldots \&\ d = 1,2,3,\cdots \end{cases}$

The decomposition of the windmill graph W_n^m contains a complete graph K_n.

This implies that

$\psi_{NP} \supseteq \left\{ K_n, K_n, \ldots, K_n (m\,times) \right\}$

That is $|\psi_{NP}| \ge\ m\,|K_n|$

Hence $\pi_{NP}(W_n^m) \ge m.$

We claim that ψ_{NP} *is a NPD* of W_n^m.

Let 'λ' be any vertex of K_n.

Case (i): Let $H_1 = K_n, n \ge 3$

Define a function $\phi^{+} : V(H_1) \to \left\{1, 2, 3, \cdots, n\right\}$ by

$\phi^{+}(v_i) = i,\quad 1 \le i \le n$

Let $\lambda = \{v_i / 1 \le i \le n\}$ with $\deg(\lambda) \ge 3$.

Then gcd$\{\ \phi^{+}(w) / w \in N_V(\lambda)\ \} = 1$

Hence the decomposition of the windmill W_n^m is NP graph.

3. Conclusion

In this paper , we investigate neighborhood prime decomposition (NPD) of cycle related graphs. In future we will investigate various number of labeling using various graphs.

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