EFFECT OF COUPLE-STRESS ON THE MICROPOLAR FLUID FLOW SATURATING A POROUS MEDIUM WITH SUSPENDED PARTICLES

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Abstract:
In this paper, we discussed the effect of couple-stress micro on the polar fluid layer heated from below in the presence of varying gravitational field in a porous medium with suspended particles. The dispersion relation has been analyzed using normal mode, it is found that the medium permeability and suspended particle have destabilizing effect. The couple-stress parameter, coupling parameter, heat conduction parameter and micro-polar coefficient have stabilizing effect. The sufficient condition for the non-existence of over stability has also been obtained.

Keywords: Couple-Stress Micro-Polar Fluid; Porous Medium; Suspended particle

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1. Introduction

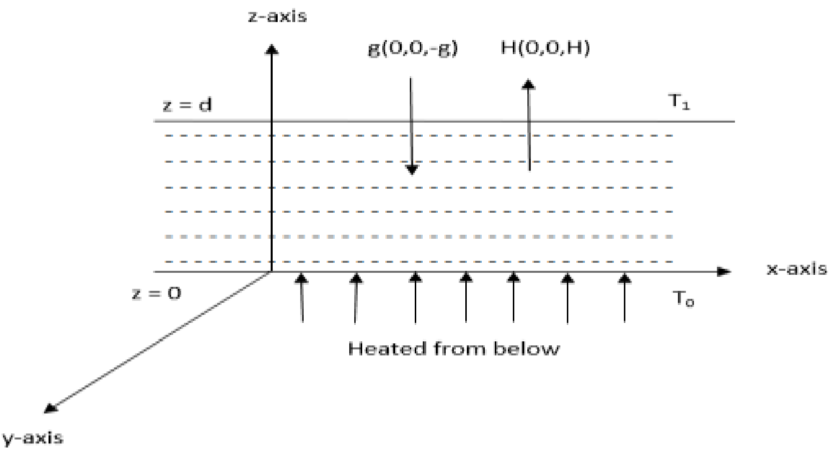
The micro-polar fluid is one of the significant fluid types found in technology fields. One of the most difficult and fascinating areas of technology that draws scientists and academics is micro-polar fluids. The general theory of micro polar fluid was introduced Eringen [2, 3]. Sharma and Gupta [11] investigated the thermal convection on micro polar fluid in porous medium. Sharma and Gupta [12, 13] have studied the numerically and analytically effect on hydrodynamics flow of a suspended and rotating micropolar fluid layer heated from below saturating a porous medium. Kumawat et al. [5-8] analyzed the effect of the couple-stress, magnetic field, permeability, rotation and suspended particles on micro polar and visco-elastic fluid flow. Mittal and Rana [9], and Singh [15] investigated the medium permeability, suspended particles and other parameters on the micro-polar ferromagnetic fluid flow saturating a porous medium.

Stokes [16] study the classical theory of couple-stress fluid. Kumar [4] et al. discussed the rotation on thermal instability in couple-stress viscous elastic fluid. Banyal and Singh [1] investigated the rotation on the couple-stress fluid in a porous medium. Shivakumara et al. [14] analyzed the onset of convection in a couple-stress fluid flow saturating a porous medium by the Galerkin method. Pundir [10] et al. analyzed the effect of medium permeability, couple-stress parameter and magnetization on ferromagnetic fluid layer heated from below in a porous medium with hall current.

In this paper, we attempt to study of couple-stress on the micro polar fluid flow saturating a porous medium with suspended particles. To my knowledge this problem has not yet been investigated using the generalized Darcy's model.

2. Mathematical Formulation

An infinite, horizontal, incompressible electrically non-conducting couple-stress micro-polar fluid layer of thickness d is assumed and has porosity $\in$ and medium permeability k_1. The upper limit $z = d$ and lower limit $z = 0$ are maintained at constant but varying temperatures T_0 and T_1 such that a study adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ has been maintained. The whole system is acted on by a gravity $\vec{g} = (0, 0, g)$ and uniform magnetic field $\vec{H} = (0, 0, H_0)$ is applied along z-axis.



The surface porosity is ∈ and 1-∈ is the fraction occupied by solid, then by the Darcy's law.

The equation of continuity is

$$\nabla.\vec{q} = 0 \quad (1)$$

The equation of momentum is

$$\frac{\rho_0}{\in}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right] = -\nabla P - \rho g \hat{e}_z + \left(\mu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q} - \frac{1}{k_1}(\mu+\varsigma)\vec{q} + \varsigma(\nabla\times\vec{v}) + \frac{K'N}{\in}(\vec{q}_d - \vec{q}) \quad (2)$$

The equation of internal angular momentum is

$$\rho_0 J\left[\frac{\partial\vec{v}}{\partial t}+\frac{1}{\in}(\vec{v}.\nabla)\vec{v}\right] = (\alpha'+\beta')\nabla(\nabla.\vec{v}) + \gamma'\nabla^2\vec{v} + \frac{\varsigma}{\in}(\nabla\times\vec{q}) - 2\varsigma\vec{v} \quad (3)$$

Where ρ – Fluid density, ρ_0 – Reference density, $\vec{q}$ – Filter velocity, $\vec{\nu}$ – Spin (micro rotation), μ - Shear kinematic viscosity coefficient, ς – Coupling viscosity coefficient, P – Pressure, μ' – Couple stress viscosity, $\hat{e}_z$ – Unit vector in z-direction, α' – – Bulk spin viscosity coefficient, β' – Shear spin viscosity coefficient, γ' – Micro-polar viscosity coefficient, J – Micro inertia constant, t – time, $\vec{q}_d = (X,t)$ – Filter velocity of suspended particles, $N = (X,t)$ – Number density of suspended particles, $X = (x,y,z)$ and $K' = 6\pi\mu r, r$ being the particle radius, is stokes drag coefficient.

The equation of temperature is

$$\left[\in \rho_0 C_v + (1-\in)\rho_s C_s\right]\frac{\partial T}{\partial t} + \rho_0 C_v(\vec{q}.\nabla)T + mNC_{Pt}\left(\in\frac{\partial T}{\partial t} + \vec{q}_d.\nabla T\right) = \chi\nabla^2 T + \delta(\nabla\times\vec{\nu}).\nabla T \quad (4)$$

The equation of state is

$$\rho = \rho_0\left[1-\alpha(T-T_a)\right] \quad (5)$$

Where C_v – Specific heat at constant volume and magnetic field, C_s – Specific heat of solid (Porous Material Matrix), ρ_s – Density of solid matrix, χ – Thermal conductivity, T – Temperature, δ – Micro-polar heat conduction coefficient, α – Coefficient of thermal expansion, T_a – Average temperature given by $T_a = \frac{(T_0+T_1)}{2}$ and mN – Mass of suspended particles per unit volume.

Now ignoring the pressure, gravity and forces on the suspended particles, then the equations of motion and continuity for the suspended particles are

$$mN\left[\frac{\partial\vec{q}_d}{\partial t} + \frac{1}{\in}(\vec{q}_d.\nabla)\vec{q}_d\right] = K'N(\vec{q}-\vec{q}_d) \quad (6)$$

$$\in\frac{\partial N}{\partial t} + \nabla.(N\vec{q}_d) = 0 \quad (7)$$

3. Basic state of the problem

The basic state is given by

$$\vec{q} = \vec{q}_b(0,0,0), \vec{q}_d = (\vec{q}_d)_b(0,0,0), \vec{\nu} = \vec{\nu}_b(0,0,0), \rho = \rho = \rho_b(z) \text{ and } P = P_b(z)$$

Under this basic state, equation (1) to (7) become

$$\frac{dP_b}{dz} + \rho_b g = 0 \quad (8)$$

$$T = T_b(z) = -\beta z + T_a; \text{ where } \beta = \frac{(T_1-T_0)}{d} \text{ and } N = N_b = N_0 \quad (9)$$

$$\rho_b = \rho_0(1+\alpha\beta z) \quad (10)$$

4. Perturbation Equations

$\vec{q} = \vec{q}_b + \vec{q}', \vec{q}_d = (\vec{q}_d)_b + \vec{q}_d', \vec{\nu} = \vec{\nu}_b + \vec{\nu}', \rho = \rho_b + \rho', P = P_b(z)$ and $T = T_b + \theta$ $N = N_b + N_0$ Where $\vec{q}', \vec{\nu}', \vec{q}_d', P', \rho', \theta$ and N_0 are the denote perturbations in $\vec{q}, \vec{\nu}, \vec{q}_d, P, \rho, T$ and N respectively, we get

$$\nabla.\vec{q}' = 0 \quad (11)$$

$$\frac{\rho_0}{\in}\left[\frac{\partial\vec{q}'}{\partial t} + \frac{1}{\in}(\vec{q}'.\nabla)\vec{q}'\right] = -\nabla P' - \rho' g\hat{e}_z + \left(\mu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q}' - \frac{1}{k_1}(\mu+\varsigma)\vec{q}' + \varsigma(\nabla\times\vec{\nu}') + \frac{K'N_0}{\in}(\vec{q}_d' - \vec{q}') \quad (12)$$

$$\rho_0 J\left[\frac{\partial\vec{\nu}'}{\partial t} + \frac{1}{\in}(\vec{\nu}'.\nabla)\vec{\nu}'\right] = (\alpha'+\beta')\nabla(\nabla.\vec{\nu}') + \gamma'\nabla^2\vec{\nu}' + \frac{\varsigma}{\in}(\nabla\times\vec{q}') - 2\varsigma\vec{\nu}' \quad (13)$$

$$\left[\in \rho_0 C_v + (1-\in)\rho_s C_s\right]\frac{\partial \theta}{\partial t} + (\vec{q}\,'.\nabla)(T_b+\theta)\rho_0 C_v + mN_0 C_{Pt}\left(\in \frac{\partial}{\partial t} + \vec{q}_d.\nabla\right)(T_b+\theta)$$
$$= \chi \nabla^2\theta + \delta(\nabla\times\vec{v}\,').\nabla\theta + \delta(\nabla\times\vec{v}\,').\nabla T_b \qquad (14)$$

$$mN_0\left[\frac{\partial \vec{q}_d\,'}{\partial t} + \frac{1}{\in}(\vec{q}_d\,'.\nabla)\vec{q}_d\,'\right] = K'N_0(\vec{q}\,' - \vec{q}_d\,') \qquad (15)$$

$$\in \frac{\partial N_0}{\partial t} + \nabla.(N_0\vec{q}_d\,') = 0 \qquad (16)$$

$$\rho' = \rho_0\alpha\theta \qquad (17)$$

To linearize above equation, ignoring the terms $(\vec{q}\,'.\nabla)\vec{q}\,', (\vec{v}\,'.\nabla)\vec{v}\,', (\vec{q}\,'.\nabla)\vec{v}\,', (\vec{q}_d\,'.\nabla)\vec{q}_d\,',$ $(\vec{q}_d\,'.\nabla)\theta, (\vec{q}\,'.\nabla)\theta, \vec{v}\,'(\vec{q}\,'.\nabla), (\nabla\times\vec{v}).\nabla\theta, \vec{v}\,'\frac{\partial\theta}{\partial t}$ then we have

$$\nabla.\vec{q}\,' = 0 \qquad (18)$$

$$\frac{\rho_0}{\in}\frac{\partial \vec{q}\,'}{\partial t} = -\nabla P' - \rho' g\hat{e}_z + \left(\mu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q}\,' - \frac{1}{k_1}(\mu+\varsigma)\vec{q}\,' + \varsigma(\nabla\times\vec{v}\,') + \frac{K'N_0}{\in}(\vec{q}_d\,' - \vec{q}\,') \qquad (19)$$

$$\rho_0 J\frac{\partial \vec{v}\,'}{\partial t} = (\alpha' + \beta')\nabla(\nabla.\vec{v}\,') + \gamma'\nabla^2\vec{v}\,' + \frac{\varsigma}{\in}(\nabla\times\vec{q}\,') - 2\varsigma\vec{v}\,' \qquad (20)$$

$$\left[\in + \frac{(1-\in)\rho_s C_s}{\rho_0 C_v}\right]\frac{\partial\theta}{\partial t} + (\vec{q}\,'.\nabla)T_b + \frac{mN_0C_{Pt}}{\rho_0 C_v}\frac{\partial\theta}{\partial t} + \frac{mN_0C_{Pt}}{\rho_0 C_v}\vec{q}_d.\nabla T_b = \frac{\chi\nabla^2\theta}{\rho_0 C_v} + \frac{\delta}{\rho_0 C_v}(\nabla\times\vec{v}\,').\nabla T_b \qquad (21)$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)\vec{q}_d = \vec{q} \qquad (22)$$

$$\in\frac{\partial N_0}{\partial t} + N_0(\nabla.\vec{q}_d\,') = 0 \qquad (23)$$

$$\rho' = -\rho_0\alpha\theta \qquad (24)$$

Now

$$\nabla.\vec{q}\,' = 0 \qquad (25)$$

$$L\left[\frac{\rho_0}{\in}\frac{\partial\vec{q}\,'}{\partial t}\right] = L\left[-\nabla P' + \rho_0\alpha\theta g\hat{e}_z + \left(\mu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q}\,' - \frac{1}{k_1}(\mu+\varsigma)\vec{q}\,' + \varsigma(\nabla\times\vec{v}\,')\right] - \frac{mN_0}{\in}\frac{\partial\vec{q}\,'}{\partial t} \qquad (26)$$

$$\rho_0 J\frac{\partial \vec{v}\,'}{\partial t} = (\alpha' + \beta')\nabla(\nabla.\vec{v}\,') + \gamma'\nabla^2\vec{v}\,' + \frac{\varsigma}{\in}(\nabla\times\vec{q}\,') - 2\varsigma\vec{v}\,' \qquad (27)$$

$$L[E + b'\in]\frac{\partial\theta}{\partial t} = L\left[k_T\nabla^2\theta - \frac{\delta}{\rho_0 C_v}(\nabla\times\vec{v}\,')_z\beta + \beta(\vec{q}\,')_z\right] + b'\beta(\vec{q}\,')_z \qquad (28)$$

$$\rho' = -\rho_0\alpha\theta \qquad (29)$$

Where $E = \in + \frac{(1-\in)\rho_s C_s}{\rho_0 C_v}$, $L = \left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)$, $k_T = \frac{\chi}{\rho_0 C_v}$ and $b' = \frac{mN_0 C_{Pt}}{\rho_0 C_v}$ – Thermal diffusivity.

Converting equation (25) to (29) to non-dimensional from by the following transform and dropping the strings $x = dx^*, y = dy^*, z = dz^*, \vec{q}\,' = \frac{k_T}{d}\vec{q}^*, P' = \frac{\mu k_T}{d^2}P^*, \vec{v}\,' = \frac{k_T}{d^2}\vec{v}^*, t = \frac{\rho_0 d^2}{\mu}t^*, \nabla = \frac{\nabla^*}{d}, \theta = \beta d\,\theta^*,$ $L^* = \tau\frac{\partial}{\partial t^*} + 1\, and\ \tau = \frac{m\mu}{K'\rho_0 d^2}$, then we have

$$\nabla.\vec{q} = 0 \qquad (30)$$

$$L\frac{1}{\in}\frac{\partial \vec{q}}{\partial t} = L\left[-\nabla P + R\theta\hat{e}_z + \left(1-F\nabla^2\right)\nabla^2\vec{q} - \frac{1}{K_1}\left(1+K\right)\vec{q} + K\left(\nabla\times\vec{v}\right)\right] - \frac{f}{\in}\frac{\partial \vec{q}}{\partial t} \quad (31)$$

$$\bar{J}\frac{\partial \vec{v}\,'}{\partial t} = C_1\nabla\left(\nabla.\vec{v}\right) - C_0\nabla\left(\nabla\times\vec{v}\right) + K\left\{\frac{1}{\in}\left(\nabla\times\vec{q}\right) - 2\vec{v}\right\} \quad (32)$$

$$EP_r\frac{\partial\theta}{\partial t} = \nabla^2\theta + W \quad (33)$$

$$LE_rP_r\frac{\partial\theta}{\partial t} = L\left[\nabla^2\theta - \bar{\delta}\left(\nabla\times\vec{v}\right)_z + \left(\vec{q}\right)_z\right] + b'\left(\vec{q}\right)_z \quad (34)$$

Where $R = \frac{\rho_0 g\alpha\beta d^4}{\mu k_T}$ – Thermal Rayleigh number, $P_r = \frac{\mu}{\rho_0 k_T}$ – Prandtl number, $E_r = E + b'\in$, $f = \frac{mN_0}{\rho_0}, \bar{J} = \frac{J}{d^2}, K_1 = \frac{k_1}{d^2}, \bar{\delta} = \frac{\delta}{\rho_0 C_{v,H} d^2}, C_0 = \frac{\gamma'}{\mu d^2}, C_1 = \frac{\alpha' + \beta' + \gamma'}{\mu d^2}$ and $W = \vec{q}.\hat{e}_z$.

5. Boundary conditions

The boundary condition is $W = \frac{d^2W}{dz^2} = 0, \theta = 0\ at\ z = 0\ and\ z = d. \quad (35)$

6. Dispersion Relation

Taking curl on both side equation (31) then we have

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t} + \left(\frac{1+K}{K_1}\right) - \left(1-F\nabla^2\right)\nabla^2\right\}L + \frac{f}{\in}\frac{\partial}{\partial t}\right]\left(\nabla\times\vec{q}\right) = L\left[R\left(\frac{\partial\theta}{\partial x}\hat{e}_x + \frac{\partial\theta}{\partial x}\hat{e}_y\right) + K\nabla\times\left(\nabla\times\vec{v}\right)\right] \quad (36)$$

$$Let\ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, D = \frac{\partial}{\partial z}, \zeta_z = \left(\nabla\times\vec{q}\right)_z, \xi_z = \left(\nabla\times\vec{q}_d\right)_z\ and\ \Omega_z' = \left(\nabla\times\vec{v}\right)_z$$

Taking curl and z-component on both sides of equation (36) and (32), then we have

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t} + \left(\frac{1+K}{K_1}\right) - \left(1-F\nabla^2\right)\nabla^2\right\}L + \frac{f}{\in}\frac{\partial}{\partial t}\right]\nabla^2 W = L\left[R\nabla_1^2\theta + K\nabla^2\Omega_z'\hat{e}_z\right] \quad (37)$$

$$\bar{J}\frac{\partial\Omega_z'}{\partial t} = C_0\nabla^2\Omega_z' - K\left[\frac{1}{\in}\nabla^2 W + 2\Omega_z'\right] \quad (38)$$

Taking z-component on both sides of equation (34) then we have

$$LE_rP_r\frac{\partial\theta}{\partial t} = L\left[\nabla^2\theta - \bar{\delta}\,\Omega_z' + W\right] + b'W \quad (39)$$

6.1 Normal Mode Analysis

Let $\left[W, \zeta_z, \theta, \Omega_z'\right] = \left[W(z), X(z), \Theta(z), G(z)\right]\exp.\left[i k_x x + i k_y y + \sigma t\right]$

Applying above normal mode analysis to the equation (37) to (39), then we have

$$\left[\left\{\frac{\sigma}{\in} + \left(\frac{1+K}{K_1}\right) + F\left(D^2-a^2\right)^2 - \left(D^2-a^2\right)\right\}\left(1+\tau\sigma\right) + \frac{f}{\in}\sigma\right]\left(D^2-a^2\right)W = \left(1+\tau\sigma\right)\left[-Ra^2\Theta + K\left(D^2-a^2\right)G\right] \quad (40)$$

$$\left[m\sigma + 2A - \left(D^2-a^2\right)\right]G = -\frac{A}{\in}\left(D^2-a^2\right)W \quad (41)$$

$$\left[\left\{E_rP_r\sigma - \left(D^2-a^2\right)\right\}\left(1+\tau\sigma\right)\right]\Theta = \left(1+\tau\sigma\right)\left[-\bar{\delta}G + W\right] + b'W \quad (42)$$

Where $a^2 = k_x^2 + k_y^2$ is the wave number, $\sigma = \sigma_r + i\,\sigma_r$ is the stability parameter, and $m = \frac{\bar{J}A}{K}$, $A = \frac{K}{C_0}$, A is the ratio between the micro-polar viscous effect and micro-polar diffusion effects.

Now the boundary condition becomes

$$W = D^2W = 0 = X = DX = G, \Theta = 0\ at\ z = 0\ to\ z = 1 \quad (43)$$

$D^{2n}W = 0\ at\ z = 0\ to\ z = 1$, Where n is positive integer.

Thus, the proper solution satisfying (43) can be taken as
$W = W_0 \sin \pi z$, W_0 is a constant.

Eliminating Θ *and* G from (30) to (34) and put the value of W & $b = \pi^2 + a^2$, then we have

$$b\left[\left\{\frac{\sigma}{\in}+\left(\frac{1+K}{K_1}\right)+Fb^2+b\right\}(1+\tau\sigma)\right]\left[m\sigma+2A+b\right]\left[E_r P_r \sigma+b\right]$$
$$=Ra^2\left[(1+\tau\sigma)\left((m\sigma+2A+b)-\frac{\bar{\delta}Ab}{\in}\right)+b'(m\sigma+2A+b)\right]+\frac{KAb^2}{\in}(1+\tau\sigma)\left[E_r P_r \sigma+b\right] \qquad (44)$$

7. Stationary Convection

Put them $\sigma = 0$ in equation (44), then we have

$$R=\frac{1}{a^2}\left[\frac{b^2(2A+b)\left(\frac{1+K}{K_1}+Fb^2+b\right)-\frac{KAb^3}{\in}}{\left((2A+b)(1+b')-\frac{\bar{\delta}Ab}{\in}\right)}\right] \qquad (45)$$

To investigate the behavior of medium permeability, suspended particle, couple-stress parameter coupling parameter, micro-polar coefficient and micro-polar heat conduction parameter, we find the nature of $\frac{dR}{dK_1}$, $\frac{dR}{db'}$, $\frac{dR}{dF}$, $\frac{dR}{dK}$, $\frac{dR}{dA}$ and $\frac{dR}{d\bar{\delta}}$ respectively, then

$$\frac{dR}{dK_1}=\frac{-b^2(2A+b)(1+K)}{a^2K_1^2\left((2A+b)(1+b')-\frac{\bar{\delta}Ab}{\in}\right)} \qquad (46)$$

$$\frac{dR}{dK_1}<0 \ if \ \bar{\delta}<\frac{\in b'}{A}$$

From equation (46), we can say that the medium permeability has destabilizing effect when $\bar{\delta}<\frac{\in b'}{A}$.

$$\frac{dR}{db'}=-\frac{(2A+b)\left[b^2\left(\frac{1+K}{K_1}+Fb^2+b\right)(2A+b)-\frac{KAb^3}{\in}\right]}{a^2\left((2A+b)(1+b')-\frac{\bar{\delta}Ab}{\in}\right)^2} \qquad (47)$$

$$\frac{dR}{db'}<0 \ if \left(\frac{1+K}{K_1}+Fb^2+b\right)>\frac{KA}{\in}$$

From equation (47), we can say that the suspended particle has stabilizing effect when the $\left(\frac{1+K}{K_1}+Fb^2+b\right)>\frac{KA}{\in}$.

$$\frac{dR}{dF}=\frac{b^4(2A+b)}{a^2\left((2A+b)(1+b')-\frac{\bar{\delta}Ab}{\in}\right)} \qquad (48)$$

$$\frac{dR}{dF}>0 \ if \ \bar{\delta}<\frac{\in b'}{A}$$

From equation (48), we can say that the couple-stress parameter has stabilizing effect when the $\bar{\delta} < \frac{\in b'}{A}$.

$$\frac{dR}{dK} = \frac{\left[Ab^2 \left(\frac{2}{K_1} - \frac{b}{\in} \right) + \frac{b^3}{K_1} \right]}{a^2 \left((2A+b)(1+b') - \frac{\bar{\delta} Ab}{\in} \right)} \quad (49)$$

$$\frac{dR}{dK} > 0 \ if \ \frac{2}{K} > \frac{b}{\in} \ and \ \bar{\delta} < \frac{\in b'}{A}$$

From equation (49), we can say that the coupling parameter has stabilizing effect when the $\frac{2}{K} > \frac{b}{\in}$ *and* $\bar{\delta} < \frac{\in b'}{A}$.

$$\frac{dR}{dA} = \frac{\frac{b^4}{\in}\left[\bar{\delta} \left(\frac{1+K}{K_1} + Fb^2 + b \right) - K(1+b') \right]}{a^2 \left((2A+b)(1+b') - \frac{\bar{\delta} Ab}{\in} \right)^2} \quad (50)$$

$$\frac{dR}{dA} > 0 \ if \ \bar{\delta} \left(\frac{1+K}{K_1} + Fb^2 + b \right) > K(1+b')$$

From equation (50), we can say that the micro-polar coefficient has stabilizing effect when the $\bar{\delta} \left(\frac{1+K}{K_1} + Fb^2 + b \right) > K(1+b')$.

$$\frac{dR}{d\bar{\delta}} = \frac{Ab}{\in} \frac{\left[b^2 \left(\frac{1+K}{K_1} + Fb^2 + b \right)(2A+b) - \frac{KAb^3}{\in} \right]}{a^2 \left((2A+b)(1+b') - \frac{\bar{\delta} Ab}{\in} \right)^2} \quad (51)$$

$$\frac{dR}{d\bar{\delta}} > 0 \, if \left(\frac{1+K}{K_1} + Fb^2 + b \right) > \frac{KA}{\in}$$

From equation (51), we can say that the micro-polar heat conduction parameter has stabilizing effect when the $\left(\frac{1+K}{K_1} + Fb^2 + b \right) > \frac{KA}{\in}$.

8. Oscillatory convection

Putting $\sigma = i\,\sigma_i$ in equation (44) then we get real & imaginary part and eliminating R between them, then we have

$$f_0\sigma_i^4 + f_1\sigma_i^2 + f_2 = 0$$

Put $s = \sigma_i^2$ then we have $f_0 s^2 + f_1 s + f_2 = 0 \quad (52)$

Where $f_0 = a_1 q_1 - p_1 b_1$

$f_1 = a_2 q_1 - p_2 b_1 - p_1 b_2$

$f_2 = a_3 q_1 - p_2 b_2$

$$a_1 = \frac{E_r P_r m b \tau}{\in}, \; b_1 = -\frac{m a^2 \tau (1+b')}{\in} \; \& \; b_2 = a^2 \left[(2A+b)(1+b') - \frac{\bar{\delta} Ab}{\in} \right]$$

$$a_2 = -\left[\left\{\frac{b\tau}{\in}\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b}{\in}+\frac{fb}{\in}\right\}\left\{(2A+b)E_rP_r+mb\right\}+\left(\frac{1+K}{K_1}+Fb^2+b\right)E_rP_rmb\right]+\frac{2KAb^2E_rP_r}{\in^2}$$

$$a_3 = (2A+b)b^2\left(\frac{1+K}{K_1}+Fb^2+b\right)-\frac{KAb^3}{\in}$$

$$P_1 = -\left[\frac{b\tau}{\in}\left\{(2A+b)E_rP_r+mb\right\}+E_rP_rm\left\{b\tau\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b}{\in}+\frac{fb}{\in}\right\}\right]$$

$$P_2 = \left[(2A+b)b\left\{b\tau\left(\frac{1+K}{K_1}+Fb^2+b\right)+\frac{b}{\in}+\frac{fb}{\in}\right\}+b\left(\frac{1+K}{K_1}+Fb^2+b\right)\left\{(2A+b)E_rP_r+mb\right\}\right]-\frac{KAb^2}{\in}(b\tau+E_rP_r)$$

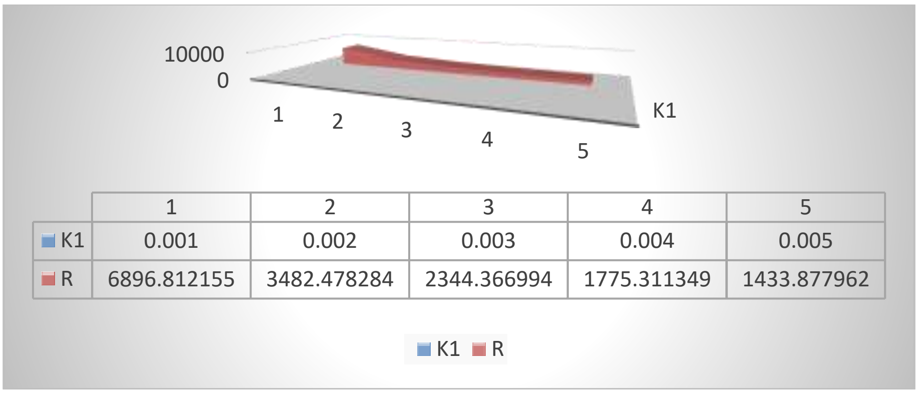
$$q_1 = a^2\left[\tau(2A+b)(1+b')+m(1+b')-\frac{\bar{\delta}Ab\tau}{\in}\right]$$

From (52), we observed that $s=\sigma_i^2$ which is always positive, therefore the sum of roots equation of (52) is positive but this is impossible if $f_0>0$ and $f_1>0$, the sum of roots of equation (52) is $-f_1/f_0$ Thus, $f_0>0$ and $f_1>0$ are the sufficient condition for the non-existence of over stability.

Now $f_0>0$ and $f_1>0$ when $\bar{\delta}<\frac{\in b'}{A}, KE_rP_r<2, K<2\in bF \text{ and } KA\tau<m\in$.

9. Numerical Calculation

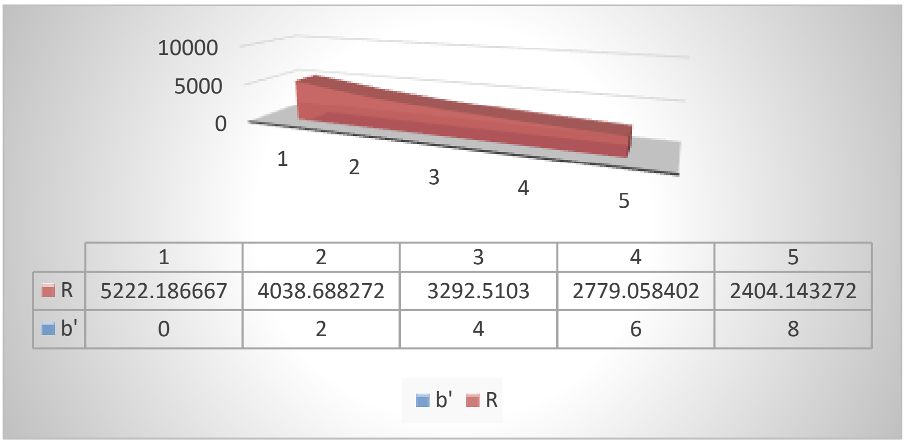
Now we show numerically the effect of medium permeability, rotation, coupling parameter, micro-polar coefficient and micro-polar heat conduction coefficient.



	1	2	3	4	5
K1	0.001	0.002	0.003	0.004	0.005
R	6896.812155	3482.478284	2344.366994	1775.311349	1433.877962

Figure1

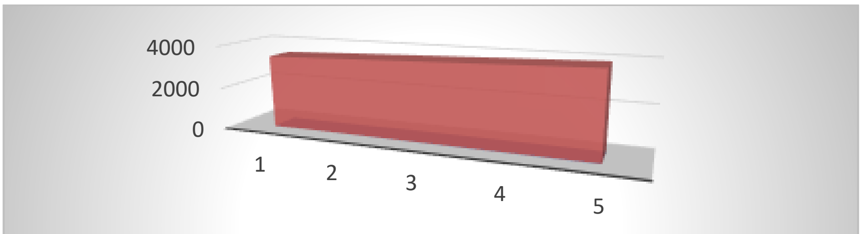
Where $E_r=1, P_r=2, \in=0.5, A=0.1, F=2, K=0.2, b'=3 \text{ and } \bar{\delta}=0.05$.



	1	2	3	4	5
R	5222.186667	4038.688272	3292.5103	2779.058402	2404.143272
b'	0	2	4	6	8

Figure 2

Where $E_r = 1, \mathrm{P}_r = 2, \in = 0.5, A = 0.1, F = 2, K = 0.2, K_1 = 0.002$ *and* $\bar{\delta} = 0.05.$

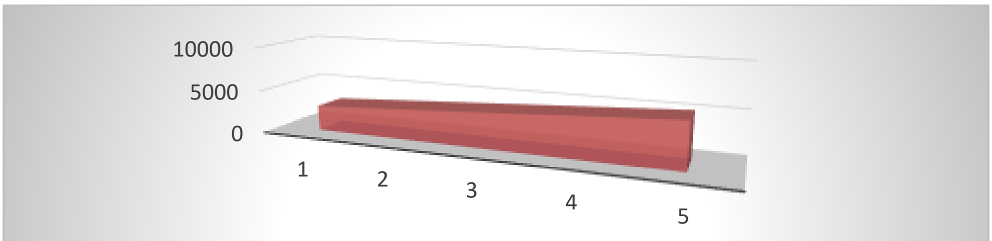


	1	2	3	4	5
R	3436.953833	3550.764962	3664.576091	3778.38722	3892.198349
F	0	5	10	15	20

F R

Figure 3

Where $E_r = 1, \mathrm{P}_r = 2, \in = 0.5, A = 0.1, b' = 3, K = 0.2, K_1 = 0.002$ *and* $\bar{\delta} = 0.05.$

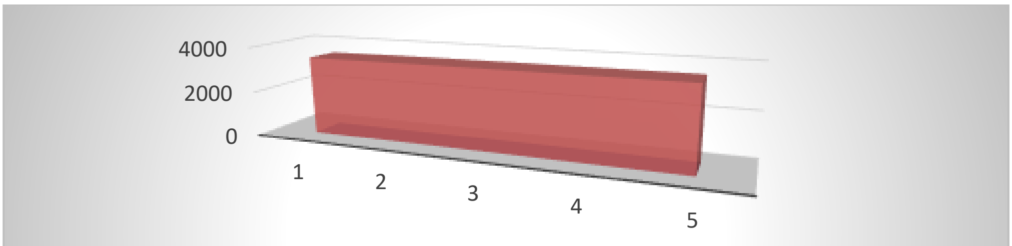


	1	2	3	4	5
R	2913.564903	3482.478284	4051.391665	4620.305047	5189.218428
K	0	0.2	0.4	0.6	0.8

K R

Figure 4

Where $E_r = 1, \mathrm{P}_r = 2, \in = 0.5, A = 0.1, b' = 3, F = 2, K_1 = 0.002$ *and* $\bar{\delta} = 0.05.$



	1	2	3	4	5
R	3467.206226	3496.066842	3519.196374	3538.147736	3553.959274
A	0	0.2	0.4	0.6	0.8

A R

Figure 5

Where $E_r = 1, \mathrm{P}_r = 2, \in = 0.5, K = 0.2, b' = 3, F = 2, K_1 = 0.002$ *and* $\bar{\delta} = 0.05.$

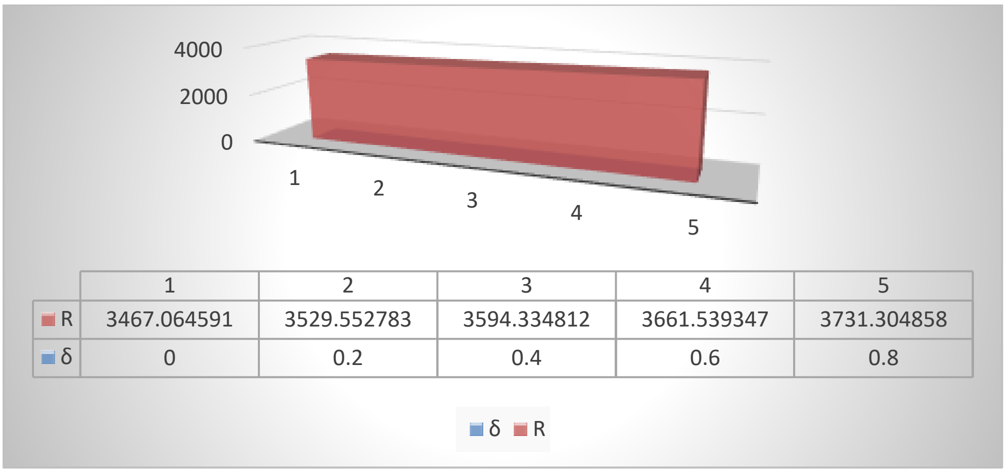


Figure 6

Where $E_r = 1, P_r = 2, \in = 0.5, K = 0.2, b' = 3, F = 2, K_1 = 0.002 \; and \; A = 0.1.$

10. Conclusions

A. For Stationary Convection

- ❖ $\frac{dR}{dK_1} < 0$, Thus the effect of medium permeability is destabilizing.
- ❖ $\frac{dR}{db'} < 0$, Thus the effect of suspended particle is destabilizing.
- ❖ $\frac{dR}{dF} > 0$, Thus the effect of couple-stress parameter is stabilizing.
- ❖ $\frac{dR}{dK} > 0$, Thus the effect of coupling parameter is stabilizing.
- ❖ $\frac{dR}{dA} > 0$, Thus the effect of micro-polar coefficient is stabilizing.
- ❖ $\frac{dR}{d\bar{\delta}} > 0$, Thus the effect of micro-polar heat conduction is stabilizing.

B. For Oscillatory Convection

The sufficient condition for the non-existence of over stability

$$\bar{\delta} < \frac{\in b'}{A}, KE_r P_r < 2, K < 2 \in bF \; and \; KA\tau < m \in .$$

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