Dynamics of One-Phytoplankton Two-Zooplankton System with Square Root Functional Response and Time Lag

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Abstract: In this paper, the effects of Phytoplankton and two competing Zooplanktons are studied using a mathematical model. The associated state variables are Phytoplankton H, Zooplankton C_1 and Zooplankton C_2. The assumption is that if the ratio of mortality rates of Zooplankton to the radical of the carrying volume of the Phytoplankton inhabitants is larger than their respective conversion efficiency and predation rates, the Phytoplankton inhabitants will survive as well as Zooplankton inhabitants will tend to extinction. The addition of delay disrupted the system's interior, axial and boundary equilibrium, and at the critical point of the delay parameter, Hopf bifurcation occurs. Sensitivity analysis is performed on to the model parameters also. MATLAB is used to assist analytical findings with numerical simulation.

Keywords: Phytoplankton, Zooplankton, Toxic Material, Hopf bifurcation, Stability, Interior Equilibrium.

Introduction

Many animals establish groups in nature, and they also move in groups from one location to another. Zooplanktons benefit from the development of groups (herds) because it improves their effectiveness at foraging and reduces their danger of predation. As a result, the herd's actions prevent the extinction of Zooplankton, which group together to protect themselves from predators. One of the most fascinating population dynamics phenomena is the aggregation of Zooplanktons. Many authors' models have taken into account various functional responses of Phytoplankton-Zooplankton forming groups Cosner et al. (1999) and Venturino (2011). Ajraldi et al. (2011) investigated that employing a two-breed system where the members of first breed live in groups while those belonging to the other breed live alone. They've explained everything from competitiveness to symbiosis to predation in populations. Limit cycles emerge naturally in the Phytoplankton-Zooplankton interaction Braza (2012). Beretta and Kuang, (1998) also looked at the Zooplankton-Phytoplankton model with square root functional response, and found in that community behavior more in the area of the emergence than other typical models that don't include herd behavior. Due to the square root word, this makes ecological sense.

Time lags occur in practically every biological scenario and are responsible for regular changes in demography, therefore models including delay are more realistic. Many writers have conducted comprehensive investigations involving time delay in a variety of biological systems Cushing (1977); Gopalswamy (1992); Kuang (1993); MacDonald (1976); Wangersky and Cunningham (1957); Chakraborty et al. (2011); Ajraldi et al. (2011). Many different forms of functional responses have been

examined in Phytoplankton-Zooplankton models Kar and Matsuda (2007); Khare et al. (2011); Holt and Lawto (1994).

The zooplankton in the natural environment is not solely dependent on the density of phytoplankton, but also on the presence and quantity of the host species as examined by Srinivasu and Prasad (2010); Srinivasu et al. (2007); Merfield et al. (2004).Several studies have demonstrated the importance of the host-commensal interaction in the survival or extinction of a variety of organisms as specified by Nouhuys and Kraft (2012); Vargas- Leon and Alcaraz (2013); Wang (2013); Zhang (2012); Bhattacharyya and Pal (2013). In the real world, harvesting has led to the extinction of some species, and as a result, other species that rely on them have suffered as well. Many publications have been published in recent years analyzing Zooplankton-Phytoplankton models with harvesting Ghosh and Kar (2013); Yuan and Pei (2013); Zhang et al. (2013); Khan et al. (2021); Chenilla et al. (2021); Ruan and Wei (2001).

Mathematical Model

The current work arose from theoretical and experimental findings on the interplay of hazardous algal blooms with various types of phytoplankton-zooplankton interactions. Following system of differential equations governs the dynamics.

$$\frac{dH}{dT} = \gamma H \left(1 - \frac{H(T-\tau)}{K} \right) - \widehat{\beta_1}\sqrt{H}C_1 - \widehat{\beta_2}\sqrt{H}C_2 \tag{1}$$

$$\frac{dC_1}{dT} = \widehat{\alpha_1}\widehat{\beta_1}\sqrt{H}C_1 - \widehat{d_1}C_1 \tag{2}$$

$$\frac{dC_2}{dT} = \widehat{\alpha_2}\widehat{\beta_2}\sqrt{H}C_2 - \widehat{d_2}C_2 - \hat{i}C_2{}^2 \tag{3}$$

with initial conditions $H(0) > 0, C_1(0) > 0, C_2(0) > 0$.

We use following for non- dimentionalising the equations:

$\hat{h} = \frac{H}{S}, y_1 = \frac{C_1\widehat{\beta_1}}{\sqrt{S}}, y_2 = \frac{C_2\widehat{\beta_2}}{\gamma\sqrt{S}}, t = \frac{\gamma T}{2}$ and setting out the new variables such as $k_1 = \frac{2\widehat{\alpha_1}\widehat{\beta_1}\sqrt{S}}{\gamma}$

, $k_2 = \frac{2\widehat{\alpha_2}\widehat{\beta_2}\sqrt{S}}{\gamma}, d_1 = \frac{\widehat{d_1}}{\widehat{\alpha_1}\widehat{\beta_1}\sqrt{S}}, d_2 = \frac{\widehat{d_2}}{\widehat{\alpha_2}\widehat{\beta_2}\sqrt{S}}, d_3 = \frac{\widehat{d_3}}{\widehat{\alpha_1}\widehat{\beta_2}{}^2}.$

The non- dimentionalised system of equations are as follows:

$$\frac{d\hat{h}}{dt} = \hat{h}\left(1 - \hat{h}(t-\tau) \right) - \sqrt{\hat{h}}y_1 - \sqrt{\hat{h}}y_2, \tag{4}$$

$$\frac{dy_1}{dt} = k_1 y_1 \left(\sqrt{\hat{h}} - d_1 \right) \tag{5}$$

$$\frac{dy_2}{dt} = k_2 y_2 \left(\sqrt{\hat{h}} - d_2 - d_3 y_2 \right) \tag{6}$$

Parameter	Description

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		Table 1.
$\hat{h}$	Density of Phytoplankton	**Descriptio**
y_1	Density of First Zooplankton	**n of**
y_2	Density of Second Zooplankton	**parameter**
$\widehat{d_\iota}$	Natural mortality rate	**s.**
$\widehat{\beta_\iota}$	Predatoriness rates	
$\hat{k}$	rate of intraspecies competition	
γ	Phytoplankton's intrinsic growth rate	
S	Carrying capacity	
$\widehat{\alpha_\iota}$	Rate of conversion	

The indicant i may have a rational value of 1 or 2.

To better analyze it, we let $\hat{h} = h^2$,

$$\frac{dh}{dt} = h\left(1 - h^2(t - \tau)\right) - y_1 - y_2 \tag{7}$$

$$\frac{dy_1}{dt} = k_1 y_1(h - d_1) \tag{8}$$

$$\frac{dy_2}{dt} = k_2 y_2(h - d_2 - d_3 y_2) \tag{9}$$

with initial conditions h (0) >0, y_1(0) >0, y_2(0) >0.

Equilibrium of the Model

The points of balance of the system (7)-(9) are given by:

$$h\left(1 - h^2(t - \tau)\right) - y_1 - y_2 = 0 \tag{10}$$

$$k_1 y_1(h - d_1) = 0 \tag{11}$$

$$k_2 y_2(h - d_2 - d_3 y_2) = 0 \tag{12}$$

On solving (10)-(12), following five points of balance or equilibrium for the system (7)-(9) are obtained:

1. The origin $E_0 = (0,0,0)$.

2. The axial equilibrium point $E_A = (1,0,0)$.

3. The boundary equilibrium points $E_{B_1} = \left(d_1, d_1\left(1 - d_1{}^2\right), 0\right)$ and $E_{B_2} = (\tilde{h}, 0, \widetilde{y_2})$

where $\widetilde{y_2} = \tilde{h}(1 - \tilde{h}^2)$ and $\tilde{h}$ is the positive root of the cubic equation

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$$d_3\tilde{h}^3 + (1 - d_3)\tilde{h} - d_2 = 0. \tag{13}$$

The boundary equilibrium point E_{B_1} exists if $d_1 < 1$ and boundary equilibrium point E_{B_2} exists if $\tilde{h} < 1$.

4. The interior equilibrium point $E^* = (x_1{}^*, y_1{}^*, y_2{}^*)$ where $x_1{}^* = d_1, y_2{}^* = \frac{d_1 - d_2}{d_3}$ and $y_1{}^* = d_1(1 - d_1{}^2) - \frac{d_1 - d_2}{d_3}$. The interior equilibrium point E^* exists if $d_1 > d_2$ and $d_1 d_3 + d_2 > d_1(1 + d_1{}^2 d_3)$.

Dynamical actions when $\tau = 0$

We will now investigate the system's dynamical behaviour in relation to each of the five possible equilibria. The system's variational matrix (7)-(9) is

$$V = \begin{bmatrix} 1 - 3h^2 & -1 & -1 \\ k_1 y_1 & k_1(h - d_1) & 0 \\ k_2 y_2 & 0 & k_2(h - d_2 - 2d_3 y_2) \end{bmatrix}$$

The characteristic V at the equilibrium point equation$E_0$is

$$(1 - \lambda)(k_1 d_1 + \lambda)(k_1 d_1 + \lambda)$$

The V-specific characteristic equation at E_A is
$$(2 + \lambda)(k_1(1 - d_1 + \lambda)(k_1 d_1 + \lambda) = 0.$$

The eigenvalues at E_A are -2, $k_1(1 - d_1)$ and $k_2(1 - d_2)$, $d_1 > 1$ and $d_2 > 1$.
If the equilibrium point E_A is stable then further the pointsE_{B_1}, E_{B_2} and E^*do not exist.
If E_{B_1}, E_{B_2} and E^*exist then point E_A is not stable.
The point E_{B_1} will have the equation:

$$(k_2(d_1 - d_2) - \lambda)(\lambda^2 + (3d_1{}^2 - 1)\lambda + k_1 d_1) = 0. \tag{14}$$

The points at E_{B_1} for (7)-(9) are locally asymptotically stable if $d_2 > d_1$ and $d_1 > \frac{1}{\sqrt{3}}$.

Also, if E_{B_1} is stable then E^* does not exist else it stable.
The point E_{B_2}will have the equation:

$$(k_1(\tilde{h} - d_1) - \lambda)(\lambda^2 + (3\tilde{x}^2 + k_2 d_3 \tilde{y}_2 - 1)\lambda + k_2 \tilde{y}_2 \tag{15}$$

The points at E_{B_2} for (7)-(9) are locally asymptotically stable if $d_1 > \tilde{h}$ and $\tilde{h} > \frac{1}{\sqrt{3}}$.

The point E^* will have the equation:

$$\lambda^3 + A_1 \lambda^2 + A_2 \lambda + A_3 = 0 \tag{16}$$

$$A_1 = k_2 d_3 y_2{}^* + 3d_1{}^2 - 1$$

$$A_2 = (3d_1{}^2 - 1)k_2 d_3 y_2{}^* + k_2 y_2{}^* + k_1 y_1{}^*$$

$$A_3 = k_1 k_2 y_1{}^* d_3 y_2{}^*$$

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Dynamical actions when $\tau > 0$

The characteristic equation for (7)-(9) around at any equilibrium point is

$$\lambda^3 + P_1\lambda^2 + P_2\lambda + P_3 + e^{-\lambda\tau}(Q_1\lambda^2 + Q_2\lambda + Q_3) = 0 \qquad (17)$$

Where

$P_1 = -1 + k_1d_1 + k_2d_2 - k_1h - k_2h + h^2 + 2k_2d_3y_2, Q_1 = 2h^2,$

$P_2 = -k_1d_1 - k_2d_2 + k_1k_2d_1d_2 + k_1h + k_2h - k_1k_2d_1h - k_1k_2d_2h + k_1k_2h^2 + k_1d_1h^2 + k_2d_2h^2 - k_1h^3 - k_2h^3 + k_1y_1 + k_2y_2 - 2k_2d_3y_2 + 2k_1k_2d_1d_3y_2 - 2k_1k_2d_3hy_2 + 2k_2d_3h^2y_2$

$Q_2 = 2k_1d_1h^2 + 2k_2d_2h^2 - 2k_1h^3 - 2k_2h^3 + 4k_2d_3h^2y_2,$

$P_3 = -k_1k_2d_1d_2 + k_1k_2d_1h + k_1k_2d_2h - k_1k_2h^2 + k_1k_2d_1d_2h^2 - k_1k_2d_1h^3 - k_1k_2d_2h^3 + k_1k_2h^4 + k_1k_2d_2y_1 - k_1k_2hy_1 + k_1k_2d_1y_2 - 2k_1k_2d_1d_3y_2 - k_1k_2hy_2 + 2k_1k_2d_3hy_2 + 2k_1k_2d_1d_3h^2y_2 - 2k_1k_2d_3h^3y_2 + 2k_1k_2d_3y_1y_2.$

$Q_3 = 2k_1k_2d_1d_2h^2 - 2k_1k_2d_1h^3 - 2k_1k_2d_2h^3 + 2k_1k_2h^4 + 4k_1k_2d_1d_3h^2y_2 - 4k_1k_2d_3x^3y_2.$

All the eigenvalues of the characteristic equation (17) should have a negative real part in order for the equilibrium point to be stable. It is challenging to determine the circumstances in which all of equation (17)'s roots will have a negative real part. When $\tau = 0$, equation (17) changes to

$$\lambda^3 + (P_1 + Q_1)\lambda^2 + (P_2 + Q_2)\lambda + (P_3 + Q_3) = 0 \qquad (18)$$

By Routh-Hurwitz criterion,

If $(P_1 + Q_1) > 0, (P_3 + Q_3) > 0, (P_1 + Q_1)(P_2 + Q_2) > (P_3 + Q_3)$, then all roots of equation (10) will have negative real parts.

If we consider $\lambda = 0$ to be solution of (18), then $(P_3 + Q_3) = 0$.Thus this condition contradicts the second condition. Hence $\lambda = 0$ is not a solution of (18). Assume that for some $\tau \geq 0, i\omega$ with $\omega > 0$ is a solution of (18), so

$$-i\omega^3 - P_1\omega^2 + iP_2\omega + P_3 + (cos\omega\tau - isin\omega\tau)(-Q_1\omega + iQ_2\omega + Q_3) = 0. \qquad (19)$$

Putting the fictional and actual components apart,

$$P_3 - P_1\omega^2 + (Q_3 - Q_1\omega^2)cos\omega\tau + Q_2\omega sin\omega\tau = 0 \qquad (20)$$

$$P_2\omega - \omega^3 + Q_2\omega cos\omega\tau - (Q_3 - Q_1\omega^2)sin\omega\tau = 0 \qquad (21)$$

which leads to $\omega^6 + m\omega^4 + n\omega^2 + s = 0 \qquad (22)$

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Where

$$m = P_1{}^2 - Q_1{}^2 - 2P_2, n = P_2{}^2 - Q_2{}^2 + 2Q_1Q_3 - 2P_1P_3, s = P_3{}^2 - Q_3{}^2.$$

Let $u = \omega^2$, then equation (22) becomes

$$u^3 + mu^2 + nu + s = 0 \tag{23}$$

Lemma 1: For the polynomial equation (23), we have the following results:
   (1)   If $s < 0$, then equation (15) has at least one positive root.
   (2)   If $s \geq 0$ and $(m^2 - 3n) \leq 0$, then equation (23) has no positive root.
   (3)   If $s \geq 0$ and $(m^2 - 3n) > 0$, then equation (23) has positive roots if and
            only if $u_1{}^* = \frac{-m+\sqrt{m^2-3n}}{3} > 0$ and $h(u_1{}^*) \leq 0$.

Lemma2: Assume $h(u_0) = (3u_0{}^2 + 2mu_0 + n) \neq 0$ and the conditions in (y_1) are satisfied. For (j=0,1...), denote $\lambda(\tau) = \alpha(\tau) + i\omega(\tau)$ be the root of equation (24) satisfying $\alpha(\tau_j) = 0, \omega(\tau_j) = \omega_0$, where

$$\tau_j = \frac{1}{\omega_0}\left(\arccos\frac{-\left(Q_2\omega_0{}^2(P_2-\omega_0{}^2)+(P_3-P_1\omega_0{}^2)(Q_3-Q_1\omega_0{}^2)\right)}{(Q_3-Q_1\omega_0{}^2)^2+(Q_2Q_3-Q_1\omega_0)^2} + 2j\Pi\right)$$

Then $\pm i\omega_0$ are simple roots. If the transversality condition

$$(y_2)\ \alpha'(\tau_j) = \left.\frac{Re\lambda(\tau)}{d\tau}\right|_{\lambda=i\omega_0} \neq 0$$

holds, a Hopf bifurcation occurs for the system (7)-(9) at any equilibrium point and

$$\tau = \tau_j$$

The proof of Lemmas have been conveyed in the publication given by Ruan, S. & Wei; On the Zeros of a Third-Degree Exponential Polynomial with Applications to A Delayed Model for The Control of Testosterone Secretion, IMA J. Math. Appl. Medic. Biol., 18, pp. 41-52.

Numerical Representation

We use numerical simulations run in Matlab to support all of the prior analytical conclusions. Regarding the group of parameters, $k_1 = 1, k_2 = 1, d_3 = 0.58$, the parameter-dependent stability areas; d_1 and d_2. Plots showing the system's equilibrium points are shown. Here, the starting point is (.41,.11,.11).
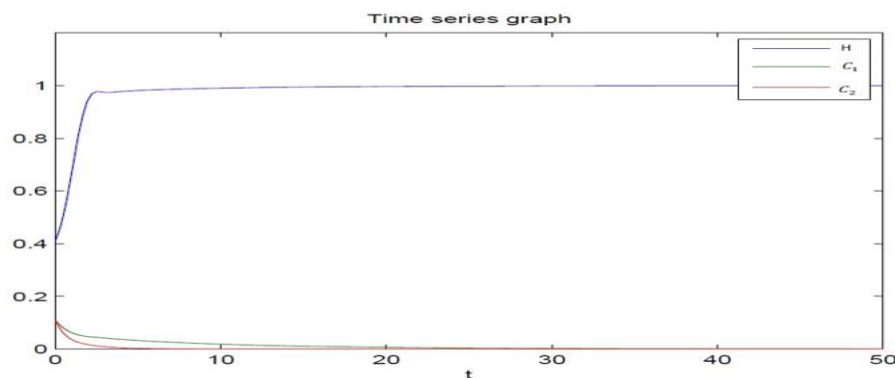

Figure 1: For $d_1 = .76, d_2 = 1.6$ and $E^* = (0.7600, 0.0452, 0.2760)$,
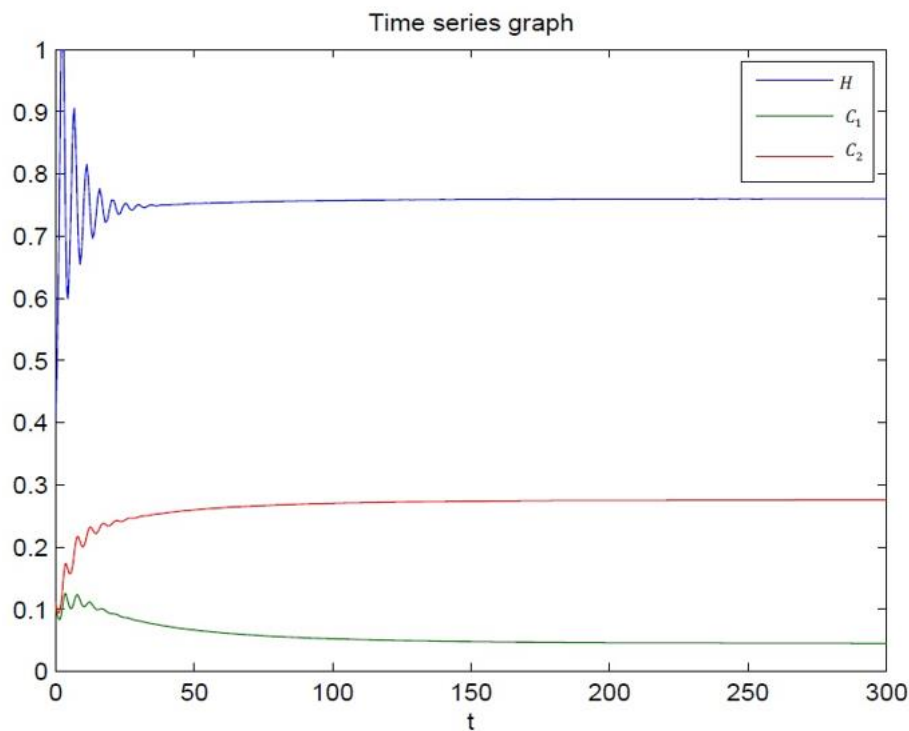
the system exhibits asymptotic stability.

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Figure2: For $d_1 = .76, d_2 = .6 \ and \ E^* = (0.7600 0.0452 \ 0.2760), \ \tau = 0.95 \ and \tau_0 = 0.920973.$ If $\tau < \tau_0$, the system exhibits asymptotic stability.
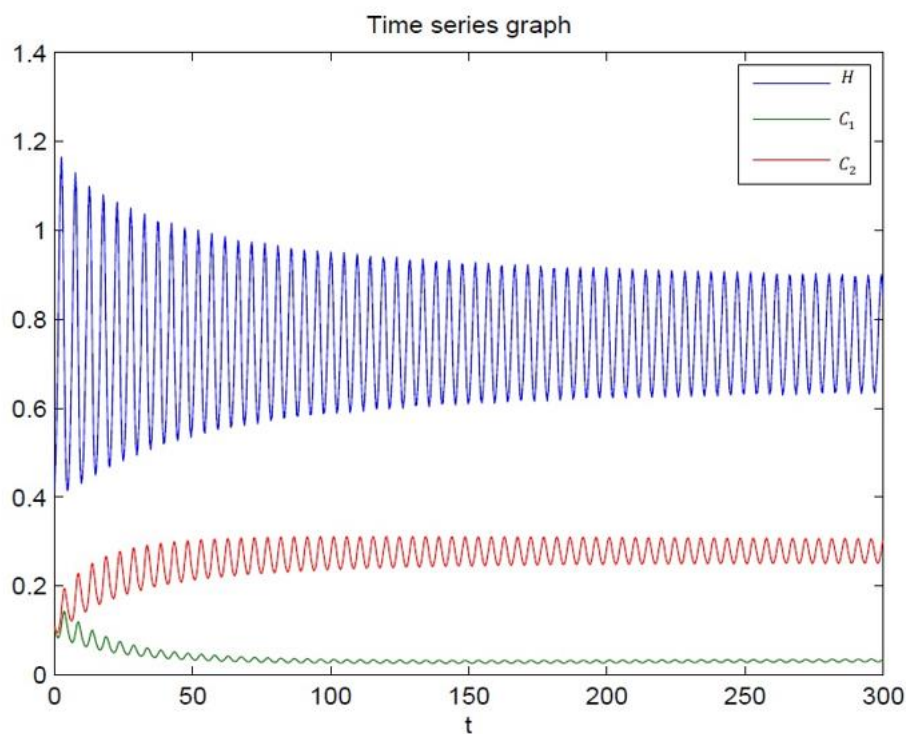

Figure 3: For $d_1 = .76, d_2 = 0.6 \ and \ E^* = (0.7600 0.0452 \ 0.2760), \ \tau = .924 \ and \tau_0 = 0.920973.$ If $\tau > \tau_0$, the system becomes unstable,

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a Hopf-bifurcation takes place.

Similar result have been exhibited using Boundary and Axial Equilibrium. It is evident based on numerical simulation for a particular set of parameters that when only the system's Phytoplankton population survives, the delay is shortest; when all sysqa a aatem species survive, it is longest.

Analysis of the State Variables' Sensitivity to the Model's Parameters

The model in this study has constant values. For approximation of the general coefficients, the "Direct Method" is utilized as conveyed in the publication given by Rihan. Here the All parameters are taken to be constants, and the original framework is used to simultaneously solve sensitivity equations. Then doing partial differentiation of the solution relating to each parameter may be all that is required for sensitivity analysis in this scenario if all of the parameters that appear in the system model (7)-(9) are assumed to be constants.

The solution (H, C_1, C_2) of partial differentiation with respect to t, for example, result in the following set of sensitivity equations:

$$\frac{\partial S_1}{\partial t} = (K_h - K_h C_2) - \alpha H(t - \tau)C_1 - \delta_1 H$$

$$\frac{\partial S_2}{\partial t} = \beta H(t - \tau)y - \delta_2 C_2$$

$$\frac{\partial S_3}{\partial t} = I - \gamma H C_1 - \delta_3 C_2$$

Where $S_1 = \frac{\partial H}{\partial \widehat{d_1}}, S_2 = \frac{\partial C_1}{\partial \widehat{d_1}}$ and $S_3 = \frac{\partial C_2}{\partial \widehat{d_1}}$.

Variable Sensitivity to Parameter $\widehat{d_1}$

Figures 4 demonstrates that as we decrease the value of $\widehat{d_1}$ from 0.9 to 0.5, here is slight variation and shift in the state variables' values concentration of H and C_1, which ultimately remain constant and tend to zero. It predicts that state variables H and C_1 will be less sensitive to the parameter $\widehat{d_1}$. But for the same range of $\widehat{d_1}$ values, the state variable C_2 experiences a significant change. It displays a decline in the delayed value. It also continues to be stable.
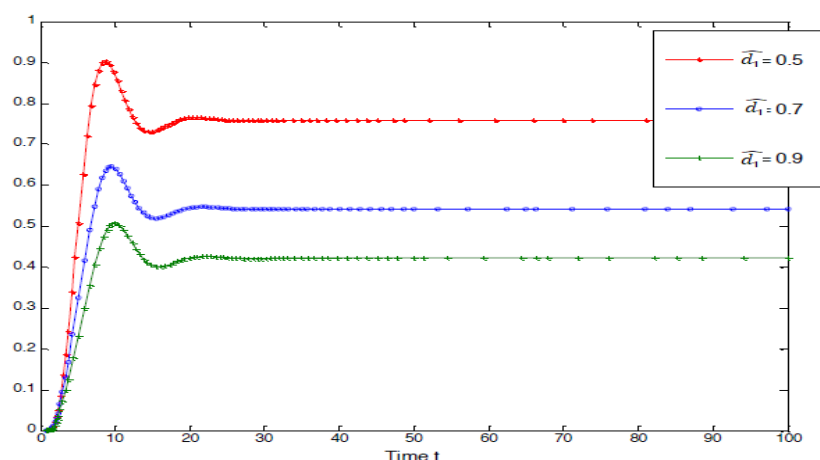

Figure 4: **An illustration of the time series between minor variations for various values of coefficient $\widehat{d_1}$**

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Hence, we conclude that with increase in mortality rate overall population of Phytoplankton and Zooplankton is decreasing.

Conclusion

The system exhibits asymptotic stability around the positive interior equilibrium, which represents the coexistence of all species, for a specific threshold of the system parameters, according to both our theoretical and numerical findings. Further, it is determined from the analysis of the equilibrium points' stability that when the ratio of a Zooplankton's mortality rate along with the multiplication of its conversion effective performance and spoliation is more than the radical of one-third of the Phytoplankton inhabitants' carrying capacity, the system is said to be coexisting. Limit cycles arise at all symmetry points when the time delay crosses a threshold value after taking the effective time delay on the body. This study also uses the direct method to examine the responsiveness of model solutions to modifications of delay differential system parameters. It is demonstrated that the sensitivity functions allow one to identify particular parameters and enhance one's comprehension of the role that particular model parameters play. The oscillation and value changes that go along with state variable sensitivity to parameter changes indicate the parameter is essential to the model and that the solution is sensitive to changes in the parameter. The state variable nutrient concentration H is the parameter that is least sensitive, according to sensitivity analysis. While rates of plant biomass decrease with a decline in the delayed value of utilization coefficient and become unstable, rates of plant biomass increase with a decline in the delayed value of consumption coefficient and remain stable.

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