



DIFFERENT TYPES OF DOMINATION TOPOLOGICAL INDICES OF SOME CHEMICAL DRUGS

Raju S¹, Puttaswamy², Ammar Alsinai^{3*}

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Abstract

The biological and physical properties of chemical compounds are important and necessary in the preparation and design of any drug. As it is possible to study these properties by finding the values of the topological indices. In this research work, we establish new domination topological properties of some chemical structures that are used for the treatment of COVID-19. The obtained results can help in studying some physical properties of the compounds studied in this paper.

Keywords: Domination Topological indices, Domination degree, Chloroquine, Hydroxy-chloroquine, Minimal domination set, COVID-19.

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^{1,2}PES College of Engineering, Mandya, University of Mysore, India.

Email: ¹rajus24@gmail.com, ²prof.puttaswamy@gmail.com

³Department of Mathematics, University of Mysore, Manasagangotri, Mysore, India .

***Corresponding Author:**

Ammar Alsinai^{3*}

^{3*}Department of Mathematics, University of Mysore, Manasagangotri, Mysore, India .

^{3*}aliammar1985@gmail.com

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1. INTRODUCTION

Historically, many infectious diseases have been recorded and because of them, millions have died in the past few centuries. The most common infectious diseases were the plague, cholera, etc. In the latter part of 2019, a new virus (COVID-19) began to multiply and spread, which disrupted the global economy and human health around the world.

One of the most important branches of mathematics in which graphs are dealt with is graph theory. Graph theory can also be considered one of the most important branches of mathematics that uses graph theory techniques on molecular graphs as it is used in the modeling of chemical compounds. The study of the topological index is very common and important in chemical graph theory. In this paper, we assume that Φ is a connected graph without loops. With the vertex set $V(\Phi)$ and edge set $E(\Phi)$. There are two classified topological indices generally into two kinds: degree-based indices, and distance-based indices. In this paper, we assume that Φ is a connected graph without loops. With the vertex set $V(\Phi)$ and edge set $E(\Phi)$. The first and second $M_1(\Phi)$ and $M_2(\Phi)$ Zagreb indices are the old topological indices that were extensively investigated. These have been introduced by [17, 18], and are defined as:

$$M_1(\Phi) = \sum_{f \in V(\Phi)} d_f^2 \quad \text{and} \quad M_2(\Phi) = \sum_{fg \in E(\Phi)} d_f d_g.$$

For more discussion, see [9, 13, 15, 16, 21, 22, 23]. A graph Φ is called connected if there is a path between any two vertices of Φ . Otherwise, Φ is called disconnected. A set $S \subseteq V(\Phi)$ is called a dominating set of Φ , if for any vertex $f \in V(\Phi) - S$, there exists a vertex $g \in S$ such that f and g are adjacent.

A dominating set $S = \{f_1, f_2, \dots, f_r\}$ is minimal domination set if $S - f_i$ is not a dominating set. In [20], the authors used the notation $Y_m(\Phi)$ to denote the number of minimal domination sets.

In [20, 1] (2021) Hanan Ahmed et al, have defined a new degree based topological indices based on minimal dominating sets called domination topological indices and they are defined as follows:

$$\begin{aligned} DM_1(\Phi) &= \sum_{f \in V(\Phi)} d_{df}^2 & DM_2(\Phi) &= \sum_{fg \in E(\Phi)} d_{df} d_{dg} \\ DM_1^*(\Phi) &= \sum_{fg \in E(\Phi)} [d_{df} + d_{dg}], & DF(\Phi) &= \sum_{f \in V(\Phi)} d_{df}^3 \\ DH(\Phi) &= \sum_{fg \in E(\Phi)} [d_{df} + d_{dg}]^2, & DF^*(\Phi) &= \sum_{fg \in E(\Phi)} (d_{df}^2 + d_{dg}^2) \end{aligned}$$

Where d_{df} is the domination degree of the vertex $f \in V(\Phi)$ which is defined as:

Definition 1.1. [20] For any vertex $f \in V(\Phi)$, the domination degree denoted by d_{df} and defined as the number of minimal dominating sets of Φ which contains f .

For more details of domination topological indices and their applications see ([2, 3, 4, 5, 8, 10, 25]).

In 2022, the authors [7] defined the domination Sombor index as

$$DSO(\Phi) = \sum_{fg \in E(\Phi)} \sqrt{d_{df}^2 + d_{dg}^2} \quad (1.1)$$

In this paper, we define new domination topological indices as domination harmonic index, domination inverse sum index, domination atom bond connectivity index, domination geometric arithmetic index, and domination arithmetic geometric index which define as follows:

$$Dh(\Phi) = \sum_{fg \in E(\Phi)} \frac{2}{d_{df} + d_{dg}} \quad (1.2)$$

$$DISI(\Phi) = \sum_{fg \in E(\Phi)} \frac{d_{df} d_{dg}}{d_{df} + d_{dg}} \quad (1.3)$$

$$DABC(\Phi) = \sum_{fg \in E(\Phi)} \sqrt{\frac{d_{df} + d_{dg} - 2}{d_{df}d_{dg}}} \quad (1.4)$$

$$DGA(\Phi) = \sum_{fg \in E(\Phi)} \frac{2\sqrt{d_{df}d_{dg}}}{d_{df} + d_{dg}} \quad (1.5)$$

$$DAG(\Phi) = \sum_{fg \in E(\Phi)} \frac{d_{df} + d_{dg}}{2\sqrt{d_{df}d_{dg}}} \quad (1.6)$$

In this paper, we establish new domination topological properties of some chemical structures that are used for the treatment of COVID-19. The obtained results can help in studying some physical properties of the compounds studied in this paper.

2. MATERIALS AND METHODS

In this research work we are using the domination topological indices for study the properties of some chemical structures that are used for the treatment of COVID-19. By considering that Chemical structure of Chloroquine and hydroxy-chloroquine are a graph It consists of vertices and edges, then we applied the domination topological indices on this graphs.

2.1. Materials: In this paper we just using Latex and MATLAB SOFTER and some chemical structures.

2.2. Methods: In this research, the researcher used mathematical investigativ methods in calculating the topological indices of the chemical structures that were used at the beginning of the emergence of the Covid 19 pandemic, as medicines were used to treat the symptoms of the disease, and this research was investigative based on mathematical analysis and did not depend on experiments, so there ar no chemical devices or tools used in this research, but the researcher used mathe- matical investigative methods by calculating control domination topological indices for chemical compounds

3. RESULTS AND DISCUSSION

3.1. Results

In this section, we find the minimal dominating sets and determine the domination degree and domination value of all vertices of G. Using this new degree we calculate domination and γ -domination topological indices and ϕP -polynomial of the molecular structure of chloroquine.

Suppose Φ_1 is the molecular graph of chloroquine. This graph is of order 22 and size 23.

Theorem 2.1. [2] If Φ_1 is the molecular graph of chloroquine, then the total number of minimal and minimum dominating sets is 648 and 6 respectively.

The domination degree of the vertices of Φ_1 is given in the following table.

d_{df}	324	216	240	264	288	408	384	144	297
Number of the vertices	7	5	2	2	2	1	1	1	1

Table 1: The domination degrees of all vertices of the molecular graph of chloroquine.

Theorem 2.2. Let Φ_1 be the molecular graph of chloroquine. Then

1. $DSO(\Phi_1) = 9000.657042$
2. $Dh(\Phi_1) = 0.08695476066$
3. $DISI(\Phi_1) = 3079.536021$
4. $DABC(\Phi_1) = 2.010188204$
5. $DGA(\Phi_1) = 25.17572622$
6. $DAG(\Phi_1) = 12.8302725$

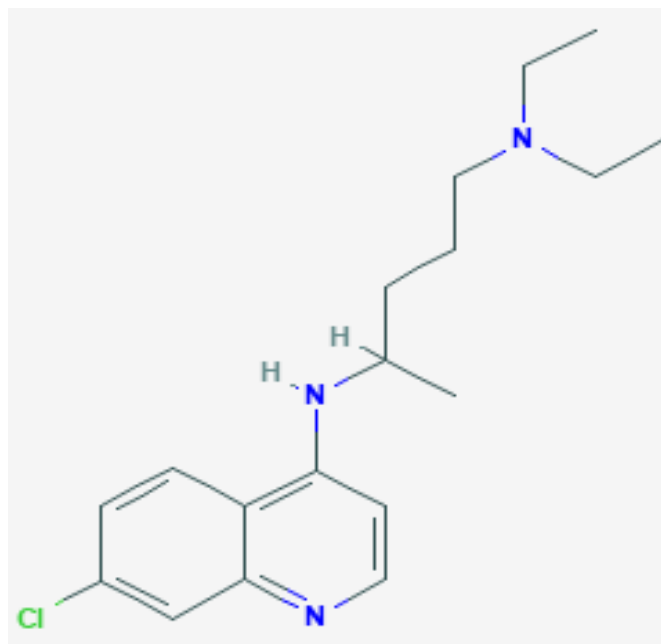


Figure 1: Chemical structure of Chloroquine

Proof: Suppose Φ_1 is the molecular graph of chloroquine, the set edges of Φ_1 can divide as

$$\begin{aligned}
 E_1 &= \{fg \in E(\Phi_1): d_{df} = 217, & d_{dg} = 288\}, & |E_1| = 2 \\
 E_2 &= \{fg \in E(\Phi_1): d_{df} = 216, & d_{dg} = 264\}, & |E_2| = 2 \\
 E_3 &= \{fg \in E(\Phi_1): d_{df} = 216, & d_{dg} = 216\}, & |E_3| = 2 \\
 E_4 &= \{fg \in E(\Phi_1): d_{df} = 324, & d_{dg} = 324\}, & |E_4| = 4 \\
 E_5 &= \{fg \in E(\Phi_1): d_{df} = 297, & d_{dg} = 324\}, & |E_5| = 2 \\
 E_6 &= \{fg \in E(\Phi_1): d_{df} = 240, & d_{dg} = 408\}, & |E_6| = 1 \\
 E_7 &= \{fg \in E(\Phi_1): d_{df} = 240, & d_{dg} = 264\}, & |E_7| = 1 \\
 E_8 &= \{fg \in E(\Phi_1): d_{df} = 144, & d_{dg} = 204\}, & |E_8| = 1 \\
 E_9 &= \{fg \in E(\Phi_1): d_{df} = 246, & d_{dg} = 288\}, & |E_9| = 1 \\
 E_{10} &= \{fg \in E(\Phi_1): d_{df} = 144, & d_{dg} = 384\}, & |E_{10}| = 1 \\
 E_{11} &= \{fg \in E(\Phi_1): d_{df} = 288, & d_{dg} = 384\}, & |E_{11}| = 1 \\
 E_{12} &= \{fg \in E(\Phi_1): d_{df} = 216, & d_{dg} = 240\}, & |E_{12}| = 1 \\
 E_{13} &= \{fg \in E(\Phi_1): d_{df} = 216, & d_{dg} = 240\}, & |E_{13}| = 1 \\
 E_{14} &= \{fg \in E(\Phi_1): d_{df} = 240, & d_{dg} = 324\}, & |E_{14}| = 1 \\
 E_{15} &= \{fg \in E(\Phi_1): d_{df} = 216, & d_{dg} = 324\}, & |E_{15}| = 1 \\
 E_{16} &= \{fg \in E(\Phi_1): d_{df} = 216, & d_{dg} = 297\}, & |E_{16}| = 1
 \end{aligned}$$

Hence,

$$\begin{aligned}
 1. DSO(\Phi_1) &= \sum_{fg \in E(\Phi_1)} \sqrt{d_{df}^2 + d_{dg}^2} \\
 &= 2\sqrt{216^2 + 288^2} + 2\sqrt{216^2 + 264^2} + 2\sqrt{216^2 + 216^2} + 4\sqrt{324^2 + 324^2} \\
 &\quad + 2\sqrt{297^2 + 324^2} + \sqrt{240^2 + 408^2} + \sqrt{240^2 + 264^2} + \sqrt{144^2 + 208^2} \\
 &\quad + \sqrt{246^2 + 288^2} + \sqrt{144^2 + 384^2} + \sqrt{288^2 + 384^2} + \sqrt{216^2 + 384^2} \\
 &\quad + \sqrt{216^2 + 240^2} + \sqrt{246^2 + 324^2} + \sqrt{216^2 + 324^2} + \sqrt{216^2 + 297^2} \\
 &= 2\sqrt{129600} + 2\sqrt{116352} + 2\sqrt{93312} + 4\sqrt{209952} + 2\sqrt{193185} + \\
 &\quad \sqrt{224064} + \sqrt{127296} + \sqrt{62352} + \sqrt{143460} + \sqrt{168192} + \sqrt{230400} + \\
 &\quad \sqrt{194112} + \sqrt{104256} + \sqrt{165492} + \sqrt{151632} + \sqrt{134865}
 \end{aligned}$$

$$\begin{aligned}
 &= 2(360) + 48\sqrt{202} + 2\sqrt{93312} + 4\sqrt{209952} + 54\sqrt{265} + 24\sqrt{389} + 24\sqrt{221} \\
 &\quad + 12\sqrt{433} + \sqrt{143460} + 48\sqrt{73} + 480 + 24\sqrt{337} + 24\sqrt{181} + \sqrt{165492} + \sqrt{151632} \\
 &\quad + 27\sqrt{185} \\
 &= 9000.657042
 \end{aligned}$$

$$\begin{aligned}
 2. Dh(\Phi_1) &= \sum_{fg \in E(\Phi_1)} \frac{2}{d_{df} + d_{dg}} \\
 &= \frac{4}{216+288} + \frac{4}{216+264} + \frac{4}{216+216} + \frac{8}{324+324} + \frac{4}{297+324} + \frac{2}{240+408} \\
 &\quad + \frac{4}{240+264} + \frac{4}{144+204} + \frac{4}{246+288} + \frac{8}{144+384} + \frac{4}{288+384} + \frac{2}{216+384} \\
 &\quad + \frac{4}{216+240} + \frac{4}{240+324} + \frac{4}{216+324} + \frac{2}{216+247} \\
 &= \frac{4}{504} + \frac{4}{480} + \frac{4}{432} + \frac{4}{648} + \frac{4}{621} + \frac{4}{648} + \frac{4}{504} + \frac{4}{348} + \frac{4}{534} + \frac{4}{528} + \frac{4}{672} + \frac{4}{600} + \frac{4}{456} \\
 &\quad + \frac{4}{564} + \frac{4}{540} + \frac{4}{463} \\
 &= \frac{1}{126} + \frac{1}{120} + \frac{1}{108} + \frac{1}{81} + \frac{4}{621} + \frac{1}{324} + \frac{1}{252} + \frac{1}{174} + \frac{1}{267} + \frac{1}{264} + \frac{1}{336} + \frac{1}{300} + \frac{1}{226} \\
 &\quad + \frac{1}{282} + \frac{1}{270} + \frac{2}{463} \\
 &= 0.08695476066
 \end{aligned}$$

$$\begin{aligned}
 3. DISI(\Phi_1) &= \sum_{fg \in E(\Phi_1)} \frac{d_{df} d_{dg}}{d_{df} + d_{dg}} = \frac{2 \times 216 \times 288}{216 + 288} + \frac{2 \times 216 \times 264}{216 + 264} + \frac{2 \times 216 \times 216}{216 + 216} \\
 &\quad + \frac{4 \times 324 \times 324}{324 + 324} + \frac{2 \times 297 \times 324}{297 + 324} + \frac{240 \times 408}{240 + 408} + \frac{240 \times 264}{240 + 264} + \frac{144 \times 204}{144 + 204} \\
 &\quad + \frac{246 \times 288}{246 + 288} + \frac{144 \times 384}{144 + 384} + \frac{288 \times 384}{288 + 384} + \frac{216 \times 384}{216 + 384} + \frac{216 \times 240}{216 + 240} + \frac{240 \times 324}{240 + 324} \\
 &\quad + \frac{216 \times 324}{216 + 324} + \frac{216 \times 297}{216 + 297} \\
 &= \frac{124416}{70848} + \frac{114048}{55296} + \frac{93312}{1110592} + \frac{419904}{82944} + \frac{192456}{51840} + \frac{97920}{77760} + \frac{63360}{69984} + \frac{29376}{64152} \\
 &\quad + \frac{504}{1728} + \frac{480}{1188} + \frac{432}{216} + \frac{648}{7128} + \frac{421}{1360} + \frac{648}{880} + \frac{504}{2448} + \frac{348}{11808} + \frac{348}{1152} + \frac{348}{1152} \\
 &\quad + \frac{3456}{25} + \frac{2160}{19} + \frac{6480}{47} + \frac{648}{5} + \frac{64152}{463} \\
 &= 3079.536021
 \end{aligned}$$

$$\begin{aligned}
 4. DABC(\Phi_1) &= \sum_{fg \in E(\Phi_1)} \sqrt{\frac{d_{df} + d_{dg} - 2}{d_{df} d_{dg}}} = 2 \sqrt{\frac{216 + 288 - 2}{216 \times 288}} + 2 \sqrt{\frac{216 + 264 - 2}{216 \times 264}} \\
 &\quad + 2 \sqrt{\frac{216 + 216 - 2}{216 \times 216}} + 4 \sqrt{\frac{324 + 324 - 2}{324 \times 324}} + 2 \sqrt{\frac{297 + 324 - 2}{297 \times 324}} + \sqrt{\frac{240 + 408 - 2}{240 \times 408}} \\
 &\quad + \sqrt{\frac{240 + 264 - 2}{240 \times 264}} + \sqrt{\frac{144 + 204 - 2}{144 \times 204}} + \sqrt{\frac{246 + 288 - 2}{246 \times 288}} + \sqrt{\frac{144 + 384 - 2}{144 \times 384}} \\
 &\quad + \sqrt{\frac{288 + 384 - 2}{288 \times 384}} + \sqrt{\frac{216 + 384 - 2}{216 \times 384}} + \sqrt{\frac{216 + 240 - 2}{216 \times 240}} + \sqrt{\frac{240 + 324 - 2}{240 \times 324}} \\
 &\quad + \sqrt{\frac{216 + 324 - 2}{216 \times 324}} + \sqrt{\frac{216 + 297 - 2}{216 \times 297}}
 \end{aligned}$$

$$\begin{aligned}
 &= 2\sqrt{\frac{502}{62208}} + 2\sqrt{\frac{478}{57024}} + 2\sqrt{\frac{430}{46656}} + 4\sqrt{\frac{646}{104976}} + 2\sqrt{\frac{619}{96228}} + \sqrt{\frac{646}{97920}} + \sqrt{\frac{502}{63360}} \\
 &+ \sqrt{\frac{346}{29376}} + \sqrt{\frac{532}{70848}} + \sqrt{\frac{526}{55296}} + \sqrt{\frac{670}{110592}} + \sqrt{\frac{598}{82944}} + \sqrt{\frac{454}{51840}} + \sqrt{\frac{562}{77760}} \\
 &+ \sqrt{\frac{538}{69984}} + \sqrt{\frac{511}{641152}} \\
 &= 2\sqrt{\frac{251}{31104}} + 2\sqrt{\frac{239}{28512}} + \sqrt{\frac{215}{23328}} + 4\sqrt{\frac{323}{52488}} + 2\sqrt{\frac{619}{96228}} + \sqrt{\frac{19}{2880}} + \sqrt{\frac{251}{31680}} \\
 &+ \sqrt{\frac{173}{14688}} + \sqrt{\frac{133}{17712}} + \sqrt{\frac{263}{27648}} + \sqrt{\frac{335}{55296}} + \sqrt{\frac{299}{41472}} + \sqrt{\frac{227}{25920}} + \sqrt{\frac{281}{3880}} \\
 &+ \sqrt{\frac{269}{34992}} + \frac{\sqrt{511}}{641152} \\
 &= 2.010188204
 \end{aligned}$$

$$\begin{aligned}
 5. DGA(\Phi_1) &= \sum_{fg \in E(\Phi_1)} \frac{2\sqrt{d_{df}d_{dg}}}{d_{dg} + d_{df}} \\
 &= \frac{4\sqrt{216 \times 288}}{216 + 288} + \frac{4\sqrt{216 \times 264}}{216 + 264} + \frac{4\sqrt{216 \times 216}}{216 + 216} + \frac{8\sqrt{324 \times 324}}{324 + 324} + \frac{4\sqrt{297 \times 324}}{297 + 324} \\
 &+ \frac{2\sqrt{240 \times 408}}{240 + 408} + \frac{2\sqrt{240 \times 264}}{240 + 264} + \frac{2\sqrt{144 \times 204}}{144 + 204} + \frac{2\sqrt{246 \times 288}}{246 + 288} + \frac{2\sqrt{144 \times 384}}{144 + 384} \\
 &+ \frac{2\sqrt{288 \times 348}}{288 + 348} + \frac{2\sqrt{216 \times 384}}{216 + 384} + \frac{2\sqrt{216 \times 240}}{216 + 240} + \frac{2\sqrt{240 \times 324}}{240 + 324} + \frac{2\sqrt{216 \times 324}}{216 + 324} \\
 &+ \frac{2\sqrt{216 \times 297}}{216 + 297} \\
 &= \frac{4\sqrt{62208}}{504} + \frac{4\sqrt{57024}}{480} + \frac{4\sqrt{46656}}{432} + \frac{8\sqrt{104976}}{648} + \frac{4\sqrt{96228}}{621} + \frac{2\sqrt{97920}}{648} + \frac{2\sqrt{63360}}{504} \\
 &+ \frac{2\sqrt{29376}}{398} + \frac{2\sqrt{70848}}{534} + \frac{2\sqrt{55296}}{528} + \frac{2\sqrt{110592}}{672} + \frac{2\sqrt{82944}}{600} + \frac{2\sqrt{51840}}{456} \\
 &+ \frac{2\sqrt{77760}}{564} + \frac{2\sqrt{69984}}{540} + \frac{2\sqrt{641152}}{463} \\
 &= 25.17572622
 \end{aligned}$$

$$\begin{aligned}
 6. DAG(\Phi_1) &= \sum_{fg \in E(\Phi_1)} \frac{d_{df} + d_{dg}}{2\sqrt{d_{df}d_{dg}}} \\
 &= \frac{216 + 288}{4\sqrt{216 \times 288}} + \frac{216 + 264}{4\sqrt{216 \times 264}} + \frac{216 + 216}{4\sqrt{216 \times 216}} + \frac{324 + 324}{8\sqrt{324 \times 324}} + \frac{297 + 324}{2\sqrt{297 \times 324}} \\
 &+ \frac{240 + 410}{2\sqrt{240 \times 408}} + \frac{240 + 264}{2\sqrt{240 \times 264}} + \frac{144 + 204}{2\sqrt{144 \times 204}} + \frac{246 + 288}{2\sqrt{246 \times 288}} + \frac{144 + 384}{2\sqrt{144 \times 384}} \\
 &+ \frac{288 + 384}{2\sqrt{288 \times 384}} + \frac{216 + 384}{2\sqrt{216 \times 384}} + \frac{216 + 324}{2\sqrt{216 \times 240}} + \frac{216 + 297}{2\sqrt{216 \times 297}} \\
 &+ \frac{504}{4\sqrt{62208}} + \frac{480}{4\sqrt{57024}} + \frac{432}{4\sqrt{46656}} + \frac{468}{8\sqrt{96228}} + \frac{648}{2\sqrt{97920}} + \frac{504}{2\sqrt{63360}} + \frac{348}{2\sqrt{29376}} \\
 &+ \frac{398}{2\sqrt{70848}} + \frac{534}{2\sqrt{55296}} + \frac{528}{2\sqrt{110592}} + \frac{672}{2\sqrt{82944}} + \frac{600}{2\sqrt{51840}} + \frac{456}{2\sqrt{77760}} \\
 &+ \frac{564}{2\sqrt{69984}} + \frac{463}{2\sqrt{641152}} \\
 &= 12.8302725
 \end{aligned}$$

3.2. RESULTS AND DISCUSSION: HYDROXY-CHLOROQUINE

Suppose Φ_2 is the molecular graph of hydroxy-chloroquine. This graph has an order 23 and size 24.

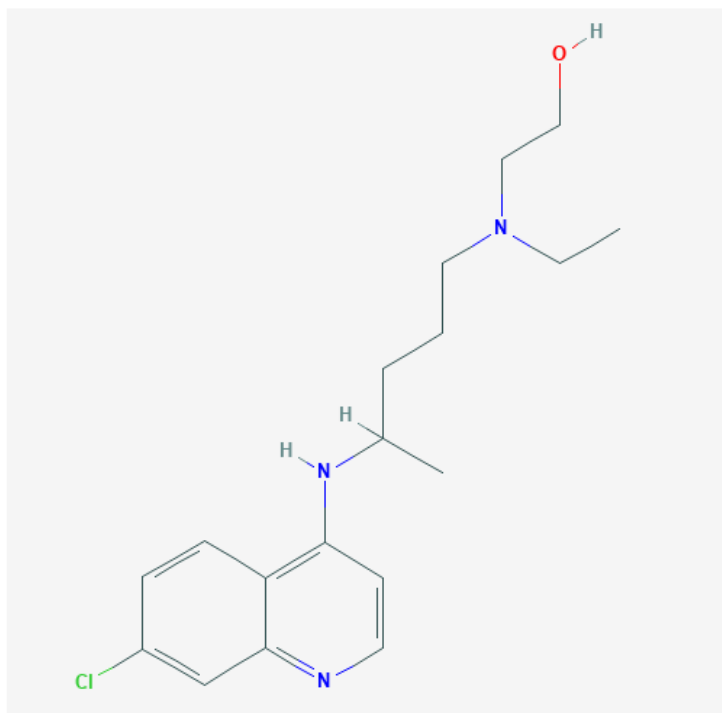


Figure 2: Chemical structure of hydroxy-chloroquine.

Theorem 3.1. [2] The total number of minimal dominating and minimum dominating sets of the molecular graph of hydroxy-chloroquine is 945 and 2 respectively.

The domination degree of the vertices of Φ_2 is given in the following table.

$d_d(v)$	459	324	385	315	420	486	621	567
Number of vertices	3	3	2	2	2	2	1	1
$d_d(v)$	297	340	317	210	425	350	595	.
Number of vertices	1	1	1	1	1	1	1	.

Table 2: Domination degrees of vertices of Φ_2 .

Theorem 3.2. Let Φ_2 be the molecular graph of hydroxy-chloroquine. Then

1. $DSO(\Phi_2) = 13552.62867$
2. $Dh(\Phi_2) = 0.06260334274$
3. $DISI(\Phi_2) = 4439.253918$
4. $DABC(\Phi_2) = 1753630564$
5. $DGA(\Phi_2) = 22.93901499$
6. $DAG(\Phi_2) = 20.20864145$

Proof: Let Φ_2 be the molecular graph of hydroxy-chloroquine. Then the set edges of Φ_2 can divide as

$$\begin{aligned}
 E_1 &= \{ij \in E(\Phi_2): d_{di} = 210, d_{dj} = 350\}, |E_1| = 1 \\
 E_2 &= \{ij \in E(\Phi_2): d_{di} = 210, d_{dj} = 425\}, |E_2| = 1 \\
 E_3 &= \{ij \in E(\Phi_2): d_{di} = 297, d_{dj} = 324\}, |E_3| = 1 \\
 E_4 &= \{ij \in E(\Phi_2): d_{di} = 297, d_{dj} = 567\}, |E_4| = 1 \\
 E_5 &= \{ij \in E(\Phi_2): d_{di} = 315, d_{dj} = 317\}, |E_5| = 1 \\
 E_6 &= \{ij \in E(\Phi_2): d_{di} = 315, d_{dj} = 385\}, |E_6| = 1
 \end{aligned}$$

$$\begin{aligned}
 E_7 &= \{ij \in E(\Phi_2): d_{di} = 315, d_{dj} = 420\}, |E_7| = 2 \\
 E_8 &= \{ij \in E(\Phi_2): d_{di} = 315, d_{dj} = 425\}, |E_8| = 1 \\
 E_9 &= \{ij \in E(\Phi_2): d_{di} = 317, d_{dj} = 340\}, |E_9| = 1 \\
 E_{10} &= \{ij \in E(\Phi_2): d_{di} = 317, d_{dj} = 385\}, |E_{10}| = 1 \\
 E_{11} &= \{ij \in E(\Phi_2): d_{di} = 324, d_{dj} = 324\}, |E_{11}| = 1 \\
 E_{12} &= \{ij \in E(\Phi_2): d_{di} = 324, d_{dj} = 459\}, |E_{12}| = 1 \\
 E_{13} &= \{ij \in E(\Phi_2): d_{di} = 324, d_{dj} = 567\}, |E_{13}| = 1 \\
 E_{14} &= \{ij \in E(\Phi_2): d_{di} = 324, d_{dj} = 621\}, |E_{14}| = 2 \\
 E_{15} &= \{ij \in E(\Phi_2): d_{di} = 340, d_{dj} = 486\}, |E_{15}| = 1 \\
 E_{16} &= \{ij \in E(\Phi_2): d_{di} = 350, d_{dj} = 385\}, |E_{16}| = 1 \\
 E_{17} &= \{ij \in E(\Phi_2): d_{di} = 350, d_{dj} = 595\}, |E_{17}| = 1 \\
 E_{18} &= \{ij \in E(\Phi_2): d_{di} = 358, d_{dj} = 420\}, |E_{18}| = 1 \\
 E_{19} &= \{ij \in E(\Phi_2): d_{di} = 420, d_{dj} = 425\}, |E_{19}| = 1 \\
 E_{20} &= \{ij \in E(\Phi_2): d_{di} = 459, d_{dj} = 486\}, |E_{20}| = 3 \\
 E_{21} &= \{ij \in E(\Phi_2): d_{di} = 486, d_{dj} = 567\}, |E_{21}| = 1
 \end{aligned}$$

Hence,

$$\begin{aligned}
 1. DSO(\Phi_2) &= \sum_{ij \in E(\Phi_2)} \sqrt{d_{di}^2 + d_{dj}^2} = \\
 &= \sqrt{210^2 + 350^2} + \sqrt{210^2 + 425^2} + \sqrt{297^2 + 324^2} + \sqrt{297^2 + 567^2} \\
 &\quad + \sqrt{315^2 + 317^2} + \sqrt{315^2 + 385^2} + 2\sqrt{315^2 + 420^2} + \sqrt{315^2 + 425^2} \\
 &\quad + \sqrt{317^2 + 340^2} + \sqrt{317^2 + 385^2} + \sqrt{324^2 + 324^2} + \sqrt{324^2 + 459^2} \\
 &\quad + \sqrt{324^2 + 567^2} + \sqrt{324^2 + 621^2} + \sqrt{340^2 + 486^2} + \sqrt{350^2 + 385^2} \\
 &\quad + \sqrt{350^2 + 595^2} + \sqrt{385^2 + 420^2} + \sqrt{420^2 + 425^2} + 3\sqrt{459^2 + 486^2} \\
 &\quad + \sqrt{486^2 + 567^2} \\
 &= \sqrt{166600} + \sqrt{224725} + \sqrt{193185} + \sqrt{409698} + \sqrt{206114} + 2\sqrt{247450} + 2\sqrt{275625} \\
 &\quad + \sqrt{279850} + \sqrt{216089} + \sqrt{248714} + \sqrt{209952} + \sqrt{315657} + \sqrt{426465} + \sqrt{490617} \\
 &\quad + \sqrt{351796} + \sqrt{270725} + \sqrt{476525} + \sqrt{324625} + \sqrt{357025} + 3\sqrt{446877} \\
 &\quad + \sqrt{557685} \\
 &= 70\sqrt{34} + \sqrt{224725} + 27\sqrt{265} + 27\sqrt{562}\sqrt{206114} + 35\sqrt{202} + \\
 &\quad 2(525) + \sqrt{279850} + \sqrt{216089} + \sqrt{248714} + \sqrt{209952} + 27\sqrt{433} + \\
 &\quad 81\sqrt{65} + 27\sqrt{673} + \sqrt{351796} + 35\sqrt{221} + 35\sqrt{389} + 35\sqrt{265} + \\
 &\quad \sqrt{357025} + 3(27\sqrt{613}) + 81\sqrt{85} \\
 &= 13552.62867
 \end{aligned}$$

$$\begin{aligned}
 2. Dh(\Phi_2) &= \sum_{ij \in E(\Phi_2)} \frac{2}{d_{di} + d_{dj}} \\
 &= \frac{2}{210 + 350} + \frac{2}{210 + 425} + \frac{2}{297 + 324} + \frac{2}{297 + 567} + \frac{2}{315 + 317} + \frac{2}{315 + 385} \\
 &\quad + \frac{2}{315 + 420} + \frac{2}{315 + 425} + \frac{2}{317 + 340} + \frac{2}{317 + 385} + \frac{2}{324 + 324} + \frac{2}{324 + 459} \\
 &\quad + \frac{2}{324 + 567} + \frac{2}{324 + 621} + \frac{2}{340 + 486} + \frac{2}{350 + 385} + \frac{2}{350 + 595} + \frac{2}{385 + 420} \\
 &\quad + \frac{2}{420 + 452} + \frac{2}{459 + 486} + \frac{2}{486 + 567} \\
 &= \frac{2}{560} + \frac{2}{635} + \frac{2}{621} + \frac{2}{864} + \frac{2}{632} + \frac{2}{700} + \frac{2}{735} + \frac{2}{742} + \frac{2}{657} + \frac{2}{702} + \frac{2}{648} + \frac{2}{783} + \frac{2}{891} \\
 &\quad + \frac{2}{945} + \frac{2}{826} + \frac{2}{735} + \frac{2}{945} + \frac{2}{805} + \frac{2}{872} + \frac{2}{945} + \frac{2}{1053} \\
 &= \frac{1}{280} + \frac{1}{635} + \frac{1}{621} + \frac{1}{432} + \frac{1}{316} + \frac{1}{350} + \frac{1}{735} + \frac{1}{370} + \frac{1}{657} + \frac{1}{351} + \frac{1}{324} + \frac{2}{783} + \frac{2}{891} \\
 &\quad + \frac{1}{945} + \frac{1}{413} + \frac{1}{735} + \frac{1}{945} + \frac{1}{805} + \frac{1}{436} + \frac{1}{315} + \frac{1}{1053}
 \end{aligned}$$

$$= 0.06260334274$$

$$\begin{aligned}
 3. \text{DISI}(\Phi_2) &= \sum_{ij \in E(\Phi_2)} \frac{d_{d_i} d_{d_j}}{d_{d_i} + d_{d_j}} \\
 &= \frac{210 \times 350}{210 + 350} + \frac{210 \times 425}{210 + 425} + \frac{297 \times 324}{297 + 324} + \frac{297 \times 567}{297 + 567} + \frac{315 \times 317}{315 + 317} + \frac{315 \times 385}{315 + 385} \\
 &\quad + \frac{315 \times 420}{324 \times 567} + \frac{315 \times 425}{324 \times 621} + \frac{317 \times 340}{340 \times 486} + \frac{317 \times 385}{350 \times 385} + \frac{324 \times 324}{350 \times 595} + \frac{324 \times 459}{385 \times 420} \\
 &\quad + \frac{324 \times 567}{420 \times 425} + \frac{324 \times 621}{3 \times 459 \times 486} + \frac{340 \times 486}{486 \times 567} + \frac{350 \times 385}{350 + 385} + \frac{350 \times 595}{350 + 595} + \frac{385 \times 420}{385 + 420} \\
 &\quad + \frac{420 + 425}{96228} + \frac{459 + 486}{168399} + \frac{486 + 567}{99855} \\
 &= \frac{73500}{560} + \frac{635}{122045} + \frac{621}{104976} + \frac{864}{148716} + \frac{632}{183708} + \frac{700}{201204} + \frac{735}{165240} + \frac{742}{134750} + \frac{657}{107780} \\
 &\quad + \frac{702}{208250} + \frac{648}{161700} + \frac{783}{178500} + \frac{891}{669222} + \frac{945}{275562} \\
 &\quad + \frac{945}{17850} + \frac{805}{3564} + \frac{872}{6237} + \frac{945}{99855} + \frac{693}{19125} + \frac{1053}{107780} + \frac{107780}{122045} + \frac{162}{3402} \\
 &= \frac{525}{4} + \frac{127}{5508} + \frac{23}{2268} + \frac{32}{7452} + \frac{632}{82620} + \frac{4}{550} + \frac{360}{5950} + \frac{106}{4620} + \frac{657}{178500} + \frac{702}{24786} + \frac{162}{3402} \\
 &\quad + \frac{29}{11} + \frac{11}{35} + \frac{35}{413} + \frac{3}{3} + \frac{27}{23} + \frac{23}{872} + \frac{35}{35} + \frac{13}{13} \\
 &= 4439.253918
 \end{aligned}$$

$$\begin{aligned}
 4. \text{DABC}(\Phi_2) &= \sum_{ij \in E(\Phi_2)} \frac{\sqrt{d_{d_i} + d_{d_j} - 2}}{d_{d_i} d_{d_j}} \\
 &= \sqrt{\frac{210 + 350 - 2}{210 \times 350}} + \sqrt{\frac{210 + 425 - 2}{210 \times 425}} + \sqrt{\frac{297 + 324 - 2}{297 \times 324}} + \sqrt{\frac{297 + 567 - 2}{297 \times 567}} \\
 &\quad + \sqrt{\frac{315 + 317 - 2}{315 \times 317}} + \sqrt{\frac{315 + 385 - 2}{315 \times 385}} + 2 \sqrt{\frac{315 + 420 - 2}{315 \times 420}} + \sqrt{\frac{315 + 425 - 2}{315 \times 425}} \\
 &\quad + \sqrt{\frac{317 + 340 - 2}{317 \times 340}} + \sqrt{\frac{317 + 385 - 2}{317 \times 385}} + \sqrt{\frac{324 + 324 - 2}{324 \times 324}} + \sqrt{\frac{324 + 459 - 2}{324 \times 459}} \\
 &\quad + \sqrt{\frac{324 + 567 - 2}{324 \times 567}} + \sqrt{\frac{324 + 621 - 2}{324 \times 621}} + \sqrt{\frac{340 + 486 - 2}{340 \times 486}} + \sqrt{\frac{350 + 385 - 2}{350 \times 385}} \\
 &\quad + \sqrt{\frac{350 + 595 - 2}{350 \times 595}} + \sqrt{\frac{385 + 420 - 2}{385 \times 420}} + \sqrt{\frac{420 + 425 - 2}{420 \times 425}} + 3 \sqrt{\frac{459 + 486 - 2}{459 \times 486}} \\
 &\quad + \sqrt{\frac{486 + 567 - 2}{486 \times 567}} \\
 &= \sqrt{\frac{558}{73500}} + \sqrt{\frac{633}{89250}} + \sqrt{\frac{619}{96228}} + \sqrt{\frac{862}{168399}} + \sqrt{\frac{630}{99855}} + \sqrt{\frac{698}{121275}} + 2 \sqrt{\frac{733}{132300}} \\
 &\quad + \sqrt{\frac{738}{133875}} + \sqrt{\frac{655}{107780}} + \sqrt{\frac{700}{122045}} + \sqrt{\frac{646}{104976}} + \sqrt{\frac{781}{148716}} + \sqrt{\frac{889}{183708}} \\
 &\quad + \sqrt{\frac{943}{201204}} + \sqrt{\frac{824}{165240}} + \sqrt{\frac{733}{134750}} + \sqrt{\frac{943}{208250}} + \sqrt{\frac{803}{161700}} + \sqrt{\frac{843}{178500}} \\
 &\quad + 3 \sqrt{\frac{943}{223074}} + \sqrt{\frac{1051}{275562}}
 \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{\frac{93}{12250}} + \sqrt{\frac{211}{29750}} + \sqrt{\frac{619}{96228}} + \sqrt{\frac{862}{168399}} + \sqrt{\frac{2}{317}} + \sqrt{\frac{698}{121275}} + 2\sqrt{\frac{733}{132300}} + \sqrt{\frac{82}{14375}} \\
 &\quad + \sqrt{\frac{131}{21556}} + \sqrt{\frac{20}{3437}} + \sqrt{\frac{323}{52488}} + \sqrt{\frac{781}{148716}} + \sqrt{\frac{127}{26244}} + \sqrt{\frac{41}{8778}} + \sqrt{\frac{103}{20655}} \\
 &\quad + \sqrt{\frac{733}{134750}} + \sqrt{\frac{943}{208250}} + \sqrt{\frac{73}{14700}} + \sqrt{\frac{281}{59500}} + 3\sqrt{\frac{943}{223074}} + \sqrt{\frac{1051}{275562}} \\
 &= 1753630564
 \end{aligned}$$

$$\begin{aligned}
 5. DGA(\Phi_2) &= \sum_{ij \in E(\Phi_2)} \frac{2\sqrt{d_{d_i}d_{d_j}}}{d_{d_i} + d_{d_j}} \\
 &= \frac{2\sqrt{210 \times 350}}{210 + 350} + \frac{2\sqrt{210 \times 425}}{210 + 425} + \frac{2\sqrt{297 \times 324}}{297 + 324} + \frac{2\sqrt{297 \times 567}}{297 + 567} + \frac{2\sqrt{315 \times 317}}{315 + 317} \\
 &\quad + \frac{2\sqrt{315 \times 385}}{315 + 385} + \frac{4\sqrt{315 \times 420}}{315 + 420} + \frac{2\sqrt{315 \times 425}}{315 + 425} + \frac{2\sqrt{317 \times 340}}{317 + 340} + \frac{2\sqrt{317 \times 385}}{317 + 385} \\
 &\quad + \frac{2\sqrt{324 \times 324}}{324 + 324} + \frac{2\sqrt{324 \times 459}}{324 + 459} + \frac{2\sqrt{324 \times 567}}{324 + 567} + \frac{2\sqrt{324 \times 621}}{324 + 621} + \frac{2\sqrt{340 \times 486}}{340 + 486} \\
 &\quad + \frac{2\sqrt{350 \times 385}}{350 + 385} \\
 &\quad + \frac{2\sqrt{350 \times 595}}{350 + 595} + \frac{2\sqrt{385 \times 420}}{385 + 420} + \frac{2\sqrt{420 \times 425}}{420 + 425} + \frac{6\sqrt{459 \times 486}}{459 + 486} + \frac{2\sqrt{486 \times 567}}{486 + 567} \\
 &= \frac{140\sqrt{15}}{560} + \frac{2\sqrt{89250}}{635} + \frac{108\sqrt{33}}{621} + \frac{54\sqrt{231}}{864} + \frac{2\sqrt{99855}}{632} + \frac{2\sqrt{121275}}{700} + \frac{4\sqrt{132300}}{735} \\
 &\quad + \frac{30\sqrt{595}}{742} + \frac{2\sqrt{107780}}{657} + \frac{2\sqrt{122045}}{702} + \frac{648}{648} + \frac{108\sqrt{51}}{783} + \frac{2\sqrt{183708}}{891} + \frac{108\sqrt{69}}{945} \\
 &\quad + \frac{36\sqrt{510}}{826} + \frac{70\sqrt{110}}{735} + \frac{70\sqrt{170}}{945} + \frac{140\sqrt{33}}{805} + \frac{2\sqrt{178500}}{872} + \frac{486\sqrt{34}}{945} + \frac{162\sqrt{42}}{1053} \\
 &= 22.93901499
 \end{aligned}$$

$$\begin{aligned}
 6. DAG(\Phi_2) &= \sum_{ij \in E(\Phi_2)} \frac{d_{d_i} + d_{d_j}}{2\sqrt{d_{d_i}d_{d_j}}} \\
 &= \frac{210 + 350}{2\sqrt{210 \times 350}} + \frac{210 + 425}{2\sqrt{210 \times 425}} + \frac{297 + 324}{2\sqrt{297 \times 324}} + \frac{297 + 567}{2\sqrt{297 \times 567}} + \frac{315 + 317}{2\sqrt{315 \times 317}} \\
 &\quad + \frac{315 + 385}{2\sqrt{315 \times 385}} + \frac{315 + 420}{4\sqrt{315 \times 420}} + \frac{315 + 425}{2\sqrt{315 \times 425}} + \frac{317 + 340}{2\sqrt{317 \times 340}} + \frac{317 + 385}{2\sqrt{317 \times 385}} \\
 &\quad + \frac{324 + 324}{2\sqrt{324 \times 324}} + \frac{324 + 459}{2\sqrt{324 \times 459}} + \frac{324 + 567}{2\sqrt{324 \times 567}} + \frac{324 + 621}{2\sqrt{324 \times 621}} + \frac{340 + 486}{2\sqrt{340 \times 486}} \\
 &\quad + \frac{350 + 385}{2\sqrt{350 \times 385}} + \frac{350 + 595}{2\sqrt{350 \times 595}} + \frac{385 + 420}{2\sqrt{385 \times 420}} + \frac{420 + 425}{2\sqrt{420 \times 425}} + \frac{459 + 486}{6\sqrt{459 \times 486}} + \frac{486 + 567}{2\sqrt{486 \times 567}} \\
 &= \frac{560}{140\sqrt{15}} + \frac{635}{2\sqrt{89250}} + \frac{621}{108\sqrt{33}} + \frac{864}{54\sqrt{231}} + \frac{632}{2\sqrt{99855}} + \frac{700}{2\sqrt{121275}} + \frac{735}{4\sqrt{132300}} \\
 &\quad + \frac{742}{30\sqrt{595}} + \frac{657}{2\sqrt{107780}} + \frac{702}{2\sqrt{122045}} + \frac{648}{648} + \frac{783}{108\sqrt{51}} + \frac{891}{2\sqrt{183708}} + \frac{945}{108\sqrt{69}} \\
 &\quad + \frac{826}{36\sqrt{510}} + \frac{735}{70\sqrt{110}} + \frac{945}{70\sqrt{170}} + \frac{805}{140\sqrt{33}} + \frac{872}{2\sqrt{178500}} + \frac{945}{486\sqrt{34}} + \frac{1053}{162\sqrt{42}} \\
 &= 20.20864145
 \end{aligned}$$

4. CONCLUSION

This article aims to study the topological properties of the chemical structures of the drug compounds that were used to reduce the number of deaths during the Corona pandemic, and the researchers knew modern topological indices that depend on the degree set (domination degree) and the researcher concluded that these indices gave excellent results better than the indices that depend on degree distance. In this paper , by study these properties by finding the values of the topological indices the new domination topological properties of some chemical structures that are used for the treatment of COVID-19 are establish and The obtained results can help in studying some physical properties of the compounds studied in this paper.

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