Minimizing the waiting time control chart for the Multi server Markovian queuing Encouraged arrival with infinite capacity model

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Abstract:

In this research, numerous commercial scenarios involving customers have seen the use of queuing theory. In order to foster client loyalty, the business offers a service facility and works to reduce expenses and time to a minimum. To find out how many clients there are and how long they wait, a service facility has to be studied. A control chart approach may be used to evaluate the effectiveness of concerns and the services they provide. A control chart that was created to track the length of time spent in the system gives customers peace of mind by indicating the minimum, maximum, and expected system size and reduction in waiting time. So we have created of a control chart for an MSMQEA (Multi Server Markovian Queuing Encouraged Arrival) with unlimited capacity is presented in this study. Numerical results, simulation and Little's law also verified.

Keywords: encouraged arrival; quality control feedback; balking; maintaining; retention

1.Introduction

Queuing or waiting in line is a typical circumstance that happens in daily life. Bus stations, hospitals, bank desks, train reservation counters and other places frequently have queues of people waiting to enter. In general, lines develop when more people want service than there are available. The number of customers (humans or objects) in the queue, Service providers are growing increasingly concerned about the number of time customers spend in lines to obtain services.

When a system requires too much capacity from service providers, they lose money. But when there is insufficient capacity, customers wait for an inordinate amount of time. Predicting and lowering wait times, as well as modifying staffing, can help find the best option. Shewhart created control chart approaches based on information from one or more quality-related product or service attributes to determine if a manufacturing process or service is producing things that meet predetermined quality criteria.

The control chart approach is appropriate for the study of system usage time.[1] provided several examples of how control charts can be used to ensure quality in manufacturing sectors.[2] created a control chart by taking into account, the first three minutes for a random queue length, N of M/M/1queueing model. [3] created the control chart using the weighted variance approach for the M/M/1queueing model's random queue length N. [4] used a control chart to assess the waiting times in a system of the M/M/1 queuing paradigm. Using EWMA Statistics, [5] constructed Mixed Control Charts. [6] used a control chart to analyse the M/M/s queuing model's waiting time system.[7] Created an optimal design of an improved X and R control chart. [9,10] An M/M/1/N queuing system with encouraged arrivals has been studied.[11] Explored in Batch Encouraged Arrival Markovian Model Analysis.

The amount of time spent matters more than the number of people in the queue when it comes to how they are sorted for facilities. The Shewhart control chart for the M/M/s queuing model waiting time is constructed in this article. This paradigm is useful in a variety of industries, including machine assembly, aviation, ATM hubs and supermarkets where the system has several parallel

2. Description of the Multi-server Markovian queue with encouraged arrival (MSMQEA model):

The M/M/s model includes s parallel-arranged servers, each of which has an equal service time that follows the same exponential rule. Each of the open counters is available to serve customers. With First Come, First Serve (FCFS) queue management, the system has limitless capacity. The clients show up in an enthusiastic manner, with a mean encouraged arrival rate $\lambda * (1+\zeta)$, where is the percentage change in the number of customers determined from the prior or clear vision. For instance, if a company had previously offered discounts and there had been a 10% to 20% shift in the number of customers, then ζ = 0.1 to 0.2, respectively. and a mean accelerate service rate μ.

2.1 Equations of a Stable State

The likelihood that n consumers are in the system (waiting and being served) at time t is denoted by $p_k(t)$.

The following two scenarios might occur if k clients are present in the queueing system at any one time:

(I) If k<s, then there won't be a line and (s-k) servers won't be overloaded. The total service rate is going to be μk = k μ, k < s.

(II) If k ≥s, then every server will be occupied, and there will be a limit of (k-s) consumers waiting in line. Then μk = s μ, k ≥ s.

The model's governing equations are

$$\left.\begin{array}{c} p_0(t+\psi t) = p_0(t)(1-\lambda*(1+\zeta)\psi t) + p_1(t)\mu\psi t + o(\psi t) k=0 \\ p_k(t+\psi t) = p_k(t)(1-\lambda*(1+\zeta)+k\mu)\psi t) + p_{k-1}(t)\lambda*(1+\zeta)\psi t + p_{k+1}(t)(k+1)\mu\psi t + o(\psi t), k<s \\ and\ p_k(t+\psi t) = p_k(t)(1-(\lambda*(1+\zeta)+s\mu) + p_{k-1}(t)\lambda*(1+\zeta)\psi t + p_{k+1}(t)s\mu\psi t + o(\psi t), k\geq s \end{array}\right\}$$
$$---(1)$$

Equation (1) gives

$$\left.\begin{array}{c} p_0'(t) = -\lambda * (1+\zeta)p_0(t) + \mu p_1(t), k = 0 \\ p_k'(t) = -(\lambda * (1+\zeta) + k\mu)p_k(t) + \lambda * (1+\zeta)p_{k-1}(t) + (k+1)\mu p_{k+1}(t), k < s \\ and p_k'(t) = -(\lambda * (1+\zeta) + s\mu)p_k(t) + \lambda * (1+\zeta)p_{k-1}(t) + s\mu p_{k+1}(t), k \geq s \end{array}\right\} - (2)$$

The equations for stable states that correspond to (2) are

$$\left.\begin{array}{c} 0 = -\lambda * (1+\zeta)p_0 + \mu p_1, k = 0 \\ 0 = -\lambda * (1+\zeta) + k\mu)p_k + \lambda * (1+\zeta)p_{k-1} + (k+1)\mu p_{k+1}, k < s \\ 0 = -(\lambda * (1+\zeta) + s\mu)p_k + \lambda * (1+\zeta)p_{k-1} + s\mu p_{k+1}, k \geq s \end{array}\right\} ------(3)$$

Let $\frac{\lambda*(1+\zeta)}{s\mu}$ be the level of traffic. From equation (3)

$$p_n = \begin{cases} \frac{1}{k!}\left(\lambda * (1+\zeta)/_{\mu}\right)^k p_0, k < s \\ \frac{1}{s!}\left(1/_{s^{k-s}}\right)\left(\lambda * (1+\zeta)/_{\mu}\right)^k, k \geq s \end{cases} ----------------(4)$$

Where

$$p_0 = \left\{\sum_{k=0}^{s-1} \frac{\left(s\frac{\lambda * (1+\zeta)}{s\mu}\right)^k}{k!} + \frac{\left(\frac{\lambda * (1+\zeta)^s}{s\mu}\right)}{s!\left(1 - \frac{\lambda * (1+\zeta)}{s\mu}\right)}\right\}^{-1}$$

2.2 Evaluating performance

(I) E(L_q)= Average number of clients in the line

$=\sum_{k=s}^{\infty}(k-s)p_0$

$$= \frac{\frac{\lambda*(1+\zeta)}{s\mu}\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^s}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^2}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^k}{k!} + \frac{\left(\frac{\lambda*(1+\zeta)^s}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1} -----------(5)$$

(II) E(L_s)= Average consumer count in the system

$$= \frac{\frac{\lambda*(1+\zeta)}{s\mu}\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^s}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^2}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^k}{k!} + \frac{\left(\frac{\lambda*(1+\zeta)^s}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1} + \frac{\lambda*(1+\zeta)}{\mu} ------(6)$$

(III) $E(W_q)$= Average time spent by a consumer in line

$$=\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^s}{s!s\mu\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^2}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^k}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^s}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1} ------------ (7)$$

(IV) $E(W_s)$= Average client wait time in the system

$$=\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^s}{s!s\mu\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^2}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^k}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^s}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}+\frac{1}{\mu}-----------(8)$$

3.Waiting time control chart, W

Let W stand for a customer's total wait time in the system, which consists of both the wait time and the service time. The random variable's pdf as described by Gross and Harris [8] is

$$g(t)=\frac{1}{\left(s-s\frac{\lambda*(1+\zeta)}{s\mu}-1\right)}\left[\left(s(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)-1+\right.$$

λ∗1+ζsμsss!1−λ∗1+ζsμk=0s−1sλ∗1+ζsμkk!+λ∗1+ζsμss!1−λ∗1+ζsμ−1μe−μt−λ∗1 +ζsμsss!1−λ∗1+ζsμk=0s−1sλ∗1+ζsμkk!+λ∗1+ζsμss!1−λ∗1+ζsμ−1(sμ−λ∗(1+ζ)e−(sμ−λ∗1+ζt,t>0

$$E(W)=\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^s}{s!s\mu\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^2}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^k}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^s}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}+\frac{1}{\mu}$$

$$Var(W)=\frac{1}{\mu^2}\left[2\left(1+\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}s\right)^s\left(\mu(S+1)-\lambda*(1+\zeta)\right)}{\mu s!s^2\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^3}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^k}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^s}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}\right)-\right.$$

1+λ∗1+ζsμ(s)ss!s1−λ∗(1+ζ)sμ2k=0s−1sλ∗(1+ζ)sμkk!+λ∗(1+ζ)sμss!1−λ∗(1+ζ)sμ −12

When a normal distribution is used to approximate the statistic under consideration, the parameters of Shewhart type control charts are given by

Upper Control Limit= $\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^{s}}{s!s\mu\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^{2}}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^{k}}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^{s}}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}+$

$$\frac{1}{\mu}+3\sqrt{\frac{1}{\mu^{2}}\left[2\left(1+\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}s\right)^{s}(\mu(S+1)-\lambda*(1+\zeta))}{\mu s!s^{2}\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^{3}}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^{k}}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^{s}}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}\right)-\left(1+\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^{s}}{s!s\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^{2}}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^{k}}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^{s}}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}\right)^{2}\right]}$$

Central Limit= $\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^{s}}{s!s\mu\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^{2}}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^{k}}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^{s}}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}+\frac{1}{\mu}$

Lower Control Limit= $\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^{s}}{s!s\mu\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^{2}}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^{k}}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^{s}}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}+\frac{1}{\mu}-$

$$3\sqrt{\frac{1}{\mu^{2}}\left[2\left(1+\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}s\right)^{s}(\mu(S+1)-\lambda*(1+\zeta))}{\mu s!s^{2}\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^{3}}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^{k}}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^{s}}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}\right)-\left(1+\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^{s}}{s!s\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^{2}}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^{k}}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^{s}}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}\right)^{2}\right]}$$

The parameters of the control chart for the customer's wait time and system size for the M/M/s encouraged arrival queuing model are given by

Upper Control Limit= $\frac{1}{\mu}\left[\left(1+\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^{s}}{s!s\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^{2}}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^{k}}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^{s}}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}\right)+\right.$

321+λ∗1+ζsμssμS+1−λ∗1+ζμs!s21−λ∗1+ζsμ3k=0s−1sλ∗1+ζsμkk!+λ∗1+ζsμss!1−λ∗1+ζsμ−1−1+λ∗1+ζsμ(s)ss!s1−λ∗(1+ζ)sμ2k=0s−1sλ∗(1+ζ)sμkk!+λ∗(1+ζ)sμss!1−λ∗(1+ζ)sμ−1212

$$\text{Central Limit}=\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^{s}}{s!s\mu\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^{2}}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^{k}}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^{s}}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}+\frac{1}{\mu}$$

$$\text{Lower Control Limit}=\frac{1}{\mu}\left[\left(1+\frac{\left(\frac{\lambda*(1+\zeta)}{s\mu}(s)\right)^{s}}{s!s\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)^{2}}\left\{\sum_{k=0}^{s-1}\frac{\left(s\frac{\lambda*(1+\zeta)}{s\mu}\right)^{k}}{k!}+\frac{\left(\frac{\lambda*(1+\zeta)^{s}}{s\mu}\right)}{s!\left(1-\frac{\lambda*(1+\zeta)}{s\mu}\right)}\right\}^{-1}\right)-\right.$$

321+λ∗1+ζsμssμS+1−λ∗1+ζμs!s21−λ∗1+ζsμ3k=0s−1sλ∗1+ζsμkk!+λ∗1+ζsμss!1−λ∗1+ζsμ−1−1+λ∗1+ζsμ(s)ss!s1−λ∗(1+ζ)sμ2k=0s−1sλ∗(1+ζ)sμkk!+λ∗(1+ζ)sμss!1−λ∗(1+ζ)sμ−1212

4.Numerical illustration

Now in order to show the use of a control chart we provide numerical examples by finding the system size and waiting time for a few specially selected values of $\lambda * (1+\zeta)$ and μ.ζ represents offered value 0.1 (or 10%). The parameters of the control chart are shown in Tables 1, 2, and 3 for various values of the arrival rate, a constant service rate of 6, and the number of servers, s, which are, respectively, 2, 4, and 6.

Table.1 lists the control chart's parameters for the values s = 2, μ= 6 and ζ=10%

S.no	$\boldsymbol{\lambda * (1+\zeta)}$	$\boldsymbol{\mu}$	$\boldsymbol{\rho}$	s	Control limit 10%discounts	Upper control limit 10%discounts
1	1.1	6	0.0917	2	0.1681	0.6706
2	1.65	6	0.1375	2	0.1669	0.6761
3	2.2	6	0.1833	2	0.1725	0.6846
4	2.75	6	0.2292	2	0.1759	0.6968
5	3.3	6	0.2750	2	0.1803	0.7135
6	3.85	6	0.3208	2	0.1858	0.7358
7	4.4	6	0.3667	2	0.1926	0.7651
8	4.95	6	0.4125	2	0.2008	0.8031
9	5.5	6	0.4583	2	0.2110	0.8523
10	6.05	6	0.5042	2	0.2235	0.9159
11	6.60	6	0.5500	2	0.2389	0.9982
12	7.15	6	0.5958	2	0.2584	1.1055
13	7.7	6	0.6417	2	0.2833	1.2473
14	8.25	6	0.6875	2	0.3160	1.4382
15	8.8	6	0.7333	2	0.3606	1.7030

Table.1a lists the little's law 10% discounts verification values

S. No	$\lambda * (1 + \zeta)$	L_q	L_s	W_q	W_s	$L_q = \lambda * (1+\zeta) W_q$	$L_s = \lambda * (1+\zeta)\, W_s$
1	1.1	0.0016	0.1849	0.0014	0.1681	0.0016	0.1849
2	1.65	0.0053	0.2803	0.0032	0.1699	0.0053	0.2803
3	2.2	0.0128	0.3794	0.0058	0.1725	0.0128	0.3794
4	2.75	0.0254	0.4837	0.0092	0,1759	0.0254	0.4837
5	3.3	0.0450	0.5950	0.0136	0.1803	0.0450	0.5950
6	3.85	0.0736	0.7153	0.0191	0.1858	0.0736	0.7153
7	4.4	0.1139	0.8472	0.0259	0.1926	0.1139	0.8472
8	4.95	0.1692	0.9942	0.0342	0.2008	0.1692	0.9942
9	5.5	0.2438	1.1604	0.0443	0.2110	0.2438	1.1604
10	6.05	0.3437	1.3520	0.0568	0.2235	0.3437	1.3520
11	6.60	0.4771	1.5771	0.0723	0.2389	0.4771	1.5771
12	7.15	0.6559	1.8476	0.0917	0.2584	0.6559	1.8476
13	7.7	0.8982	2.1816	0.1167	0.2833	0.8982	2.1816
14	8.25	1.2324	2.6074	0.1494	0.3160	1.2324	2.6074
15	8.8	1.7064	3.1731	0.1939	0.3606	1.7064	3.1731

Figure:1 We offer visual representations and 10 percent arrival, s = 2, μ= 6 Control Diagram

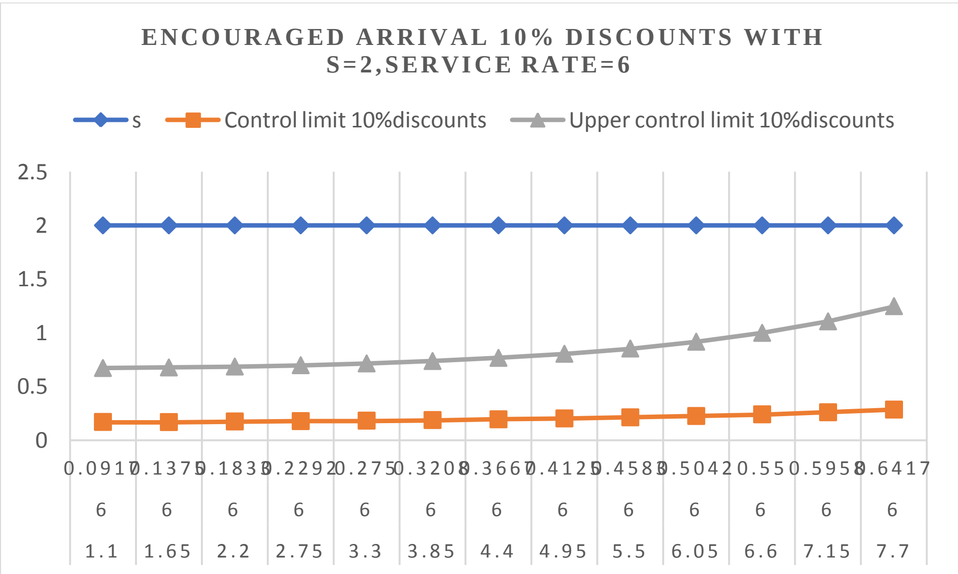


Table.2 lists the control chart's parameters for the values s = 4, μ= 6 and ζ=10%

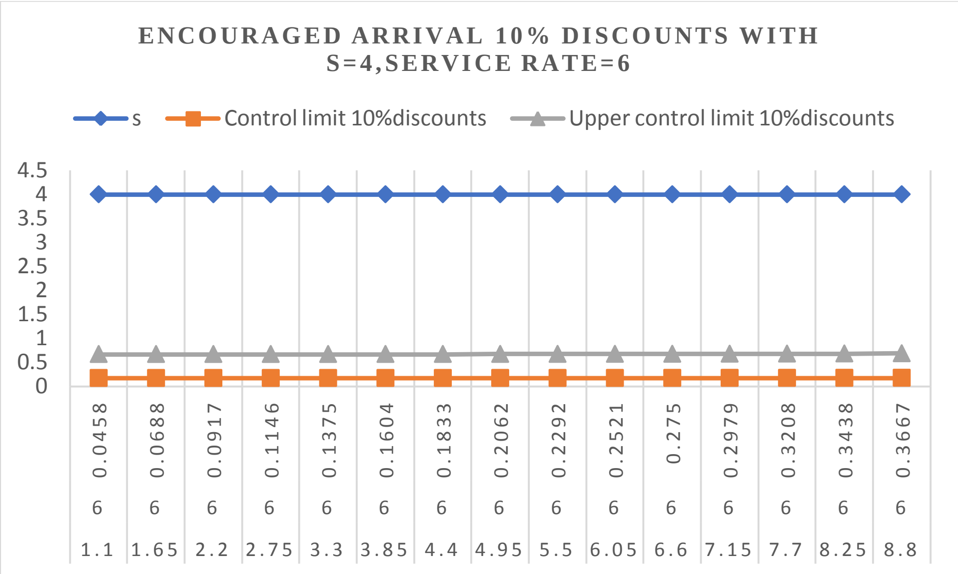
S.no	$\lambda * (1+\zeta)$	μ	ρ	S	Control limit 10%discounts	Upper control limit 10%discounts
1	1.1	6	0.0458	4	0.1667	0.6667
2	1.65	6	0.0688	4	0.1667	0.6667
3	2.2	6	0.0917	4	0.1667	0.6667

4	2.75	6	0.1146	4	0.1667	0.6668
5	3.3	6	0.1375	4	0.1668	0.6669
6	3.85	6	0.1604	4	0.1669	0.6672
7	4.4	6	0.1833	4	0.1670	0.6675
8	4.95	6	0.2062	4	0.1672	0.6681
9	5.5	6	0.2292	4	0.1675	0.6688
10	6.05	6	0.2521	4	0.1678	0.6699
11	6.6	6	0.2750	4	0.1683	0.6713
12	7.15	6	0.2979	4	0.1688	0.6731
13	7.7	6	0.3208	4	0.1695	0.6755
14	8.25	6	0.3438	4	0.1703	0.6785
15	8.8	6	0.3667	4	0.1712	0.6823

Table.2a lists the little's law 10% discounts verification values

S. No	$\boldsymbol{\lambda} * (\mathbf{1} + \zeta)$	$\boldsymbol{L_q}$	$\boldsymbol{L_s}$	$\boldsymbol{W_q}$	$\boldsymbol{W_s}$	$\boldsymbol{L_q} = \boldsymbol{\lambda} * (\mathbf{1} + \zeta)\boldsymbol{W_q}$	$\boldsymbol{L_s} = \boldsymbol{\lambda} * (\mathbf{1} + \zeta)\ \boldsymbol{W_s}$
1	1.1	0.0000	0.1833	0.0000	0.1667	0.0000	0.1833
2	1.65	0.0000	0.2750	0.0000	0.1667	0.0000	0.2750
3	2.2	0.0001	0.3667	0.0000	0.1667	0.0001	0.3667
4	2.75	0.0002	0.4585	0.0001	0.1667	0.0002	0.4585
5	3.3	0.0004	0.5504	0.0001	0.1668	0.0004	0.5504
6	3.85	0.0008	0.6425	0.0002	0.1669	0.0008	0.6425
7	4.4	0.0016	0.7349	0.0004	0.1670	0.0016	0.7349
8	4.95	0.0028	0.8278	0.0006	0.1672	0.0028	0.8278
9	5.5	0.0045	0.9212	0.0008	0.1675	0.0045	0.9212
10	6.05	0.0071	1.0154	0.0012	0.1678	0.0071	1.0154
11	6.60	0.0106	1.1106	0.0016	0.1683	0.0106	1.1106
12	7.15	0.0154	1.2070	0.0022	0.1688	0.0154	1.2070
13	7.7	0.0217	1.3050	0.0028	0.1695	0.0217	1.3050
14	8.25	0.0299	1.4049	0.0036	0.1703	0.0299	1.4049
15	8.8	0.0403	1.5070	0.0046	0.1712	0.0403	1.5070

Figure:2



We offer visual representations and 10 percent arrival, s = 4, μ= 6 Control Diagram

Table.3 lists the control chart's parameters for the values s = 6, μ= 6 and ζ=10%

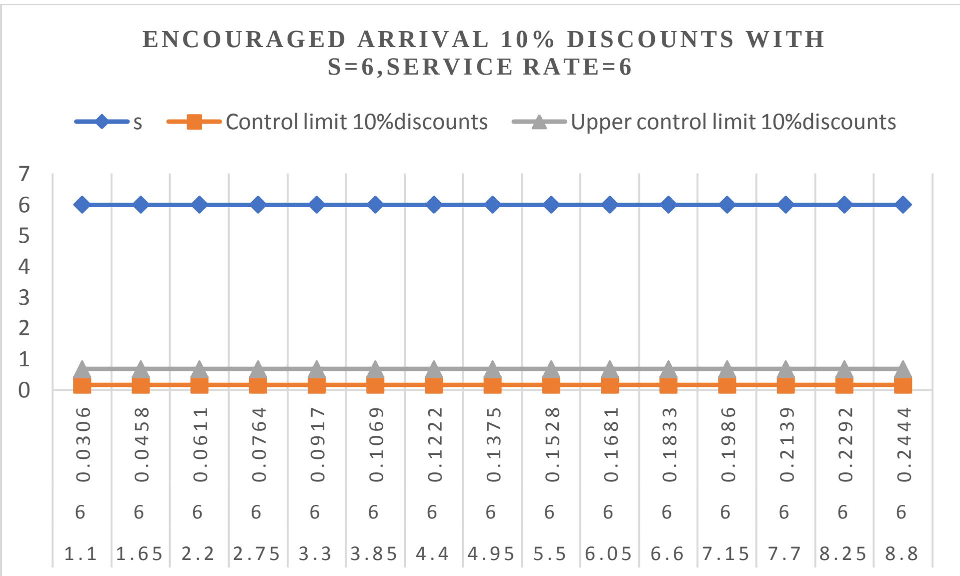
S.no	$\lambda * (1+\zeta)$	μ	ρ	s	Control limit 10%discounts	Upper control limit 10%discounts
1	1.1	6	0.0306	6	0.1667	0.6667
2	1.65	6	0.0458	6	0.1667	0.6667
3	2.2	6	0.0611	6	0.1667	0.6667
4	2.75	6	0.0764	6	0.1667	0.6667
5	3.3	6	0.0917	6	0.1667	0.6667
6	3.85	6	0.1069	6	0.1667	0.6667
7	4.4	6	0.1222	6	0.1667	0.6667
8	4.95	6	0.1375	6	0.1667	0.6667
9	5.5	6	0.1528	6	0.1667	0.6667
10	6.05	6	0.1681	6	0.1667	0.6667
11	6.6	6	0.1833	6	0.1667	0.6667
12	7.15	6	0.1986	6	0.1667	0.6668
13	7.7	6	0.2139	6	0.1667	0.6668
14	8.25	6	0.2292	6	0.1668	0.6669
15	8.8	6	0.2444	6	0.1668	0.6670

Table.3a lists the little's law 10% discounts verification value

S. No	$\lambda * (1+\zeta)$	L_q	L_s	W_q	W_s	$L_q = \lambda * (1+\zeta) W_q$	$L_s = \lambda * (1+\zeta)\, W_s$
1	1.1	0.0000	0.1833	0.0000	0.1667	0.0000	0.1833
2	1.65	0.0000	0.2750	0.0000	0.1667	0.0000	0.2750

3	2.2	0.0000	0.3667	0.0000	0.1667	0.0000	0.3667
4	2.75	0.0000	0.4583	0.0000	0.1667	0.0000	0.4583
5	3.3	0.0000	0.5500	0.0000	0.1667	0.0000	0.5500
6	3.85	0.0000	0.6417	0.0000	0.1667	0.0000	0.6417
7	4.4	0.0000	0.7333	0.0000	0.1667	0.0000	0.7333
8	4.95	0.0000	0.8250	0.0000	0.1667	0.0000	0.8250
9	5.5	0.0001	0.9167	0.0000	0.1667	0.0001	0.9167
10	6.05	0.0001	1.0085	0.0000	0.1667	0.0001	1.0085
11	6.60	0.0002	1.1002	0.0000	0.1667	0.0002	1.1002
12	7.15	0.0004	1.1920	0.0001	0.1667	0.0004	1.1920
13	7.7	0.0006	1.2839	0.0001	0.1667	0.0006	1.2839
14	8.25	0.0009	1.3759	0.0001	0.1668	0.0009	1.3759
15	8.8	0.0014	1.4680	0.0002	0.1668	0.0014	1.4680

Figure:3



We offer visual representations and 10 percent arrival, s = 6, μ= 6 Control Diagram

Remark:1

The following details are revealed by the numerical values of Tables and figures 1, 2 and :

- increase in the encouraged arrival rate. The average waiting time and the anticipated maximum waiting time increase by 10% with discounts and constant service rates.
- increasing arrival rate When there are more servers, there is a 10% reduction in both the average wait time and the expected maximum wait time. This model provides more efficient waiting and service times when compared to the Poisson arrival [6].

Tables and figures4, 5 and 6 provide the control chart's parameters for a constant arrival rate discount of 10% , as well as various values for the service rate and the number of servers (s = 2, 4, and 6).

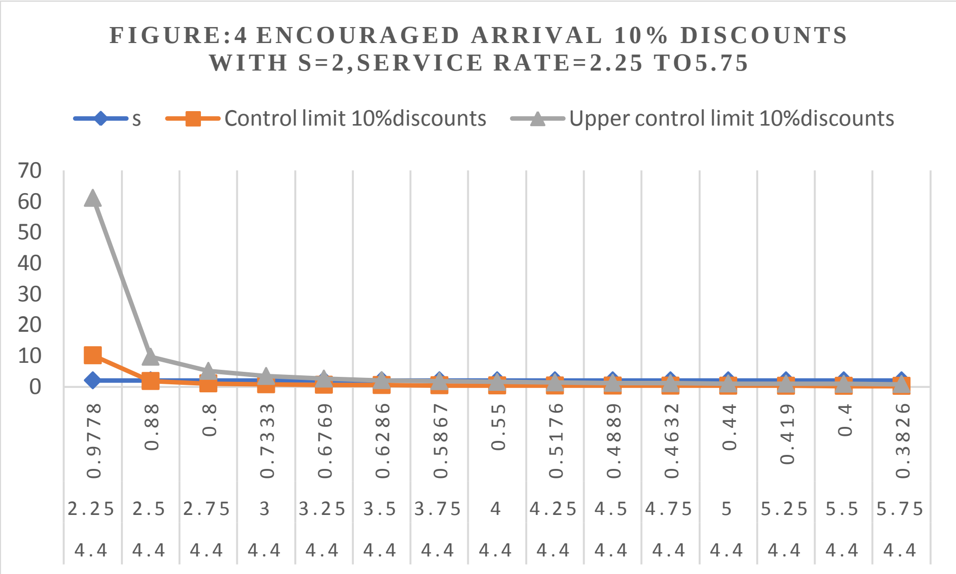
Table.4 lists the control chart's parameters for the values s = 2, $\lambda * (1+\zeta)$= 4.4 and ζ=10%

S.no	$\lambda * (1+\zeta)$	μ	ρ	s	Control limit 10%discounts	Upper control limit 10%discounts
1	4.4	2.25	0.9778	2	10.1124	61.1255
2	4.4	2.50	0.8800	2	1.7730	9.6415
3	4.4	2.75	0.8000	2	1.0101	5.0684
4	4.4	3.00	0.7333	2	0.7212	3.4061
5	4.4	3.25	0.6769	2	0.5679	2.5637
6	4.4	3.50	0.6286	2	0.4723	2.0615
7	4.4	3.75	0.5867	2	0.4066	1.7306
8	4.4	4.00	0.5500	2	0.3584	1.4973
9	4.4	4.25	0.5176	2	0.3214	1.3241
10	4.4	4.50	0.4889	2	0.2920	1.1905
11	4.4	4.75	0.4632	2	0.2680	1.0841
12	4.4	5.00	0.4400	2	0.2480	0.9973
13	4.4	5.25	0.4190	2	0.2310	0.9250
14	4.4	5.50	0.4000	2	0.2165	0.8637
15	4.4	5.75	0.3826	2	0.2037	0.8110

Table.4a lists the little's law 10% discounts verification values

S. No	$\lambda * (1+\zeta)$	L_q	L_s	W_q	W_s	$L_q = \lambda * (1+\zeta) W_q$	$L_s = \lambda * (1+\zeta) W_s$
1	4.4	42.5388	44.4944	9.6679	10.1124	42.5388	44.4944
2	4.4	6.0414	7.8014	1.3730	1.7730	6.0414	7.8014
3	4.4	2.8444	4.4444	0.6465	1.0101	2.8444	4.4444
4	4.4	1.7064	3.1731	0.3878	0.7212	1.7064	3.1731
5	4.4	0.1451	2.4989	0.2602	0.5679	0.1451	2.4989
6	4.4	0.8211	2.0783	0.1866	0.4723	0.8211	2.0783
7	4.4	0.6158	1.7891	0.1399	0.4099	0.6158	1.7891
8	4.4	0.4771	1.5771	0.1084	0.3584	0.4771	1.5771
9	4.4	0.3790	1.4143	0.0861	0.3214	0.3790	1.4143
10	4.4	0.3071	1.2849	0.0698	0.2920	0.3071	1.2849
11	4.4	0.2530	1.1793	0.0575	0.2680	0.2530	1.1793
12	4.4	0.2113	1.0913	0.0480	0.2480	0.2113	1.0913
13	4.4	0.1785	1.0166	0.0406	0.2310	0.1785	1.0166
14	4.4	0.1524	0.9524	0.0346	0.2165	0.1524	0.9524
15	4.4	0.1312	0.8964	0.0298	0.2037	0.1312	0.8964

Figure:4



We offer visual representations and 10 percent arrival, s=2, μ=2.25 to 5.75 Control Diagram

Table.5 lists the control chart's parameters for the values s = 4, $\lambda * (1+\zeta)$= 4.4 and ζ=10%

S.no	$\boldsymbol{\lambda} * (\mathbf{1}+\boldsymbol{\zeta})$	$\boldsymbol{\mu}$	$\boldsymbol{\rho}$	s	Control limit 10%discounts	Upper control limit 10%discounts
1	4.4	2.00	0.5500	4	0.5630	2.3091
2	4.4	2.25	0.4889	4	0.4799	1.9321
3	4.4	2.50	0.4400	4	0.4215	1.6849
4	4.4	2.75	0.4000	4	0.3774	1.5047
5	4.4	3.00	0.3667	4	0.3425	1.3647
6	4.4	3.25	0.3385	4	0.3140	1.2512
7	4.4	3.50	0.3143	4	0.2902	1.1567
8	4.4	3.75	0.2933	4	0.2699	1.0763
9	4.4	4.00	0.2750	4	0.2524	1.0069
10	4.4	4.25	0.2588	4	0.2371	0.9462
11	4.4	4.50	0.2444	4	0.2236	0.8927
12	4.4	4.75	0.2316	4	0.2116	0.8450
13	4.4	5.00	0.2200	4	0.2009	0.8022
14	4.4	5.25	0.2095	4	0.1912	0.7636
15	4.4	5.50	0.2000	4	0.1824	0.7236

Table.5a lists the little's law 10% discounts verification values

S. No	$\boldsymbol{\lambda} * (\mathbf{1}+\zeta)$	$\boldsymbol{L_q}$	$\boldsymbol{L_s}$	$\boldsymbol{W_q}$	$\boldsymbol{W_s}$	$\boldsymbol{L_q} = \boldsymbol{\lambda} * (\mathbf{1}+\zeta)\boldsymbol{W_q}$	$\boldsymbol{L_s} = \boldsymbol{\lambda} * (\mathbf{1}+\zeta)\, \boldsymbol{W_s}$
1	4.4	0.2772	2.4772	0.0630	0.5630	0.2772	2.4772
2	4.4	2.1116	2.1116	0.0355	0.4799	2.1116	2.1116
3	4.4	1.8546	1.8546	0.0215	0.4215	1.8546	1.8546
4	4.4	1.6605	1.6605	0.0137	0.3774	1.6605	1.6605
5	4.4	0.0403	1.5070	0.0092	0.3425	0.0403	1.5070

6	4.4	0.0278	1.3816	0.0063	0.3140	0.0278	1.3816
7	4.4	0.0197	1.2768	0.0045	0.2902	0.0197	1.2768
8	4.4	0.0143	1.1876	0.0033	0.2699	0.0143	1.1876
9	4.4	0.0106	1.1106	0.0024	0.2524	0.0106	1.1106
10	4.4	0.0080	1.0433	0.0018	0.2371	0.0080	1.0433
11	4.4	0.0061	0.9839	0.0014	0.2236	0.0061	0.9839
12	4.4	0.0048	0.9311	0.0011	0.2116	0.0048	0.9311
13	4.4	0.0037	0.8837	0.0009	0.2009	0.0037	0.8837
14	4.4	0.0030	0.8411	0.0007	0.1912	0.0030	0.8411
15	4.4	0.0024	0.8024	0.0005	0.1824	0.0024	0.8024

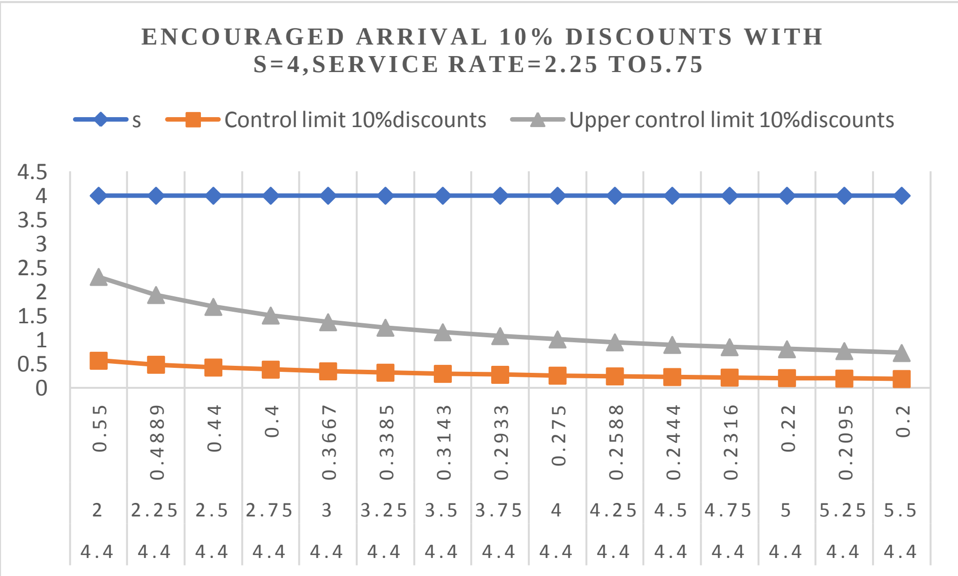


Figure:5

We offer visual representations and 10 percent arrival, s=4, μ=2.25 to 5.75 Control Diagram

Table.6 lists the control chart's parameters for the values s = 6, $\lambda * (1+\zeta)$= 4.4 and ζ=10%

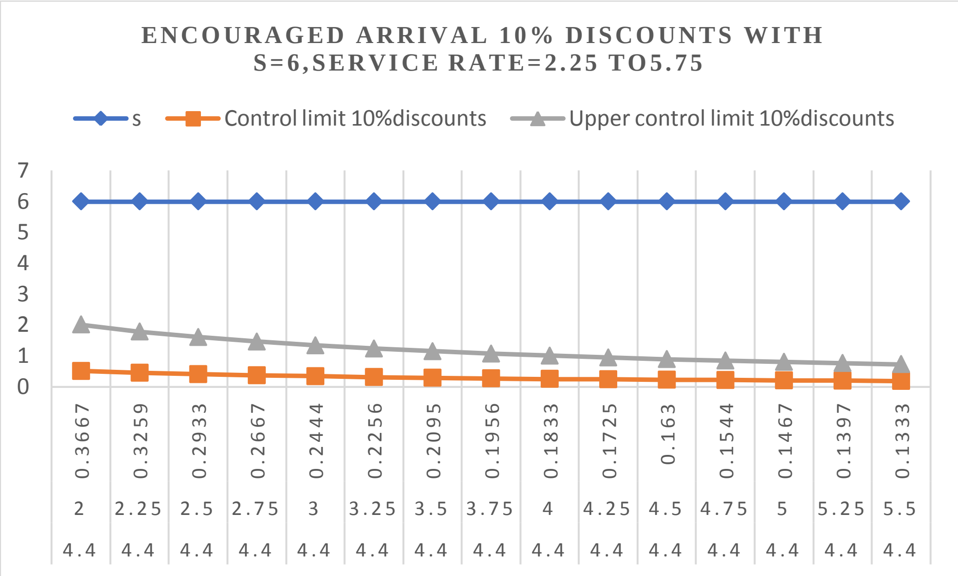
S.no	$\boldsymbol{\lambda} * (\mathbf{1}+\boldsymbol{\zeta})$	$\boldsymbol{\mu}$	$\boldsymbol{\rho}$	s	Control limit 10%discounts	Upper control limit 10%discounts
1	4.4	2.00	0.3667	6	0.5036	2.0116
2	4.4	2.25	0.3259	6	0.4462	1.7830
3	4.4	2.50	0.2933	6	0.4009	1.6026
4	4.4	2.75	0.2667	6	0.3642	1.4559
5	4.4	3.00	0.2444	6	0.3336	1.3341
6	4.4	3.25	0.2256	6	0.3079	1.2312
7	4.4	3.50	0.2095	6	0.2858	1.1431
8	4.4	3.75	0.1956	6	0.2667	1.0668
9	4.4	4.00	0.1833	6	0.2501	1.0001
10	4.4	4.25	0.1725	6	0.2353	0.9413

11	4.4	4.50	0.1630	6	0.2222	0.8889
12	4.4	4.75	0.1544	6	0.2105	0.8421
13	4.4	5.00	0.1467	6	0.2000	0.8000
14	4.4	5.25	0.1397	6	0.1905	0.7619
15	4.4	5.50	0.1333	6	0.1818	0.7273

Table.6a lists the little's law 10% discounts verification values

S. No	$\boldsymbol{\lambda} * (\mathbf{1} + \zeta)$	$\boldsymbol{L_q}$	$\boldsymbol{L_s}$	$\boldsymbol{W_q}$	$\boldsymbol{W_s}$	$\boldsymbol{L_q} = \boldsymbol{\lambda} * (\mathbf{1} + \zeta)\boldsymbol{W_q}$	$\boldsymbol{L_s} = \boldsymbol{\lambda} * (\mathbf{1} + \zeta)\, \boldsymbol{W_s}$
1	4.4	0.0159	2.2159	0.0036	0.5036	0.0159	2.2159
2	4.4	0.0079	1.9634	0.0018	0.4462	0.0079	1.9634
3	4.4	0.0042	1.7642	0.0009	0.4009	0.0042	1.7642
4	4.4	0.0023	1.6023	0.0005	0.3642	0.0023	1.6023
5	4.4	0.0014	1.4680	0.0003	0.3336	0.0014	1.4680
6	4.4	0.0008	1.3547	0.0002	0.3079	0.0008	1.3547
7	4.4	0.0005	1.2577	0.0001	0.2858	0.0005	1.2577
8	4.4	0.0003	1.1737	0.0001	0.2667	0.0003	1.1737
9	4.4	0.0002	1.1002	0.0001	0.2501	0.0002	1.1002
10	4.4	0.0002	1.0354	0.0000	0.2353	0.0002	1.0354
11	4.4	0.0001	0.9779	0.0000	0.2222	0.0001	0.9779
12	4.4	0.0001	0.9264	0.0000	0.2105	0.0001	0.9264
13	4.4	0.0001	0.8801	0.0000	0.2000	0.0001	0.8801
14	4.4	0.0000	0.8381	0.0000	0.1905	0.0000	0.8381
15	4.4	0.0000	0.8000	0.0000	0.1818	0.0000	0.8000

Figure:6



We offer visual representations and 10 percent arrival, s=6, μ=2.25 to 5.75 Control Diagram

Remark:2

The following results are found from Tables and figures 4,5 and 6:

- The average waiting time and the anticipated maximum waiting time both decrease with an increase in service rate and constant encouraged arrival rate.
- The average wait time and anticipated maximum wait time are reduced with an increase in servers.
- This model's arrival procedure is more efficient than the Poisson model in terms of service time and waiting reduction [6].

5.Result and discussion:

- increase in the encouraged arrival rate. The average waiting time and the anticipated maximum waiting time increase by 10% with discounts and constant service rates.
- increasing arrival rate When there are more servers, there is a 10% reduction in both the average wait time and the expected maximum wait time.
- The average waiting time and the anticipated maximum waiting time both decrease with an increase in service rate and constant encouraged arrival rate.
- The average wait time and anticipated maximum wait time are reduced with an increase in servers.
- When comparing this model to the Poisson arrival, it has more effective waiting and service times [6].

6.Conclusion:

This model can be used in real-world systems, including computer networks, telephones, stock markets, ATM hubs, supermarkets and manufacturing. The number of arrivals is more in line with the Poisson arrival process in tables 3 and 6, which both offer discounts of 10% for encouraged arrivals. due to the research model's increased emphasis on minimising waiting time. The MSMQEA model, which is currently operating with fewer customers, can be expanded, resulting in a decrease in waiting times as well as an improvement in customer satisfaction. When comparing this model to the Poisson arrival [6], it has more effective waiting and service times.

Competing interest

The authors have no relevant financial or non-financial interests to disclose.

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