Analysis of the reliability of the paper manufacturing system using a hexadecagonal fuzzy number and a boolean function based on the Weibull distribution

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***Abstract*—In this paper, we assessed the reliability of a paper plant using the weibull distribution and a hexadecagonal fuzzy number. Many difficulties in life have ambiguity, uncertainty, impreciseness, and so on, thus we must understand these phenomena. The paper factory under study is a complicated system comprised of numerous subsystems. Based on the Boolean function approach, the mathematical model was created.**

In this study, we will employ a hexadecagonal fuzzy Weibull lifetime distribution, in which the lifespan parameters are assumed to be fuzzy due to data ambiguity and inaccuracy. Expressions for fuzzy reliability, fuzzy mean time to failure, fuzzy hazard function, and their α-cut have been addressed when the system follows the hexadecagonal fuzzy weibull lifetime distribution. An examination of the numbers is offered at the end to highlight the system's fuzzy reliability characteristics.

Keywords*- *Fuzzy reliability, Fuzzy weibull distribution, Fuzzy hazard function, Mean time to failure, and Fuzzy logic approach

I. Introduction

In the design of engineering systems, reliability has always played a crucial role. The survival function, often known as the reliability function, is the one that is most frequently employed in lifetime data analysis and reliability engineering. Indicated by this function is the likelihood that the given object will perform properly for a specific period of time. On accurate data, the traditional methods of estimating the weibull distribution's parameters are predicated. The assumption is typically made that the observed data are exact real numbers. Over the past many years, a number of scholars have studied the various aspects of reliability technology of the subsystems or systems in process industries at various levels. Singh et al. are only a few of the researchers that have produced research articles in this area. A supply problem for fertiliser production has been reliably identified by Singh et al. [1]. A numerical analysis of the availability and dependability of the serial operations in the butter-oil manufacturing plant was developed by Gupta et al. [2]. It is assumed that component lives are random variables. The Weibull distribution has been shown to be adaptable and versatile for representing monotonic failure rate data among the many distributions. However, Cai et al. [4] presented a different insight by introducing the possibility assumption and fuzzy state assumption to replace the probability and binary state assumptions, demonstrating that the weibull distribution is insufficient for many contemporary complex systems that exhibit bathtub shaped failure rates. The application of fuzzy technique in system failure engineering has been introduced by Cai et al. [5]. A paper manufacturing plant's mathematical models and digestion system were powered by queuing theory according to Pervaiz et al. [6]. Two fuzzy dependability models based on the weibull fuzzy distribution were created by Karpisek et al. [7]. A mathematical model for the problem of employment scheduling was developed by Pervaiz et al. [8] for a small-scale assessment of the paper manufacturing industry. Utilizing a fuzzy exponential lifespan distribution, Baloui et al. [9] analysed the reliability function. A mathematical modeling and performance analysis of the stock preparation unit in the paper plant industry using genetic algorithms was given by Pervaiz et al. [10]. A hexadecagonal fuzzy number serves as the representation for the system parameter. The dependability analysis of a sugar manufacturing plant utilizing a boolean function technique is presented by Agarwal et al. [11] In this study, we use the fuzzy weibull distribution to build the fuzzy reliability function of systems and its α-cut set.Hexadecagonal fuzzy numbers serve as the system's representation of the lifetime distribution.

II. Preliminaries

Here we have an introduction to the fuzzy sets and fuzzy numbers (hexadecagonal fuzzy numbers) as well as some fundamental definitions of reliability context.

Fuzzy Sets [14]

Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function:

$$\mu_A : X \to [0,1].$$

That is:

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in X \\ 0, & \text{if } \notin X \\ (0,1) & \text{if } x \text{ is partly in } X. \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form

$$A = \{\langle x, \mu_A(x)\rangle : x \in X\} \text{ or } A = \left\{\left(\frac{\mu_A(x)}{x}\right) : x \in X\right\},$$

where the function

$$\mu_A(x) : X \to [0,1],$$

defines the degree of membership of the element, $x \in X$.

A. *Hexadecagonal fuzzy number*

A fuzzy number $\xi_{\tilde{\theta}}$ is a hexadecagonal fuzzy number denoted by

$$\tilde{\theta} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$$

such that we can describe a membership function $\xi_{\tilde{\theta}}(x)$ in the following manner:

$$\xi_{\tilde{\theta}}(x) = \begin{cases} 0 & , \quad x \leq a_1 \\ k_1\left(\frac{x-a_1}{a_2-a_1}\right) & , \quad a_1 \leq x \leq a_2 \\ k_1 & , \quad a_2 \leq x \leq a_3 \\ k_1 + (k_2-k_1)\left(\frac{x-a_3}{a_4-a_3}\right), & \quad a_3 \leq x \leq a_4 \\ k_2 & , \quad a_4 \leq x \leq a_5 \\ k_2 + (k_3-k_2)\left(\frac{x-a_5}{a_6-a_5}\right), & \quad a_5 \leq x \leq a_6 \\ k_3 & , \quad a_6 \leq x \leq a_7 \\ k_3 + (1-k_3)\left(\frac{x-a_7}{a_8-a_7}\right) & , \quad a_7 \leq x \leq a_8 \\ 1 & , \quad a_8 \leq x \leq a_9 \\ k_3 + (1-k_3)\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & , \quad a_9 \leq x \leq a_{10} \\ k_3 & , \quad a_{10} \leq x \leq a_{11} \\ k_2 + (k_3-k_2)\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right) & , \quad a_{11} \leq x \leq a_{12} \\ k_2 & , \quad a_{12} \leq x \leq a_{13} \\ k_1 + (k_2-k_1)\left(\frac{a_{14}-x}{a_{14}-a_{13}}\right), & \quad a_{13} \leq x \leq a_{14} \\ k_1 & , \quad a_{14} \leq x \leq a_{15} \\ k_1\left(\frac{a_{16}-x}{a_{16}-a_{15}}\right) & , \quad a_{15} \leq x \leq a_{16} \\ 0 & , \quad x \geq a_{16} \end{cases}$$

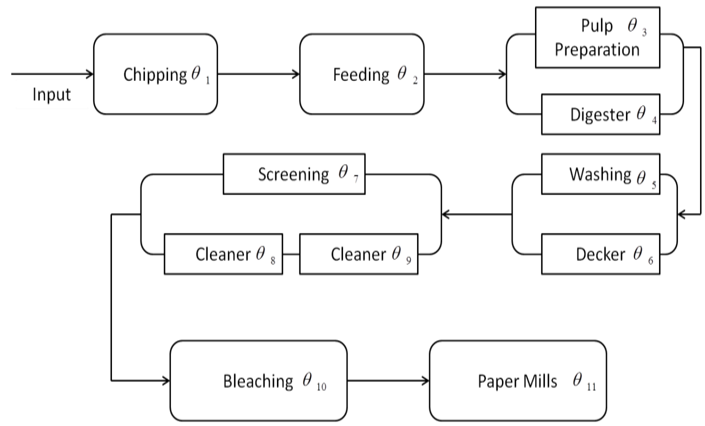

Fig. 1 Flow chart of paper milling process

Paper milling process

It primarily consists of seven subsystems. The first subsystem is the chipping unit, which accepts raw material as input. Feeding occurs when the cutters begin cutting in fine little pieces and the pieces are sent to the second subsystem. The pulp preparation unit, which has an identical unit digester in standby redundancy, is the next subsystem. The particles are then crushed and turned into pulp by being mixed with fresh water. The pulp is sent to the fourth subsystem, the washing unit, which has a standby unit Decker. Following the filtration process, the pulp is directed to the screening unit, which has a similar unit cleaner in parallel redundancy. After the contaminants have been eliminated, the purification process is completed by a succession of bleaching processes. Finally, the white pulp is sent to the final subsystem, known as paper manufacture or completed product. As illustrated in figure 1, all of the subsystems are connected in series and series. Various technology disciplines have used reliability ideas throughout the previous five decades. This technique has also been applied to a variety of industrial and transportation problems; Dhillon et al. provide a full overview. [3]

Assumptions

a) Initially, the entire system is operational.
b) Component failure rates are s-independent.
c) There is no mechanism for repairing a failed component.
d) The reliability of each component is known ahead of time.
e) This system is configured in a fuzzy manner.

List of Notations

θ_1 :The State of chipping unit.
θ_2 : The state of the feeding unit.
θ_3 : The state of the pulp preparation unit.
θ_4 : Digestive system.
θ_5 : The condition of the washing machine.
θ_6 : Decker.
θ_7 : The current state of the screening unit.
θ_8,θ_9 : The screening unit's current state. Cleaners are ranked 8th and 9th.
θ_10 :The current state of the bleaching unit.
θ_11 : Finished product / Paper mills.
$\theta_i(i = 1, 2, ...11)$: 1 in good state.
in bad state.
$\tilde{\theta}$: Fuzzy number.
$\tilde{S}(t)$: Fuzzy reliability functions of a component with time t.
$\xi\theta$: Membership functions of a fuzzy number.
$\wedge/\vee$: Conjunction/Disjunction.

B. Analysis and Mathematical Formation

The prerequisites of capability for the successful operation of the system in terms of logical matrix are expressed using the Boolean function technique as follows:

$$F(\theta_1, \theta_2, \cdots, \theta_{11}) = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_5 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_3 & \theta_5 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \\ \theta_1 & \theta_2 & \theta_3 & \theta_6 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_3 & \theta_6 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \\ \theta_1 & \theta_2 & \theta_4 & \theta_5 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_4 & \theta_5 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \\ \theta_1 & \theta_2 & \theta_4 & \theta_6 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_4 & \theta_6 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \end{bmatrix}$$

Above equation can be expressed using algebraic reasoning as,

$$F(\theta_1, \theta_2, \cdots, \theta_{11}) = [\theta_1\theta_2\theta_{10}\theta_{11} \wedge f(\theta_1, \theta_2, \cdots, \theta_{11})]$$

$$f(\theta_1, \theta_2, \cdots, \theta_{11}) = \begin{bmatrix} \theta_3 & \theta_5 & \theta_7 & \\ \theta_3 & \theta_5 & \theta_8 & \theta_9 \\ \theta_3 & \theta_6 & \theta_7 & \\ \theta_3 & \theta_6 & \theta_8 & \theta_9 \\ \theta_4 & \theta_5 & \theta_7 & \\ \theta_4 & \theta_5 & \theta_8 & \theta_9 \\ \theta_4 & \theta_6 & \theta_7 & \\ \theta_4 & \theta_6 & \theta_8 & \theta_9 \end{bmatrix}$$

Then

$$\begin{aligned} A_1 &= [\theta_3 \quad \theta_5 \quad \theta_7] \\ A_2 &= [\theta_3 \quad \theta_5 \quad \theta_8 \quad \theta_9] \\ A_3 &= [\theta_3 \quad \theta_6 \quad \theta_7] \\ A_4 &= [\theta_3 \quad \theta_6 \quad \theta_8 \quad \theta_9] \\ A_5 &= [\theta_4 \quad \theta_5 \quad \theta_7] \\ A_6 &= [\theta_4 \quad \theta_5 \quad \theta_8 \quad \theta_9] \\ A_7 &= [\theta_4 \quad \theta_6 \quad \theta_7] \\ A_8 &= [\theta_4 \quad \theta_6 \quad \theta_8 \quad \theta_9] \end{aligned}$$

III. Fuzzy Weibull Distribution

The probability density function of the Weibull distribution, which is commonly utilised in reliability, is as follows:

$$f(x) = \frac{\beta}{\theta}\left(\frac{x-\delta}{\theta}\right)^{\beta-1} e^{-}\left(\frac{x-\delta}{\theta}\right)^{\beta}$$

In nature, where θ is the scale parameter, β is the shape parameter, and δ is the location parameter are crisps.

IV. Fuzzy Reliability Function

The fuzzy set theory of Zadeh [12] is the foundation for fuzzy reliability. $\tilde{S}(t)$ is the fuzzy chance that a unit will survive beyond time t . Let X be the lifetime of a system component; additionally, let X have a density function and a fuzzy cumulative distribution function. $\tilde{F_x}(t)$=$\tilde{P}(X \leq t)$,at time t, the fuzzy reliability function is defined as:

$$\begin{aligned} \tilde{S}(t) &= \tilde{P}(X \leq t) \\ &= 1 - \tilde{F_x}(t) \\ &= [1 - F_{max}(x)[\alpha], 1 - F_{min}(x)[\alpha]], \mu_F(x) = \alpha, t > 0 \end{aligned}$$

Assume we want to calculate the reliability of a component whose lifetime random variable has a fuzzy Weibull distribution. As a result, we express parameter $\tilde{\theta}$ as a hexadecagonal fuzzy number.

$$\tilde{\theta} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$$

As a result, we can describe a membership function $\xi_{\tilde{\theta}}(x)$ as follows:

$$\xi_{\tilde{\theta}}(x) = \begin{cases} 0 & , \quad x \le a_1 \\ k_1\left(\frac{x-a_1}{a_2-a_1}\right) & , \quad a_1 \le x \le a_2 \\ k_1 & , \quad a_2 \le x \le a_3 \\ k_1 + (k_2-k_1)\left(\frac{x-a_3}{a_4-a_3}\right), & \quad a_3 \le x \le a_4 \\ k_2 & , \quad a_4 \le x \le a_5 \\ k_2 + (k_3-k_2)\left(\frac{x-a_5}{a_6-a_5}\right), & \quad a_5 \le x \le a_6 \\ k_3 & , \quad a_6 \le x \le a_7 \\ k_3 + (1-k_3)\left(\frac{x-a_7}{a_8-a_7}\right) & , \quad a_7 \le x \le a_8 \\ 1 & , \quad a_8 \le x \le a_9 \\ k_3 + (1-k_3)\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & , \quad a_9 \le x \le a_{10} \\ k_3 & , \quad a_{10} \le x \le a_{11} \\ k_2 + (k_3-k_2)\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right) & , \quad a_{11} \le x \le a_{12} \\ k_2 & , \quad a_{12} \le x \le a_{13} \\ k_1 + (k_2-k_1)\left(\frac{a_{14}-x}{a_{14}-a_{13}}\right), & \quad a_{13} \le x \le a_{14} \\ k_1 & , \quad a_{14} \le x \le a_{15} \\ k_1\left(\frac{a_{16}-x}{a_{16}-a_{15}}\right) & , \quad a_{15} \le x \le a_{16} \\ 0 & , \quad x \ge a_{16} \end{cases}$$

The α cuts $\tilde{\theta}$ is denoted as follows:

$$\tilde{\theta}[\alpha] = \left[a_1 + \frac{\alpha}{k_1}(a_2-a_1), a_{16} - \frac{\alpha}{k_1}(a_{16}-a_{15})\right] for \alpha \in [0, k_1]$$
$$\left[a_3 + \left(\frac{\alpha-k_1}{k_2-k_1}\right)(a_4-a_3), a_{14} - \left(\frac{\alpha-k_1}{k_2-k_1}\right)(a_{14}-a_{13})\right] for \alpha \in [k_1, k_2]$$
$$\left[a_5 + \left(\frac{\alpha-k_2}{k_3-k_2}\right)(a_6-a_5), a_{12} - \left(\frac{\alpha-k_1}{k_3-k_2}\right)(a_{12}-a_{11})\right] for \alpha \in [k_2, k_3]$$
$$\left[a_7 + \left(\frac{\alpha-k_3}{1-k_3}\right)(a_8-a_7), a_{10} - \left(\frac{\alpha-k_5}{1-k_5}\right)(a_{10}-a_9)\right] for \alpha \in [k_3, 1]$$

As a result, the fuzzy reliability function of a component is as follows:

$$\tilde{S}(t)[\alpha] = \left\{ \int_c^d \frac{\beta}{\theta}\left(\frac{x-\delta}{\theta}\right)^{\beta-1} e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}} dx | \theta \in \tilde{\theta}[\alpha] \right\}$$

$$= \left\{ e^{-\left(\frac{t-\delta}{\theta}\right)^{\beta}} | \theta \in \tilde{\theta}[\alpha] \right\}$$

According to that the $e^{-\left(\frac{t-\delta}{\theta}\right)^{\beta}}$ increasing θ, then the α -cuts of fuzzy reliability function is as:

$$\tilde{S}(t)[\alpha] = \tilde{S}(t)[\alpha] = \left[exp\left\{-\left(\frac{t-\delta}{a_1 + \frac{\alpha}{k_1}(a_2-a_1)}\right)^{\beta}\right\}, exp\left\{-\left(\frac{t-\delta}{a_{16} - \frac{\alpha}{k_1}(a_{16}-a_{15})}\right)^{\beta}\right\}\right]$$
$$\left[exp\left\{-\left(\frac{t-\delta}{a_3 + \frac{\alpha-k_1}{k_2-k_1}(a_4-a_3)}\right)^{\beta}\right\}, exp\left\{-\left(\frac{t-\delta}{a_{14} - \frac{\alpha-k_1}{k_2-k_1}(a_{14}-a_{13})}\right)^{\beta}\right\}\right]$$
$$\left[exp\left\{-\left(\frac{t-\delta}{a_5 + \frac{\alpha-k_2}{k_3-k_2}(a_6-a_5)}\right)^{\beta}\right\}, exp\left\{-\left(\frac{t-\delta}{a_{12} - \frac{\alpha-k_1}{k_3-k_2}(a_{12}-a_{11})}\right)^{\beta}\right\}\right]$$
$$\left[exp\left\{-\left(\frac{t-\delta}{a_7 + \frac{\alpha-k_3}{1-k_3}(a_8-a_7)}\right)^{\beta}\right\}, exp\left\{-\left(\frac{t-\delta}{a_{10} - \frac{\alpha-k_5}{1-k_5}(a_{10}-a_9)}\right)^{\beta}\right\}\right]$$

$\tilde{S}(t)[\alpha]$ is a two dimensional function in terms of α and t ,where$(0 \le \alpha \le 1$ and $t > 1)$.For t_0, this is hexadecagonal fuzzy number and membership function of $\tilde{S}(t_0)$ is as follows:

$$\xi_{\tilde{s}(t_0)}(x)=\begin{cases}\dfrac{x-exp\left\{-\left(\frac{t_0-\delta}{a_1}\right)^{\beta}\right\}}{exp\left\{-\left(\frac{t_0-\delta}{a_2}\right)^{\beta}\right\}-exp\left\{-\left(\frac{t_0-\delta}{a_1}\right)^{\beta}\right\}}, & exp\left\{-\left(\frac{t_0-\delta}{a_1}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_2}\right)^{\beta}\right\}\\ 1, exp\left\{-\left(\frac{t_0-\delta}{a_2}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\} & \\ \dfrac{x-exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\}}{exp\left\{-\left(\frac{t_0-\delta}{a_4}\right)^{\beta}\right\}-exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\}}, & exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_4}\right)^{\beta}\right\}\\ 1, exp\left\{-\left(\frac{t_0-\delta}{a_4}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\} & \\ \dfrac{x-exp\left\{-\left(\frac{t_0-\delta}{a_5}\right)^{\beta}\right\}}{exp\left\{-\left(\frac{t_0-\delta}{a_6}\right)^{\beta}\right\}-exp\left\{-\left(\frac{t_0-\delta}{a_5}\right)^{\beta}\right\}}, & exp\left\{-\left(\frac{t_0-\delta}{a_5}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_6}\right)^{\beta}\right\}\\ 1, exp\left\{-\left(\frac{t_0-\delta}{a_6}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_7}\right)^{\beta}\right\} & \\ \dfrac{x-exp\left\{-\left(\frac{t_0-\delta}{a_7}\right)^{\beta}\right\}}{exp\left\{-\left(\frac{t_0-\delta}{a_8}\right)^{\beta}\right\}-exp\left\{-\left(\frac{t_0-\delta}{a_7}\right)^{\beta}\right\}}, & exp\left\{-\left(\frac{t_0-\delta}{a_7}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_8}\right)^{\beta}\right\}\\ 1, exp\left\{-\left(\frac{t_0-\delta}{a_8}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_9}\right)^{\beta}\right\} & \\ \dfrac{exp\left\{-\left(\frac{t_0-\delta}{a_{10}}\right)^{\beta}\right\}-x}{exp\left\{-\left(\frac{t_0-\delta}{a_{10}}\right)^{\beta}\right\}-exp\left\{-\left(\frac{t_0-\delta}{a_9}\right)^{\beta}\right\}}, & exp\left\{-\left(\frac{t_0-\delta}{a_9}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_{10}}\right)^{\beta}\right\}\\ 1, exp\left\{-\left(\frac{t_0-\delta}{a_{10}}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_{11}}\right)^{\beta}\right\} & \\ \dfrac{exp\left\{-\left(\frac{t_0-\delta}{a_{12}}\right)^{\beta}\right\}-x}{exp\left\{-\left(\frac{t_0-\delta}{a_{12}}\right)^{\beta}\right\}-exp\left\{-\left(\frac{t_0-\delta}{a_{11}}\right)^{\beta}\right\}}, & exp\left\{-\left(\frac{t_0-\delta}{a_{11}}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_{12}}\right)^{\beta}\right\}\\ 1, exp\left\{-\left(\frac{t_0-\delta}{a_{12}}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_{13}}\right)^{\beta}\right\} & \\ \dfrac{exp\left\{-\left(\frac{t_0-\delta}{a_{14}}\right)^{\beta}\right\}-x}{exp\left\{-\left(\frac{t_0-\delta}{a_{14}}\right)^{\beta}\right\}-exp\left\{-\left(\frac{t_0-\delta}{a_{13}}\right)^{\beta}\right\}}, & exp\left\{-\left(\frac{t_0-\delta}{a_{13}}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_{14}}\right)^{\beta}\right\}\end{cases}$$

$$\xi_{\tilde{\theta}}(x)=\begin{cases}1, exp\left\{-\left(\frac{t_0-\delta}{a_{14}}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_{15}}\right)^{\beta}\right\} & \\ \dfrac{exp\left\{-\left(\frac{t_0-\delta}{a_{16}}\right)^{\beta}\right\}-x}{exp\left\{-\left(\frac{t_0-\delta}{a_{16}}\right)^{\beta}\right\}-exp\left\{-\left(\frac{t_0-\delta}{a_{15}}\right)^{\beta}\right\}}, & exp\left\{-\left(\frac{t_0-\delta}{a_{15}}\right)^{\beta}\right\}\le x\le exp\left\{-\left(\frac{t_0-\delta}{a_{16}}\right)^{\beta}\right\}\end{cases}$$

The expected time to failure is expressed as the fuzzy mean time to failure $(FMTTF)$.
According to Buckley [13] , the $FMTTF$ of this fuzzy system is a fuzzy number that can be determined as follows:

$$M\tilde{T}TF[\alpha]=\left\{\int_0^{\infty}xf(x)dx|\theta\in\tilde{\theta}[\alpha]\right\}$$

$$=\left\{\int_0^{\infty}S(t)dt|\theta\in\tilde{\theta}[\alpha]\right\}$$

When the failure random variable is distributed with a fuzzy Weibull distribution,then:

$$\begin{aligned}M\tilde{T}TF&=\left\{\theta\Gamma(1+\beta^{-1})|\theta\in\tilde{\theta}[\alpha]\right\}\\ &=\Big[(a_1+4\alpha(a_2-a_1))\Gamma(1+\beta^{-1}),\\ &\qquad(a_{16}-4\alpha(a_{16}-a_{15}))\Gamma(1+\beta^{-1})\Big]\\ &=\Big[(a_3+(4\alpha-1)(a_4-a_3))\Gamma(1+\beta^{-1}),\\ &\qquad(a_{14}-(4\alpha-1)(a_{14}-a_{13}))\Gamma(1+\beta^{-1})\Big]\\ &=\Big[(a_5+(4\alpha-2)(a_6-a_5))\Gamma(1+\beta^{-1}),\\ &\qquad(a_{12}-(4\alpha-2)(a_{12}-a_{11}))\Gamma(1+\beta^{-1})\Big]\\ &=\Big[(a_7+(4\alpha-3)(a_8-a_7))\Gamma(1+\beta^{-1}),\\ &\qquad(a_{10}-(4\alpha-3)(a_{10}-a_9))\Gamma(1+\beta^{-1})\Big]\end{aligned}$$

Accordingly to upper expression the following membership function is obtained:

$$\xi\tilde{s}(t_0)(x) = \begin{cases} k_1\left(\frac{x-a_1\Gamma(1+\beta^{-1})}{(a_2-a_1)\Gamma(1+\beta^{-1})}\right), & a_1\Gamma(1+\beta^{-1}) \le x \le a_2\Gamma(1+\beta^{-1}) \\ k_1, & a_2\Gamma(1+\beta^{-1}) \le x \le a_3\Gamma(1+\beta^{-1}) \\ k_1+(k_2-k_1)\left(\frac{x-a_3\Gamma(1+\beta^{-1})}{(a_4-a_3)\Gamma(1+\beta^{-1})}\right), & a_3\Gamma(1+\beta^{-1}) \le x \le a_4\Gamma(1+\beta^{-1}) \\ k_2, & a_4\Gamma(1+\beta^{-1}) \le x \le a_5\Gamma(1+\beta^{-1}) \\ k_2+(k_3-k_2)\left(\frac{x-a_5\Gamma(1+\beta^{-1})}{(a_6-a_5)\Gamma(1+\beta^{-1})}\right), & a_5\Gamma(1+\beta^{-1}) \le x \le a_6\Gamma(1+\beta^{-1}) \\ k_3, & a_6\Gamma(1+\beta^{-1}) \le x \le a_7\Gamma(1+\beta^{-1}) \\ k_3+(1-k_3)\left(\frac{x-a_7\Gamma(1+\beta^{-1})}{(a_8-a_7)\Gamma(1+\beta^{-1})}\right), & a_7\Gamma(1+\beta^{-1}) \le x \le a_8\Gamma(1+\beta^{-1}) \\ 1, & a_8\Gamma(1+\beta^{-1}) \le x \le a_9\Gamma(1+\beta^{-1}) \\ k_3+(1-k_3)\left(\frac{a_{10}\Gamma(1+\beta^{-1})-x}{(a_{10}-a_9)\Gamma(1+\beta^{-1})}\right), & a_9\Gamma(1+\beta^{-1}) \le x \le a_{10}\Gamma(1+\beta^{-1}) \\ k_3, & a_{10}\Gamma(1+\beta^{-1}) \le x \le a_{11}\Gamma(1+\beta^{-1}) \\ k_2+(k_3-k_2)\left(\frac{a_{12}\Gamma(1+\beta^{-1})-x}{(a_{12}-a_{11})\Gamma(1+\beta^{-1})}\right), & a_{11}\Gamma(1+\beta^{-1}) \le x \le a_{12}\Gamma(1+\beta^{-1}) \\ k_2, & a_{12}\Gamma(1+\beta^{-1}) \le x \le a_{13}\Gamma(1+\beta^{-1}) \\ k_1+(k_2-k_1)\left(\frac{a_{14}\Gamma(1+\beta^{-1})-x}{(a_{14}-a_{13})\Gamma(1+\beta^{-1})}\right), & a_{13}\Gamma(1+\beta^{-1}) \le x \le a_{14}\Gamma(1+\beta^{-1}) \\ k_1, & a_{14}\Gamma(1+\beta^{-1}) \le x \le a_{15}\Gamma(1+\beta^{-1}) \\ k_1\left(\frac{a_{16}\Gamma(1+\beta^{-1})-x}{(a_{16}-a_{15})\Gamma(1+\beta^{-1})}\right), & a_{15}\Gamma(1+\beta^{-1}) \le x \le a_{16}\Gamma(1+\beta^{-1}) \end{cases}$$

V. Fuzzy Hazard Function

The fuzzy hazard function is a particularly appealing concept in fuzzy reliability theory. We shall introduce the concept of a fuzzy hazard function based on the fuzzy probability measure and call it $\alpha - cuts$. The fuzzy hazard function is the fuzzy conditional probability of an item failing in the interval $\tilde{h}$ t to $t+dt$ if it has not failed by time t. The hazard function is also known as the immediate failure rate. The fuzzy hazard function is defined mathematically as follows:

$$\begin{aligned} \tilde{h}[\alpha] &= \lim_{\Delta\to 0} \frac{\tilde{P}(t < X < t+\Delta t | X > t)}{\Delta t} \\ &= \left\{ \lim_{\Delta\to 0} \frac{S(t)-S(t+\Delta t)}{\Delta t S(t)} | \theta \in \tilde{\theta}[\alpha] \right\} \\ &= \left\{ \frac{-S'(t)}{S(t)} | \theta \in \tilde{\theta}[\alpha] \right\} = \left\{ \frac{f(t)}{S(t)} | \theta \in \tilde{\theta}[\alpha] \right\} \end{aligned}$$

The fuzzy hazard function for the fuzzy Weibull distribution with $\delta = 0$ is as follows:

$$\tilde{h(t)}[\alpha] = \left\{ \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} | \theta \in \tilde{\theta}[\alpha] \right\}$$

$\tilde{h(t)}[\alpha]$ is a two-dimensional function defined by α and t ($0 \le \alpha \le 1$ and $t > 1$).

VI. Numerical Analysis

Let the component's lifetime be represented by a Weibull distribution with a fuzzy parameter $\tilde{\theta}$ and $\delta = 0$ that $\tilde{\theta} = (1.20, 1.25, 1.30, 1.35, 1.40, 1.45, 1.50, 1.55, 1.60, 1.65, 1.70, 1.75, 1.80, 1.85, 1.90, 1.95)$.Then α-cuts of the fuzzy reliability function, fuzzy hazard function, and fuzzy mean time to failure are provided by

(i)Fuzzy reliability is: $\tilde{\theta}[\alpha]$=

$$\begin{gathered} 1.20 + 4\alpha(0.05), 1.95 - 4\alpha(0.05)] \\ [1.30 + (4\alpha-1)(0.05), 1.85 - (4\alpha-1)(0.05)] \\ [1.40 + (4\alpha-2)(0.05), 1.75 - (4\alpha-2)(0.05)] \\ [1.50 + (4\alpha-3)(0.05), 1.95 - (4\alpha-3)(0.05)] \end{gathered}$$

$S(t)[\alpha]$=

$$\begin{gathered} \left[exp\left\{-\frac{t}{1.20+0.2\alpha}\right\}^{\beta}, exp\left\{-\frac{t}{1.95+0.2\alpha}\right\}^{\beta}\right] \\ \left[exp\left\{-\frac{t}{1.25+0.2\alpha}\right\}^{\beta}, exp\left\{-\frac{t}{1.90+0.2\alpha}\right\}^{\beta}\right] \\ \left[exp\left\{-\frac{t}{1.30+0.2\alpha}\right\}^{\beta}, exp\left\{-\frac{t}{1.85+0.2\alpha}\right\}^{\beta}\right] \\ \left[exp\left\{-\frac{t}{1.35+0.2\alpha}\right\}^{\beta}, exp\left\{-\frac{t}{2.1+0.2\alpha}\right\}^{\beta}\right] \end{gathered}$$

$$\begin{aligned} S(t)[\alpha] = &\left[exp\left\{-\frac{t-\delta}{1.20+0.2\alpha}\right\}^{\beta}, exp\left\{-\frac{t-\delta}{1.95+0.2\alpha}\right\}^{\beta}\right] \\ &\left[exp\left\{-\frac{t-\delta}{1.25+0.2\alpha}\right\}^{\beta}, exp\left\{-\frac{t-\delta}{1.90+0.2\alpha}\right\}^{\beta}\right] \\ &\left[exp\left\{-\frac{t-\delta}{1.30+0.2\alpha}\right\}^{\beta}, exp\left\{-\frac{t-\delta}{1.85+0.2\alpha}\right\}^{\beta}\right] \\ &\left[exp\left\{-\frac{t-\delta}{1.35+0.2\alpha}\right\}^{\beta}, exp\left\{-\frac{t-\delta}{2.1+0.2\alpha}\right\}^{\beta}\right] \end{aligned}$$

for all α

(1)if $t = 0.5$

$$\xi_{\tilde{s}(0.5)(x)} = \begin{cases} \frac{x-exp\{-(0.4166)^{\beta}\}}{exp\{-(0.4)^{\beta}\}-exp\{-(0.4166)^{\beta}\}}, \\ \quad exp\{-(0.4166)^{\beta}\} \leq x \leq exp\{-(0.4)^{\beta}\} \\ \frac{x-exp\{-(0.3846)^{\beta}\}}{exp\{-(0.3703)^{\beta}\}-exp\{-(0.3846)^{\beta}\}}, \\ \quad exp\{-(0.3846)^{\beta}\} \leq x \leq exp\{-(0.3703)^{\beta}\} \\ 1, \quad exp\{-(0.3703)^{\beta}\} \leq x \leq exp\{-(0.3846)^{\beta}\} \\ \frac{x-exp\{-(0.3571)^{\beta}\}}{exp\{-(0.3448)^{\beta}\}-exp\{-(0.3571)^{\beta}\}}, \\ \quad exp\{-(0.3571)^{\beta}\} \leq x \leq exp\{-(0.3448)^{\beta}\} \\ 1, \quad exp\{-(0.3448)^{\beta}\} \leq x \leq exp\{-(0.3333)^{\beta}\} \\ \frac{x-exp\{-(0.3333)^{\beta}\}}{exp\{-(0.3225)^{\beta}\}-exp\{-(0.3333)^{\beta}\}}, \\ \quad exp\{-(0.3333)^{\beta}\} \leq x \leq exp\{-(0.3225)^{\beta}\} \\ 1, \quad exp\{-(0.3225)^{\beta}\} \leq x \leq exp\{-(0.3125)^{\beta}\} \\ \frac{x-exp\{-(0.3030)^{\beta}-x\}}{exp\{-(0.3030)^{\beta}\}-exp\{-(0.3125)^{\beta}\}}, \\ \quad exp\{-(0.3030)^{\beta}\} \leq x \leq exp\{-(0.3030)^{\beta}\} \\ 1, \quad exp\{-(0.3030)^{\beta}\} \leq x \leq exp\{-(0.2941)^{\beta}\} \\ \frac{x-exp\{-(0.2857)^{\beta}-x\}}{exp\{-(0.2857)^{\beta}\}-exp\{-(0.2941)^{\beta}\}}, \\ \quad exp\{-(0.2857)^{\beta}\} \leq x \leq exp\{-(0.2857)^{\beta}\} \\ 1, \quad exp\{-(0.2857)^{\beta}\} \leq x \leq exp\{-(0.2777)^{\beta}\} \\ \frac{x-exp\{-(0.2702)^{\beta}-x\}}{exp\{-(0.2702)^{\beta}\}-exp\{-(0.2777)^{\beta}\}}, \\ \quad exp\{-(0.2702)^{\beta}\} \leq x \leq exp\{-(0.2702)^{\beta}\} \\ 1, \quad exp\{-(0.2702)^{\beta}\} \leq x \leq exp\{-(0.2631)^{\beta}\} \\ \frac{x-exp\{-(0.2564)^{\beta}-x\}}{exp\{-(0.2564)^{\beta}\}-exp\{-(0.2631)^{\beta}\}}, \\ \quad exp\{-(0.2564)^{\beta}\} \leq x \leq exp\{-(0.2564)^{\beta}\} \end{cases}$$

(2)if $\alpha = 0$ then reliability is as:

$$\begin{aligned} s\tilde{(t)}[0] &= \Big[exp\{-(0.8333)^{\beta}\}, exp\{-(0.512)^{\beta}\}\Big] \\ &\Big[exp\{-(0.8)^{\beta}\}, exp\{-(0.526)^{\beta}\}\Big] \\ &\Big[exp\{-(0.384)^{\beta}\}, exp\{-(0.540)^{\beta}\}\Big] \\ &\Big[exp\{-(0.740)^{\beta}\}, exp\{-(0.476)^{\beta}\}\Big] \end{aligned}$$

(ii) **Fuzzy hazard function is:**

$$\begin{aligned} h\tilde{(t)}[\alpha] = &\left\{ \frac{\beta}{1.95 - 0.2\alpha}\left(\frac{t}{1.95 - 0.2\alpha}\right)^{\beta-1}, \frac{\beta}{1.20 - 0.2\alpha}\left(\frac{t}{1.20 - 0.2\alpha}\right)^{\beta-1} \right\} \\ &\left\{ \frac{\beta}{1.90 - 0.2\alpha}\left(\frac{t}{1.90 - 0.2\alpha}\right)^{\beta-1}, \frac{\beta}{1.25 - 0.2\alpha}\left(\frac{t}{1.25 - 0.2\alpha}\right)^{\beta-1} \right\} \\ &\left\{ \frac{\beta}{1.85 - 0.2\alpha}\left(\frac{t}{1.85 - 0.2\alpha}\right)^{\beta-1}, \frac{\beta}{1.30 - 0.2\alpha}\left(\frac{t}{1.30 - 0.2\alpha}\right)^{\beta-1} \right\} \\ &\left\{ \frac{\beta}{2.1 - 0.2\alpha}\left(\frac{t}{2.1 - 0.2\alpha}\right)^{\beta-1}, \frac{\beta}{1.35 - 0.2\alpha}\left(\frac{t}{1.35 - 0.2\alpha}\right)^{\beta-1} \right\} \end{aligned}$$

(iii) **Fuzzy mean time to failure:**

$$\begin{aligned} M\tilde{T}TF[\alpha](\beta) &= \Big\{\theta\Gamma(1+\beta^{-1}) | \theta \in \tilde{\theta}[\alpha]\Big\} \\ &= \Big[(a_1 + 4\alpha(a_2 - a_1))\Gamma(1+\beta^{-1}), (a_{16} - 4\alpha(a_{16} - a_{15}))\Gamma(1+\beta^{-1})\Big] \\ &= \Big[1.20 + 4\alpha(0.05), 1.95 - 4\alpha(0.05)\Big] \\ &= \Big[(a_3 + (4\alpha - 1)(a_4 - a_3))\Gamma(1+\beta^{-1}), (a_{14} - (4\alpha - 1)(a_{14} - a_{13}))\Gamma(1+\beta^{-1})\Big] \\ &= \Big[1.30 + (4\alpha - 1)(0.05), 1.85 - (4\alpha - 1)(0.05)\Big] \\ &= \Big[(a_5 + (4\alpha - 2)(a_6 - a_5))\Gamma(1+\beta^{-1}), (a_{12} - (4\alpha - 1)(a_{12} - a_{11}))\Gamma(1+\beta^{-1})\Big] \\ &= \Big[1.40 + (4\alpha - 2)(0.05), 1.75 - (4\alpha - 2)(0.05)\Big] \\ &= \Big[(a_7 + (4\alpha - 3)(a_8 - a_7))\Gamma(1+\beta^{-1}), (a_{10} - (4\alpha - 1)(a_{10} - a_9))\Gamma(1+\beta^{-1})\Big] \\ &= \Big[1.50 + (4\alpha - 3)(0.05), 1.95 - (4\alpha - 3)(0.05)\Big] \end{aligned}$$

The value of $FMTTF$ at $\beta = 2.166$ has a minimum value, according to the behaviour of the gamma function:
$M\tilde{T}TF[\alpha](2.166)$=

$$\Big[(1.06272 + 0.17712\alpha), (1.72692 - 0.17712\alpha)\Big]$$
$$\Big[(1.107 + 0.17712\alpha), (1.68264 - 0.17712\alpha)\Big]$$
$$\Big[(1.23984 + 0.08856\alpha), (1.5498 - 0.08856\alpha)\Big]$$
$$\Big[(1.19556 + 0.17712\alpha), (1.85976 - 0.17712\alpha)\Big]$$

Accordingly the following membership function is obtained: Let

$$k_1 = \frac{1}{4}, k_2 = \frac{1}{2}, k_3 = \frac{3}{4}$$

so

$$\xi_{\tilde{s}(t_0)(x)} = \begin{cases} \frac{x-1.06272}{0.17712}, & 1.06272 \le x \le 1.23984 \\ 1, & 1.23984 \le x \le 1.5498 \\ \frac{1.72692-x}{0.17712}, & 1.5498 \le x \le 1.72692 \\ \\ \frac{x-1.107}{0.17712}, & 1.107 \le x \le 1.28412 \\ 1, & 1.28412 \le x \le 1.50552 \\ \frac{1.68264-x}{0.17712}, & 1.50552 \le x \le 1.68264 \\ \\ \frac{x-1.23984}{0.08856}, & 1.23984 \le x \le 1.3284 \\ 1, & 1.3284 \le x \le 1.46124 \\ \frac{1.5498-x}{0.08856}, & 1.46124 \le x \le 1.5498 \\ \\ \frac{x-1.19556}{0.17712}, & 1.19556 \le x \le 1.37268 \\ 1, & 1.37268 \le x \le 1.68264 \\ \frac{1.85976-x}{0.17712}, & 1.68264 \le x \le 1.85976 \end{cases}$$

VII. **Conclusion**

In this paper we have been successfully evaluated the Weibull distribution, fuzzy reliability function, fuzzy hazard function and their α-cuts. Whenever the lifetimes of a components and parameters contain randomness and fuzziness respectively, the approach of reliability theory based on traditional statistical analysis may be improper. In our method, fuzzy set theory and fuzzy probability theory are the foundations for fuzzy system reliability. The scale parameter in this study was treated as a hexadecagonal fuzzy number. The study of some crucial issues in fuzzy reliability theory, such as mean leftover life, requires additional research.

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