SURFACE WAVES IN A PORO-ELECTRO-ELASTIC PLATE

Anil K. Vashishth[1], Ashok Dahiya[2], Vishakha Gupta[3]*

Abstract
The propagation of the surface waves in a poro-electro-elastic plate studied. The problem is solved analytically as well as numerically. The computation is done numerically for Barium Titanate 6mm poro-piezo ceramic and dispersion curves are presented. The behavior of symmetric and anti-symmetric modes of vibrations and their relation to surface acoustic waves are also studied. The effects of electro-elastic interaction, wave frequency and porosity on the phase velocities of these modes are observed.

Keywords: Dispersion equations, Surface wave, Porous electro-elastic lamina, Symmetric modes, Antisymmetric modes.

[1]Department of Mathematics, Kurukshetra University, Kurukshetra-136119, INDIA.
[2]Department of Mathematics, D.A.V. College, Pehowa-136128, INDIA.
[3]*Department of Mathematics, Dyal Singh College, Karnal-132001, INDIA.

***Corresponding author:** Vishakha Gupta
*Department of Mathematics, Dyal Singh College, Karnal-132001, INDIA.
Email: guptavishu19@gmail.com

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1 Introduction

Surface waves are elastic perturbations propagating in a solid plate with traction-free boundaries having symmetrical and anti-symmetrical vibrations. The lowest-order symmetric mode has high phase velocity in comparison to the lowest-order antisymmetric mode. Propagation of Surface waves in electro-elastic materials has been subject of interest because of their application in surface acoustic wave (SAW) sensors.
Different authors [1-5] studied propagation of surface waves in different types of materials. Jin and Joshi [6] investigated symmetric and antisymmetric lower-order surface modes in a electro-elastic plate for both metallized and unmetallized surfaces. Nayfeh and Chein [7] investigated the effects of fluid mechanical loading on Surface waves. Lee and Staszewski [8-9] used the model of Surface waves for damage detection in metallic structures. The effects of different types of boundary conditions on the propagation of Surface waves in electro-elastic plates having 6mm symmetry was studied in [10].

Porous electro-elastic materials have a wide range of ultrasonic applications such as hydrophones, actuators, underwater transducers and medical diagnostics etc. In order to characterize porous electro-elastic materials, Gupta and Venkatesh determined the effect of porosity on the electromechanical response of porous electro-elastic ceramics [11]. Vashishth and Gupta derived basic equations for porous electro-elastic materials [12] and studied vibrations characteristics of such type of plate and further studied wave propagation in Porous electro-elastic materials [13]. Sharma analysed the electro-elastic effects on the phase velocities and group velocities of waves propagating in anisotropic porous electro-elastic materials [14]. The investigation of SH wave propagation in a periodically layered porous electro-elastic structure has been carried out and stop band effects are shown [15-19]. he important problem of propagation of Surface waves in a porous electro-elastic lamina has not been studied earlier.
In this paper, the characteristics of Surface waves in porous electro-elastic lamina of 6mm symmetry are studied. Two groups of waves, symmetric and anti-symmetric modes, are studied to indicate the symmetry of the particle displacements associated with the wave. Dispersion equations for both approaches are obtained and solved numerically also. The numerical results are discussed in particular for Barium Titanate.

2 Formulation of the problem

Let us consider an infinite poro-electro-elastic plate of thickness h, as shown in Fig.1. Two outer interfaces of the lamina are $z = h/2$ and $z = -h/2$ and are assumed to be stress-free and electrically shorted surfaces.

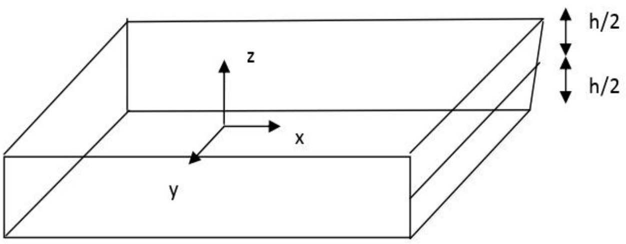

Fig 1. Geometry of the problem

The equations of motion are given as

$$\sigma_{ij,j} = (\rho_{11})_{ij}\ddot{u}_j + (\rho_{12})_{ij}\ddot{U}_j^* + b_{ij}(\dot{u}_j - \dot{U}_j^*),$$
$$\sigma_{,i}^* = (\rho_{12})_{ij}\ddot{u}_j + (\rho_{22})_{ij}\ddot{U}_j^* - b_{ij}(\dot{u}_j - \dot{U}_j^*),$$
$$D_{i,i} = 0,$$
$$D_{i,i}^* = 0, \qquad (1)$$

where u_i, U_i^* are the mechanical displacement components for solid; fluid phase. $(\rho_{11})_{ij}, (\rho_{12})_{ij}, (\rho_{22})_{ij}; b_{ij}$ are dynamical coefficients; dissipation coefficients.

The constitutive equations for poro-electro-elastic materials can be written as

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} + m_{ij}\varepsilon^* - e_{kij}E_k - \zeta_{kij}E_k^*,$$
$$\sigma^* = m_{ij}\varepsilon_{ij} + R\varepsilon^* - \tilde{\zeta}_k E_k - e_k^* E_k^*,$$
$$D_i = e_{ikl}\varepsilon_{kl} + \tilde{\zeta}_i\varepsilon^* + \xi_{il}E_l + A_{il}E_l^*,$$
$$D_i^* = \zeta_{ikl}\varepsilon_{kl} + e_i^*\varepsilon^* + A_{il}E_l + \xi_{il}^* E_l^*, \quad (i,j,k,l = 1,2,3) \qquad (2)$$

Here $\sigma_{ij}, \varepsilon_{ij}, D_i, E_i$ are the stress, strain, electric displacement and electric field components for solid phase. The corresponding components with

$*$ are corresponding to fluid phase. $c_{ijkl}, m_{ij}, R; e_{ijk}, e_i^*, \zeta_{ijk}, \tilde{\zeta}_i; \xi_{ij}, \xi_{ij}^*, A_{ij}$ are elastic; electro-elastic; dielectric coefficients.

The strain tensor components and electric field components are related to displacement components and electric potential, respectively as

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad \varepsilon^* = U_{i,i}^*,$$
$$E_i = -\phi_{,i}, \qquad E_i^* = -\phi_{,i}^*, \qquad (3)$$

3 Dispersion equation of surface wave

We consider the plane harmonic solutions of Eq (1) as follows:

$$\{u_1, u_2, u_3\} = \{u_1^0(z), u_2^0(z), u_3^0(z)\}\exp\iota(k_x x + k_y y - \omega t),$$
$$\{U_1, U_2, U_3\} = \{U_1^0(z), U_2^0(z), U_3^0(z)\}\exp\iota(k_x x + k_y y - \omega t),$$
$$\{\phi, \phi^*\} = \{\phi^0(z), \phi^{*0}(z)\}\exp\iota(k_x x + k_y y - \omega t), \qquad (4)$$

where $k = \{k_x, k_y\}^T$ is propagation vector. $|k| = k = \omega / c$, $k_x = k\cos\theta$, $k_y = k\sin\theta$. c is the phase velocity, ω is wave circular frequency and $\iota = \sqrt{-1}$. θ is considered as positive when measured counter clock wise from x-axis.

For symmetric modes, the expressions for mechanical displacements and electric potential can be written as

$$u_{1s}^0 = A_{1s}\cos(qz), u_{2s}^0 = A_{2s}\cos(qz), u_{3s}^0 = A_{3s}\sin(qz),$$
$$U_{1s}^0 = A_{4s}\cos(qz), U_{2s}^0 = A_{5s}\cos(qz), U_{3s}^0 = A_{6s}\sin(qz),$$
$$\phi_s^0 = A_{7s}\sin(qz) \quad \phi_s^{*0} = A_{8s}\sin(qz), \quad (5)$$

and corresponding expressions for antisymmetric modes are

$$u_{1a}^0 = A_{1a}\sin(qz), u_{2a}^0 = A_{2a}\sin(qz), u_{3a}^0 = A_{3a}\cos(qz),$$
$$U_{1a}^0 = A_{4a}\sin(qz), U_{2a}^0 = A_{5a}\sin(qz), U_{3a}^0 = A_{6a}\cos(qz),$$
$$\phi_a^0 = A_{7a}\cos(qz) \quad \phi_a^{*0} = A_{8a}\cos(qz), \quad (6)$$

where q is an unknown variable and the subscripts ’s’, ’a’ are associated with symmetric and antisymmetric modes, respectively.

Making use of Eqs. (3) - (6) in Eq. (1), we get

$$WS = 0, \qquad (7)$$

where $S = [A_{1\alpha}, A_{2\alpha}, A_{3\alpha}, A_{4\alpha}, A_{5\alpha}, A_{6\alpha}, A_{7\alpha}, A_{8\alpha}], (\alpha = s, a)$ and **W** is a 8×8 matrix whose elements are given in Appendix A.

The non-trivial solution of system (7) exists if $det(W) = 0$, which leads to

$$T_1 q^{12} + T_2 q^{10} + T_3 q^8 + T_4 q^6 + T_5 q^4 + T_6 q^2 + T_7 = 0 \quad (8)$$

where T_p $(p = 1,2,\ldots,7)$ are listed in appendix A. Eq.(8) has twelve roots with the condition $q_j = -q_{j+6}$ $(j = 1,2,\ldots,6)$. The six roots, which have positive imaginary parts are chosen and are named as $q_1, q_2, q_3, q_4, q_5, q_6$. These correspond to slowness wave modes propagating in porous electro-elastic lamina.

For each q_j, the amplitude ratios $(R_{1j}, R_{2j}, R_{3j}, R_{4j}, R_{5j}, R_{6j}, R_{7j})$ are defined as $R_{pj} = \frac{A_{p+1j}}{A_{1j}}, (p = 1,2,\ldots,7)$ where $A_{1j} = [A_1]_{q_j}$ and

$$R_{1i} = \frac{c(W_{82})_{q_j}}{c(W_{81})_{q_j}}, R_{2i} = \frac{c(W_{83})_{q_j}}{c(W_{81})_{q_j}}, R_{3i} = \frac{c(W_{84})_{q_j}}{c(W_{81})_{q_j}}, R_{4i} = \frac{c(W_{85})_{q_j}}{c(W_{81})_{q_j}}, R_{5i} = \frac{c(W_{86})_{q_j}}{c(W_{81})_{q_j}}, R_{6i} = \frac{c(W_{87})_{q_j}}{c(W_{81})_{q_j}}, R_{7i} = \frac{c(W_{88})_{q_j}}{c(W_{81})_{q_j}}, \ (9)$$

where $c(W_{ij})_{q_j}$ are the cofactor of W_{ij} corresponding to the eigen value q_j.

Thus, by suppressing the term $e^{i(k_x x + k_y y - \omega t)}$ for brevity, the formal solutions for the mechanical displacements and electric potentials in the porous electro-elastic lamina are expressed as

$$\{u_1^0, u_2^0, u_3^0\} = \sum_{j=1}^{6} A_{1j}\{\cos(q_j z + \psi), R_{1j}\cos(q_j z + \psi), R_{2j}\sin(q_j z + \psi)\},$$
$$\{U_1^0, U_2^0, U_3^0\} = \sum_{j=1}^{6} A_{1j}\{R_{3j}\cos(q_j z + \psi), R_{4j}\cos(q_j z + \psi), R_{5j}\sin(q_j z + \psi)\},$$
$$\{\phi^0, \phi^{*0}\} = \sum_{j=1}^{6} A_{1j}\{R_{6j}\sin(q_j z + \psi), R_{7j}\sin(q_j z + \psi)\}, \ (10)$$

where $\psi = 0$ and $\psi = \pi/2$, represents antisymmetric and symmetric Surface wave modes, and the stress components are given as

$$\sigma_{31} = -\sum_{j=1}^{6} [c_{44}(q_j - \iota k_x R_{2j}) - \iota e_{15} k_x R_{6j} - \iota\zeta_{15} k_x R_{7j}] A_{1j}\sin(q_j z + \psi)],$$
$$\sigma_{32} = -\sum_{j=1}^{6} [c_{44}(q_j R_{1j} - \iota k_y R_{2j}) - \iota e_{15} k_y R_{6j} - \iota\zeta_{15} k_y R_{7j}] A_{1j}\sin(q_j z + \psi)],$$

$$\sigma_{33} = \iota \sum_{j=1}^{6} [\, c_{13}\, k_x + c_{13}\, k_y\, R_{1j} - \iota\, c_{33}\, R_{2j}\, q_j + m_{33}(R_{3j}k_x + R_{4j}k_y - \iota R_{5j}\, q_j) - \iota\, e_{33}\, R_{6j}\, q_j - \iota \zeta_{33}\, R_{7j}\, q_j]\, A_{1j} \cos(q_j z + \psi),$$

$$\sigma^* = \iota \sum_{j=1}^{6} [m_{11}k_x + m_{11}R_{1j}k_y - \iota\, m_{33}\, R_{2j}\, q_j + R(R_{3j}k_x + k_y R_{4j} - \iota R_{5j}q_j) - \iota\tilde{\zeta}_3 R_{6j}q_j - \iota e_3^* R_{7j}q_j]A_{1j}\cos(q_j z + \psi). \quad (11)$$

The boundary conditions at the upper $(z = h/2)$ and lower surface $(z = -h/2)$ of the lamina are expressed as

$$\sigma_{31} = 0, \sigma_{32} = 0, \sigma_{33} = 0, \sigma^* = 0, \phi = 0, \phi^* = 0. \quad (12)$$

Using equations(10)-(12), we have

$$\Gamma F = 0, \quad (13)$$

where $F = [A_{11}, A_{12}, A_{13}, A_{14}, A_{15}, A_{16}]^T$ and matrix Γ is given in appendix B. The dispersion equation for $3D$ Surface wave is given as

$$|\Gamma| = 0. \quad (14)$$

4 Numerical discussion

In this section, numerical solutions of the dispersion Eqs.(14), are presented in the form of dispersion curves. For numerical computations, the material selected in the present study is Barium Titanate of thickness 1mm and its elastic, electro-elastic, dielectric constants are listed in Table 1.

Table 1. Materials Constants for $BaTiO_3$.

Elastic constants(GPa)	Electro-elastic constants(C/m^2)	Dielectric constants(nC/Vm)
$c_{11} = 150$	$e_{15} = 11.4$	$\xi_{11} = 10.8$
$c_{12} = 65.63$	$e_{31} = -4.32$	$\xi_{33} = 13.1$
$c_{13} = 65.94$	$e_{33} = 17.4$	$\xi_{11}^* = 11.8$
$c_{33} = 145.5$	$\zeta_{15} = 4.56$	$\xi_{33}^* = 13.9$
$c_{44} = 43.86$	$\zeta_{31} = -1.728$	$A_{11} = 12.8$
$m_{11} = 8.8$	$\zeta_{33} = 6.96$	$A_{33} = 15.1$
$m_{33} = 5.2$	$e_3^* = -3.6$	
$R = 20$	$\tilde{\zeta}_3 = -7.5$	

All the figures depict the dispersion of surface waves. The dimensionless frequency $\omega h/c_T$ is taken along x-axis and dimensionless phase velocity c/c_T is shown along y-axis. c_T is the characteristic velocity and direction of propagation is taken along $\theta = 30^o$. A set of Figs.2-7, shows the dispersive curves of surface modes. Three symmetric (S_0, S_1, S_2) and four antisymmetric (A_0, A_1, A_2, A_3) wave modes are shown in a finite dimensionless frequency range $0 - 10$. The modes are labeled as S, for symmetric and A for antisymmetric. Also, it is to be noted that for all modes, a cut-off frequency exists with the exception of lower order symmetric (S_0) and antisymmetric (A_0) modes.

Fig.2 depicts the symmetric surface modes in a poro-electro-elastic lamina. The fundamental mode (S_0) dispersion is different from dispersion of other modes of vibration. After the value 4 of $\omega h/c_T$, it is nearly non-dispersive in case of S_0 while not so in case of S_1 and S_2. The higher symmetric modes are highly dispersive in nature with cut-off frequencies (4 and 5). The antisymmetric Surface wave modes are shown in Fig.3. The lowest antisymmetric mode (A_0) behaves differently, in which the phase velocity starts from lowest value, increases with the frequency and finally becomes non-dispersive. The solution corresponding to third mode (A_3) are also obtained in antisymmetric case. The phase velocity decreases with frequency in A_1,A_2 and A_3 modes. The weak dispersion of fundamental modes, both in symmetric and antisymmetric modes, in high frequency range makes it distinct from higher Surface modes. In order to determine the effect of porosity, considered in the present study, in comparison to the existing theoretical electro-elastic model for ceramics, the results for electro-elastic material are also obtained.

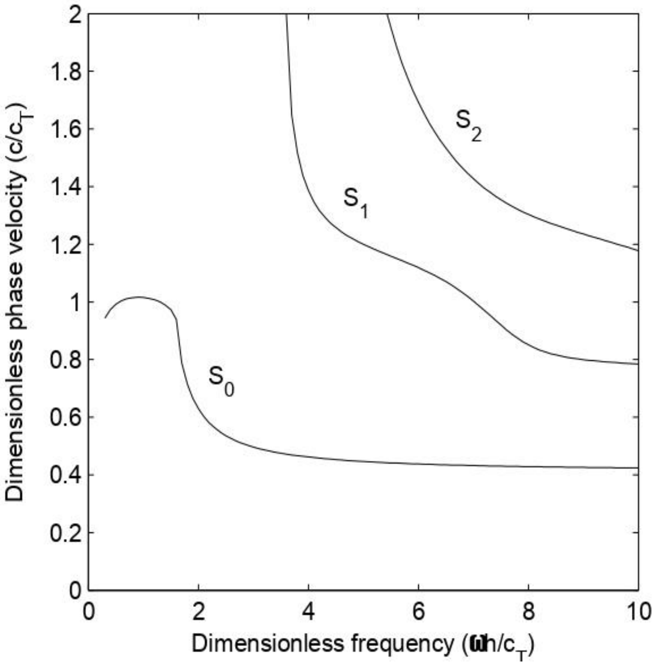

Fig 2: Dispersion Curves: symmetric modes

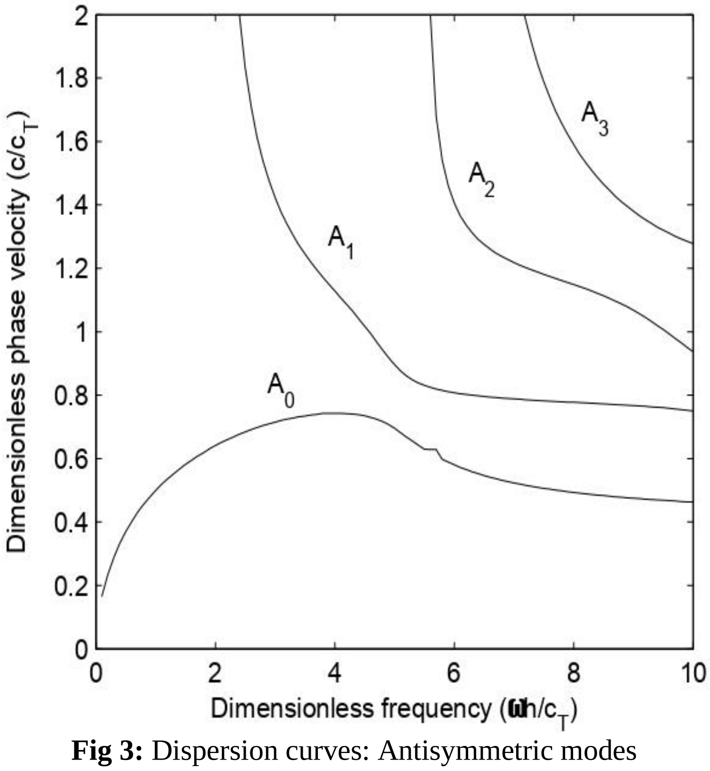

Fig 3: Dispersion curves: Antisymmetric modes

Fig.4 demonstrate the study of effects of porosity on dispersion curves of symmetric modes. The dispersion curves for electro-elastic lamina are shown by dashed curves in these figures. Fig.4 shows that the velocity of S_0 mode in porous electro-elastic lamina has higher value in comparison to electro-elastic one, within frequency range $0-1.8$ and becomes less thereafter. In the case of the higher modes, phase velocity for porous electro-elastic lamina is less than that for electro-elastic lamina before interception.

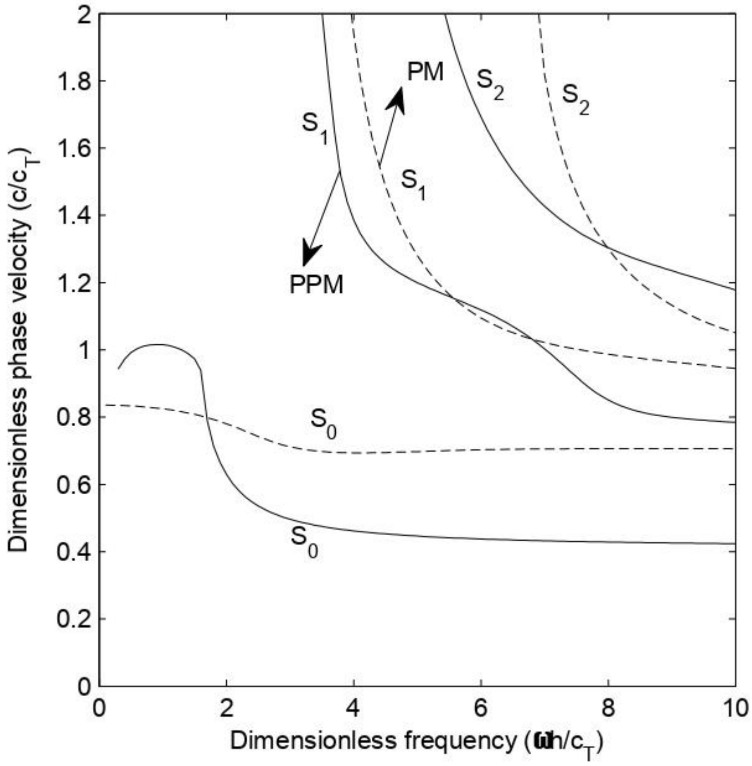

Fig 4. Effects of porosity on dispersion curves of symmetric modes

5 Conclusions

This paper has presented a theoretical and numerical investigation of the propagation of Surface waves in a porous electro-elastic lamina. This investigation reveals that the fundamental mode (S_0) dispersion is different from the dispersion of other modes of vibration and higher symmetric modes are highly dispersive in nature with cut-off frequencies. The velocity of S_0 mode in porous electro-elastic lamina has a higher value in comparison to electro-elastic one while phase velocity for porous electro-elastic lamina is less than that for electro-elastic lamina before interception. The phase velocity decreases with frequency in antisymmetric modes. Due to the difference in velocities of fundamental symmetric and antisymmetric modes, the decomposition of surface acoustic waves (SAW) into S_0 and A_0 modes is useful in explaining the beating effect. The effect of consideration of porosity in the theoretical framework of the model is distinctively observed in the dispersion curves. The results obtained by Sharma and Pal (2003) for electro-elastic materials are obtained as a particular case of study.

Appendix A

$$(\bar{\rho}_{11})_{ij} = (\rho_{11})_{ij} + \frac{\iota b_{ij}}{\omega}, (\bar{\rho}_{12})_{ij} = (\rho_{12})_{ij} - \frac{\iota b_{ij}}{\omega}, (\bar{\rho}_{22})_{ij} = (\rho_{22})_{ij} + \frac{\iota b_{ij}}{\omega}, dl(\bar{\rho}) = dl(\rho_{11})_{11} - dl(\rho_{12})_{11}^2/dl(\rho_{22})_{22}, dl(\rho_{11})_{ij} = (\bar{\rho}_{11})_{ij}\omega^2, dl(\rho_{12})_{ij} = (\bar{\rho}_{12})_{ij}\omega^2, dl(\rho_{22})_{ij} = (\bar{\rho}_{22})_{ij}\omega^2.$$

The elements of matrix W in Eq.(9) are:

$$W_{11} = c_{11}k_x^2 + c_{66}k_y^2 + c_{44}q^2 - dl(\rho_{11})_{11}, W_{12} = (c_{12} + c_{66})k_x k_y, W_{13} = -\iota(c_{13} + c_{44})k_x q,$$
$$W_{14} = m_{11}k_x^2 - dl(\rho_{12})_{11}, W_{15} = m_{11}k_x k_y, W_{16} = -\iota m_{11}k_x q, W_{17} = -\iota(e_{31} + e_{15})k_x q,$$
$$W_{18} = -\iota(\zeta_{31} + \zeta_{15})k_x q, W_{21} = (c_{12} + c_{66})k_x k_y, W_{22} = c_{66}k_x^2 + c_{11}k_y^2 + c_{44}q^2 - dl(\rho_{11})_{22}, W_{23} = -\iota(c_{13} + c_{44})k_y q, W_{24} = m_{11}k_x k_y, W_{25} = m_{11}k_y^2 - dl(\rho_{12})_{22}, W_{26} = -\iota m_{11}k_y q,$$
$$W_{27} = -\iota(e_{31} + e_{15})k_y q, W_{28} = -\iota(\zeta_{31} + \zeta_{15})k_y q, W_{31} = \iota(c_{13} + c_{44})k_x q, W_{32} = \iota(c_{13} + c_{44})k_y q, W_{33} = c_{44}k_x^2 + c_{44}k_y^2 + c_{33}q^2 - dl(\rho_{11})_{33}, W_{34} = \iota m_{33}k_x q, W_{35} = \iota m_{33}k_y q, W_{36} = m_{33}q^2 - dl(\rho_{12})_{33},$$
$$W_{37} = e_{15}(k_x^2 + k_y^2) + e_{33}q^2, W_{38} = \zeta_{15}(k_x^2 + k_y^2) + \zeta_{33}q^2, W_{41} = m_{11}k_x^2 - dl(\rho_{12})_{11}, W_{42} = m_{11}k_x k_y, W_{43} = -\iota m_{33}k_x q, W_{44} = Rk_x^2 - dl(\rho_{22})_{11}, W_{45} = Rk_x k_y, W_{46} = -\iota Rk_x q, W_{47} = -\iota\tilde{\zeta}_3 k_x q, W_{48} = -\iota e_3^* k_x q, W_{51} = m_{11}k_x k_y, W_{52} = m_{11}k_y^2 - dl(\rho_{12})_{22}, W_{53} = -\iota m_{33}k_y q, W_{54} = Rk_x k_y,$$
$$W_{55} = Rk_y^2 - dl(\rho_{22})_{22}, W_{56} = -\iota Rk_y q, W_{57} = -\iota\tilde{\zeta}_3 k_y q, W_{58} = -\iota e_3^* k_y q, W_{61} = \iota m_{11}k_x q,$$

$W_{62} = \iota m_{11} k_y q, W_{63} = m_{33}q^2 - dl(\rho_{12})_{33}, W_{64} = \iota R k_x q, W_{65} = \iota R k_y q,$
$W_{66} = Rq^2 - dl(\rho_{22})_{33}, W_{67} = \tilde{\zeta}_3 q^2, W_{68} = e_3^* q^2, W_{71} = \iota(e_{31} + e_{15})k_x q, W_{72} = \iota(e_{31} + e_{15})k_y q,$
$W_{73} = e_{15}(k_x^2 + k_y^2) + e_{33}q^2, W_{74} = \iota\tilde{\zeta}_3 k_x q, W_{75} = \iota\tilde{\zeta}_3 k_y q, W_{76} = \tilde{\zeta}_3 q^2, W_{77} = -[\xi_{11}(k_x^2 + k_y^2) + \xi_{33}q^2],$
$W_{78} = -[A_{11}(k_x^2 + k_y^2) + A_{33}q^2], W_{81} = \iota(\zeta_{31} + \zeta_{15})k_x q, W_{82} = \iota(\zeta_{31} + \zeta_{15})k_y q, W_{83} = \zeta_{15}(k_x^2 + k_y^2) + \zeta_{33}q^2, W_{84} = \iota e_3^* k_x q, W_{85} = \iota e_3^* k_y q, W_{86} = e_3^* q^2, W_{87} = -[A_{11}(k_x^2 + k_y^2) + A_{33}q^2], W_{88} = -[\xi_{11}^*(k_x^2 + k_y^2) + \xi_{33}^* q^2].$

The coefficients $T_1, T_2, T_3, T_4, T_5, T_6, T_7$ are:

$$T_1 = T_{11}T_{17}, T_2 = T_{12}T_{17} + T_{18}T_{11}, T_3 = T_{13}T_{17} + T_{18}T_{12}, T_4 = T_{14}T_{17} + T_{18}T_{13}, T_5 = T_{15}T_{17} + T_{18}T_{14}, T_6 = T_{16}T_{17} + T_{18}T_{15}, T_7 = T_{18}T_{16},$$

$$T_{11} = b_{11}d_{11} - d_{21}b_{12} + a_1 d_{31} - b_{14}d_{41}, T_{12} = b_{11}d_{12} + g_{11}d_{11} - b_{12}d_{22} - g_{12}d_{21} + a_1 d_{32} + b_{13}d_{31} - b_{14}d_{42} - g_{14}d_{41}, T_{13} = b_{11}d_{13} + g_{11}d_{12} - b_{12}d_{23} - g_{12}d_{22} + a_1 d_{33} + b_{13}d_{32} + g_{13}d_{31} - b_{14}d_{43} - g_{14}d_{42}, T_{14} = b_{11}d_{14} + g_{11}d_{13} - b_{12}d_{24} - g_{12}d_{23} + a_1 d_{34} + b_{13}d_{33} + g_{13}d_{32} - b_{14}d_{44} - g_{14}d_{43}, T_{15} = b_{11}d_{15} + g_{11}d_{14} - b_{12}d_{25} - g_{12}d_{24} + b_{13}d_{34} + g_{13}d_{33} - b_{14}d_{45} - g_{14}d_{44}, T_{16} = g_{11}d_{15} - g_{12}d_{25} + g_{13}d_{34} - g_{14}d_{45}$$

$$T_{17} = c_{44}, T_{18} = c_{66}[k_x^2 + k_y^2] - dl(\bar{\rho}),$$

$d_{11} = b_{22}(b_{44}a_2 - a_3 b_{34}) + b_{24}(b_{32}a_3 - b_{42}a_2), d_{12} = b_{22}(b_{44}b_{33} + g_{44}a_2 - b_{43}b_{34} - g_{34}a_3) + g_{22}(b_{44}a_2 - a_3 b_{34}) - b_{23}(b_{44}b_{32} - b_{42}b_{34}), +b_{24}(a_3 g_{32} + b_{43}b_{32} - g_{42}a_2 - b_{42}b_{33}) + g_{24}(b_{32}a_3 - b_{42}a_2),$
$d_{13} = b_{22}(b_{44}g_{33} + g_{44}b_{33} - g_{43}b_{34} - b_{43}g_{34}) + g_{22}(b_{44}b_{33} + g_{44}a_2 - b_{43}b_{34} - g_{34}a_3) - b_{23}(b_{44}g_{32} + g_{44}b_{32} - b_{34}g_{42} - g_{34}b_{42}) - g_{23}(b_{44}b_{32} - b_{42}b_{34}) + b_{24}(b_{43}g_{32} + g_{43}b_{32} - g_{42}b_{33} - g_{33}b_{42}) + g_{24}(a_3 g_{32} + b_{43}b_{32} - g_{42}a_2 - b_{42}b_{33}), d_{14} = b_{22}(g_{44}g_{33} - g_{43}g_{34}) + g_{22}(b_{44}g_{33} + g_{44}b_{33} - g_{43}b_{34} - b_{43}g_{34}) - b_{23}(g_{44}g_{32} - g_{42}g_{34}) - g_{23}(b_{44}g_{32} + g_{44}b_{32} - b_{34}g_{42} - g_{34}b_{42}) + b_{24}(g_{43}g_{32} - g_{33}b_{42}) + g_{24}(b_{43}g_{32} + g_{43}b_{32} - g_{42}b_{33} - g_{33}b_{42}), d_{15} = g_{22}(g_{44}g_{33} - g_{43}g_{34}) - g_{23}(g_{44}g_{32} - g_{42}g_{34}) + g_{24}(g_{43}g_{32} - g_{33}b_{42}), d_{21} = b_{21}(b_{44}a_2 - a_3 b_{34}) + b_{24}(b_{31}a_3 - b_{41}a_2), d_{22} = b_{21}(b_{44}b_{33} + g_{44}a_2 - b_{43}b_{34} - g_{34}a_3) + g_{21}(b_{44}a_2 - a_3 b_{34}) - b_{23}(b_{44}b_{31} - b_{41}b_{34}) + b_{24}(a_3 g_{31} + b_{43}b_{31} - g_{41}a_2 - b_{41}b_{33}) + g_{24}(b_{31}a_3 - b_{41}a_2), d_{23} = b_{21}(b_{44}g_{33} + g_{44}b_{33} - g_{43}b_{34} - b_{34}g_{34}) + g_{21}(b_{44}b_{33} + g_{44}a_2 - b_{43}b_{34} - g_{34}a_3) - b_{23}(b_{44}g_{31} + g_{44}b_{31} - b_{34}g_{41} + g_{43}b_{41}) - g_{23}(b_{44}b_{31} - b_{41}b_{34}) + b_{24}(b_{43}g_{31} + g_{43}b_{31} - g_{41}b_{33} - g_{33}b_{41}) + g_{24}(a_3 g_{31} + b_{43}b_{31} - g_{41}a_2 - b_{41}b_{33}), d_{24} = b_{21}(g_{44}g_{33} - g_{43}g_{34}) + g_{21}(b_{44}g_{33} + g_{44}b_{33} - g_{43}b_{34} - b_{43}g_{34}) - b_{23}(g_{44}g_{31} - g_{41}g_{34}) - g_{23}(b_{44}g_{31} + g_{44}b_{31} - b_{34}g_{41} - g_{34}b_{41}) + b_{24}(g_{43}g_{31} - g_{33}b_{41}) + g_{24}(b_{43}g_{31} + g_{43}b_{31} - g_{41}b_{33} - g_{33}b_{41}), d_{25} = g_{21}(g_{44}g_{33} - g_{43}g_{34}) - g_{23}(g_{44}g_{31} - g_{41}g_{34}) + g_{24}(g_{43}g_{31} - g_{33}b_{41}),$
$d_{31} = b_{21}(b_{32}b_{44} - b_{42}b_{34}) - b_{22}(b_{31}b_{44} - b_{41}b_{34}) + b_{24}(b_{31}b_{42} - b_{32}b_{41}), d_{32} = b_{21}(b_{32}g_{44} + g_{32}b_{44} - b_{42}g_{34} - g_{42}b_{34}) + g_{21}(b_{32}b_{44} - b_{42}b_{34}) - b_{22}(b_{31}g_{44} - g_{31}b_{44} - g_{41}b_{34} - b_{41}g_{34}) - g_{22}(b_{31}b_{44} - b_{34}b_{41}) + b_{24}(b_{31}g_{42} + g_{31}b_{42} - b_{32}g_{41} - g_{32}b_{41}) + g_{24}(b_{31}b_{42} - b_{32}b_{41}), d_{33} = b_{21}(g_{32}g_{44} - g_{42}g_{34}) + g_{21}(b_{32}g_{44} + g_{32}b_{44} - b_{42}g_{34} - g_{42}b_{34}) - b_{22}(g_{31}g_{44} - g_{41}g_{34}) - g_{22}(b_{31}g_{44} + g_{31}b_{44} - g_{34}b_{41} - b_{34}g_{41}) + b_{24}(g_{31}g_{42} - g_{41}g_{32}) + g_{24}(b_{31}g_{42} + g_{31}b_{42} - b_{32}g_{41} - g_{32}b_{41}), d_{34} = g_{21}(g_{32}g_{44} - g_{42}g_{34}) - g_{22}(g_{31}g_{44} - g_{41}g_{34}) + g_{24}(g_{31}g_{42} - g_{41}g_{32}),$
$d_{41} = b_{22}(b_{41}a_2 - a_3 b_{31}) + b_{21}(b_{32}a_3 - b_{42}a_2), d_{42} = b_{22}(b_{41}b_{33} + g_{41}a_2 - b_{43}b_{31} - g_{31}a_3) + g_{22}(b_{41}a_2 - a_3 b_{31}) - b_{23}(b_{41}b_{32} - b_{42}b_{31}) + b_{21}(a_3 g_{32} + b_{43}b_{32} - g_{42}a_2 - b_{42}b_{33}) + g_{21}(b_{32}a_3 - b_{42}a_2), d_{43} = b_{22}(b_{41}g_{33} + g_{41}b_{33} - g_{43}b_{31} - b_{43}g_{31}) + g_{22}(b_{41}b_{33} - g_{41}a_2 - b_{43}b_{31} - g_{31}a_3) - b_{23}(b_{41}g_{32} + g_{41}b_{32} - b_{31}g_{42} + g_{31}b_{42}) - g_{23}(b_{41}b_{32} - b_{42}b_{31}) + b_{21}(b_{43}g_{32} + g_{43}b_{32} - g_{42}b_{33} - g_{33}b_{42}) + g_{21}(a_3 g_{32} + b_{43}b_{32} - g_{42}a_2 - b_{42}b_{33}), d_{44} = b_{22}(g_{41}g_{33} - g_{43}g_{31}) + g_{22}(b_{41}g_{33} + g_{41}b_{33} - g_{43}b_{31} - b_{43}g_{31}) - b_{23}(g_{41}g_{32} - g_{42}g_{31}) - g_{23}(b_{41}g_{32} + g_{41}b_{32} - b_{31}g_{42} - g_{31}b_{42}) + b_{21}(g_{43}g_{32} - g_{33}b_{42}) + g_{21}(b_{43}g_{32} + g_{43}b_{32} - g_{42}b_{33} - g_{33}b_{42}) d_{45} = g_{22}(g_{41}g_{33} - g_{43}g_{31}) - g_{23}(g_{41}g_{32} - g_{42}g_{31}) + g_{21}(g_{43}g_{32} - g_{33}b_{42}),$
$g_{11} = x_8, g_{12} = x_5(m_{33}dl(\rho_{22})_{33} - Rdl(\rho_{12})_{33}) + k^2 x_4 R(e_3^* e_{15} - \tilde{\zeta}_3\zeta_{15}), ; g_{13} = Rk^2(e_{15}e_3^* - \tilde{\zeta}_3\zeta_{15}), g_{14} = k^2 R[\zeta_{15}x_5 + x_6(e_3^* e_{15} - \tilde{\zeta}_3\zeta_{15}), g_{21} = x_2(x_4 dl(\rho_{12})_{33} - x_3 dl(\rho_{22})_{33}), g_{22} = x_5 x_2 dl(\rho_{22})_{33}, g_{23} = 0, g_{24} = 0, ; g_{31} = e_{15}k^2 R x_4 + \tilde{\zeta}_3(x_4 dl(\rho_{12})_{33} - x_3 dl(\rho_{22})_{33}), g_{32} = x_5 dl(\rho_{22})_{33}\tilde{\zeta}_3 + k^2 x_4 R(A_{11}\tilde{\zeta}_3 - \xi_{11}e_3^*), g_{33} = Rk^2(\tilde{\zeta}_3 A_{11} - \xi_{11}e_3^*), , g_{34} = k^2 R[x_6(A_{11}\tilde{\zeta}_3 - \xi_{11}e_3^*) - x_5 A_{11}], g_{41} = k^2 x_4(\tilde{\zeta}_3\zeta_{15} - e_3^* e_{15}), g_{42} = k^2 x_4(e_3^{*2}\xi_{11} + \tilde{\zeta}_3^2\xi_{11}^* - 2e_3^*\tilde{\zeta}_3 A_{11}), g_{43} = k^2(\xi_{11}e_3^{*2} + \tilde{\zeta}_3^2\xi_{11}^* -$

$2e_3^*\tilde{\zeta}_3A_{11}), g_{44} = k^2[x_5(A_{11}e_3^* - \tilde{\zeta}_3\xi_{11}^*) + x_6(e_3^{*2}\xi_{11} + \tilde{\zeta}_3^2\xi_{11}^* - 2e_3^*\tilde{\zeta}_3A_{11})],$
$b_{11} = (Rc_{33} - m_{33}^2)x_4, b_{12} = x_4(e_{33}e_3^* - \zeta_{33}\tilde{\zeta}_3), b_{13} = R^2x_2c_{44}k^2(e_{15}e_3^* - \tilde{\zeta}_3\zeta_{15}) + Rx_7(e_{33}e_3^* - \zeta_{33}\tilde{\zeta}_3) + k^2Rx_5[R(c_{13} + c_{13}) - m_{33}m_{11}], b_{14} = x_5(R\zeta_{33} - m_{33}e_3^*) + x_6R(e_{33}e_3^* - \zeta_{33}\tilde{\zeta}_3), b_{21} = [x_4(Rk^2m_{33}k_y - x_2m_{33}) - x_3(k^2R^2k_y - Rx_2)], b_{22} = x_5(k^2R^2k_y - Rx_2), b_{23} = Rx_5k_yk^2[m_{11}dl(\rho_{22})_{22} - Rdl(\rho_{12})_{22}], b_{24} = x_5(Rk^2k_ye_3^* - e_3^*x_2), b_{31} = x_4(e_{33}R - m_{33}\tilde{\zeta}), b_{32} = x_4[\tilde{\zeta}_3(A_{33}R) - e_3^*(\zeta_{33}R)], b_{33} = R^2x_2c_{44}k^2(\tilde{\zeta}_3A_{11} - \xi_{11}e_3^*) + Rx_7(A_{33}\tilde{\zeta}_3 - \xi_{33}e_3^*) + k^2Rx_5[R(e_{15} + e_{31}) - \tilde{\zeta}_3m_{11}], b_{34} = x_6\tilde{\zeta}_3A_{33}R - x_6e_3^*\xi_{33}R - x_5(A_{33}R + e_3^*\tilde{\zeta}_3), \quad b_{41} = x_4(\zeta_{33}\tilde{\zeta}_3 - e_{33}e_3^*), b_{42} = x_4[e_3^{*^2}\zeta_{33} + \tilde{\zeta}_3^2\zeta_{33}^* - 2e_3^*\tilde{\zeta}_3A_{33}], b_{43} = Rx_2c_{44}k^2(\xi_{11}e_3^{*2} + \tilde{\zeta}_3^2\xi_{11}^* - 2e_3^*\tilde{\zeta}_3A_{11}) + x_7(\xi_{33}e_3^{*2} + \tilde{\zeta}_3^2\xi_{33}^* - 2e_3^*\tilde{\zeta}_3A_{33}) + k^2Rx_5[e_3^*(e_{15} + e_{31}) - \tilde{\zeta}_3(\zeta_{15} + \zeta_{31})], b_{44} = x_5(A_{11}e_3^* - \tilde{\zeta}_3\xi_{11}^*) + x_6[e_3^{*^2}\xi_{33} + \tilde{\zeta}_3^2\xi_{11}^* - 2e_3^*\tilde{\zeta}_3A_{11}],$
$x_1 = [m_{11}k^2 - dl(\rho_{12})_{22}]k_y, x_2 = [Rk^2 - dl(\rho_{22})_{22}]k_y, x_3 = -x_1m_{33} + x_2(c_{13} + c_{44}), x_4 = -x_1R + x_2m_{11}, x_5 = x_2[\tilde{\zeta}_3(\zeta_{31} + \zeta_{15}) - e_3^*(e_{31} + e_{15})], x_6 = x_2(\zeta_{31} + \zeta_{15}) - x_1e_3^*, x_7 = k^2x_2(Rc_{11} - m_{11}^2) - x_2[m_{11}dl(\rho_{12})_{22} - Rdl(\rho_{11})_{22}] - x_1[m_{11}dl(\rho_{22})_{22} - Rdl(\rho_{12})_{22}], x_8 = x_4[m_{33}dl(\rho_{12})_{33} - Rdl(\rho_{11})_{33}] - x_3[m_{33}dl(\rho_{22})_{33} - Rdl(\rho_{12})_{33}], a_1 = R^2x_2c_{44}(e_{33}e_3^* - \zeta_{33}\tilde{\zeta}_3), a_2 = R^2x_2c_{44}(A_{33}\tilde{\zeta}_3 - \xi_{33}e_3^*), a_3 = Rx_2c_{44}k^2(\xi_{33}e_3^{*2} + \tilde{\zeta}_3^2\xi_{33}^* - 2e_3^*\tilde{\zeta}_3A_{33})].$

Appendix B

The elements of matrix Γ in Eq.(15) are:

$\Gamma_{1j} = [c_{44}(q_j - ik_xR_{2j}) - ie_{15}k_xR_{6j} - i\zeta_{15}k_xR_{7j}]\sin(q_jz + \psi), \Gamma_{2j} = [c_{44}(q_jR_{1j} - ik_yR_{2j}) - ie_{15}k_yR_{6j} - i\zeta_{15}k_yR_{7j}]\sin(q_jz + \psi), \Gamma_{3j} = [c_{13}k_x + c_{13}k_yR_{1j} - ic_{33}R_{2j}q_j + m_{33}(R_{3j}k_x + R_{4j}k_y - iR_{5j}q_j) - ie_{33}R_{6j}q_j - i\zeta_{33}R_{7j}q_j]\cos(q_jz + \psi), \Gamma_{4j} = [m_{11}k_x + im_{11}R_{1j}k_y - im_{33}R_{2j}q_j + R(R_{3j}k_x + k_yR_{4j} - iR_{5j}q_j) - i\tilde{\zeta}_3R_{6j}q_j - ie_3^*R_{7j}q_j]\cos(q_jz + \psi),$

$\Gamma_{5j} = R_{6j}\sin(q_jz + \psi), \Gamma_{6j} = R_{7j}\sin(q_jz + \psi). \qquad j = (1, 2, \ldots.6)$

Appendix C

The elements of matrix $\overline{W}$ in Eq.(17) are:

$\overline{W}_{11} = c_{11}k_x^2 + c_{44}q^2 - dl(\rho_{11})_{11}, \overline{W}_{12} = 0, \overline{W}_{13} = -\iota(c_{13} + c_{44})k_xq,$
$\overline{W}_{14} = m_{11}k_x^2 - dl(\rho_{12})_{11}, \overline{W}_{15} = 0, \overline{W}_{16} = -\iota m_{11}k_xq, \overline{W}_{17} = -\iota(e_{31} + e_{15})k_xq,$
$\overline{W}_{18} = -\iota(\zeta_{31} + \zeta_{15})k_xq, \overline{W}_{21} = 0, \overline{W}_{22} = c_{66}k_x^2 + c_{44}q^2 - dl(\rho_{11})_{22}, \overline{W}_{23} = 0, \overline{W}_{24} = 0, \overline{W}_{25} = m_{11}k_y^2 - dl(\rho_{12})_{22}, \overline{W}_{26} = 0,$
$\overline{W}_{27} = 0, \overline{W}_{28} = 0, \overline{W}_{31} = \iota(c_{13} + c_{44})k_xq, \overline{W}_{32} = 0,$
$\overline{W}_{33} = c_{44}k_x^2 + c_{33}q^2 - dl(\rho_{11})_{33}, \overline{W}_{34} = \iota m_{33}k_xq, \overline{W}_{35} = 0, \overline{W}_{36} = m_{33}q^2 - dl(\rho_{12})_{33},$
$\overline{W}_{37} = e_{15}(k_x^2) + e_{33}q^2, \overline{W}_{38} = \zeta_{15}(k_x^2) + \zeta_{33}q^2, \overline{W}_{41} = m_{11}k_x^2 - dl(\rho_{12})_{11}, \overline{W}_{42} = 0,$
$\overline{W}_{43} = -\iota m_{33}k_xq, \overline{W}_{44} = Rk_x^2 - dl(\rho_{22})_{11}, \overline{W}_{45} = 0, \overline{W}_{46} = -\iota Rk_xq, \overline{W}_{47} = -\iota\tilde{\zeta}_3k_xq,$
$\overline{W}_{48} = -\iota e_3^*k_xq, \overline{W}_{51} = 0, \overline{W}_{52} = -dl(\rho_{12})_{22}, \overline{W}_{53} = 0, \overline{W}_{54} = 0,$
$\overline{W}_{55} = -dl(\rho_{22})_{22}, \overline{W}_{56} = 0, \overline{W}_{57} = 0, \overline{W}_{58} = 0, \overline{W}_{61} = \iota m_{11}k_xq,$
$\overline{W}_{62} = 0, \overline{W}_{63} = m_{33}q^2 - dl(\rho_{12})_{33}, \overline{W}_{64} = \iota Rk_xq, \overline{W}_{65} = 0,$
$\overline{W}_{66} = Rq^2 - dl(\rho_{22})_{33}, \overline{W}_{67} = \tilde{\zeta}_3q^2, \overline{W}_{68} = e_3^*q^2, \overline{W}_{71} = \iota(e_{31} + e_{15})k_xq, \overline{W}_{72} = 0, \overline{W}_{73} = e_{15}(k_x^2) + e_{33}q^2, \overline{W}_{74} = \iota\tilde{\zeta}_3k_xq, \overline{W}_{75} = 0, \overline{W}_{76} = \tilde{\zeta}_3q^2, \overline{W}_{77} = -[\xi_{11}(k_x^2) + \xi_{33}q^2],$
$\overline{W}_{78} = -[A_{11}(k_x^2) + A_{33}q^2], \overline{W}_{81} = \iota(\zeta_{31} + \zeta_{15})k_xq, \overline{W}_{82} = 0, \overline{W}_{83} = \zeta_{15}(k_x^2) + \zeta_{33}q^2, \overline{W}_{84} = \iota e_3^*k_xq, \overline{W}_{85} = 0, \overline{W}_{86} = e_3^*q^2, \overline{W}_{87} = -[A_{11}(k_x^2) + A_{33}q^2], \overline{W}_{88} = -[\xi_{11}^*(k_x^2) + \xi_{33}^*q^2].$

The coefficients $\overline{T}_1, \overline{T}_2, \overline{T}_3, \overline{T}_4, \overline{T}_5, \overline{T}_6, \overline{T}_7, \overline{T}_8$ are:

$\overline{T}_1 = T_{11}, \overline{T}_2 = T_{12}, \overline{T}_3 = T_{13}, \overline{T}_4 = T_{14}, \overline{T}_5 = T_{15}, \overline{T}_6 = T_{16}, \overline{T}_7 = T_{17}, \overline{T}_8 = c_{66}k_x^2 - dl(\bar{\rho}),$

$\bar{d}_{ij} = d_{ij},$ and a_i, b_{ij}, g_{ij} in d_{ij} arereplacedwith $\bar{a}_i, \bar{b}_{ij}, \bar{g}_{ij}$ $\bar{a}_1 = x_{30}, \bar{a}_2 = x_{48}, \bar{a}_3 = x_{57}, \bar{b}_{11} = x_{27}, \bar{b}_{12} = x_{32}, \bar{b}_{13} = x_{29}, \bar{b}_{14} = x_{34}, \bar{b}_{21} = x_{37}, \bar{b}_{22} = x_{41}, \bar{b}_{23} = x_{38}, \bar{b}_{24} = x_{43}, \bar{b}_{31} = x_{46}, \bar{b}_{32} = x_{50}, \bar{b}_{33} = x_{47}, \bar{b}_{34} = x_{52}, \bar{b}_{41} = x_{55}, \bar{b}_{42} = x_{59}, \bar{b}_{43} = x_{56}, \bar{b}_{44} = x_{61}, \bar{g}_{11} = x_{26}, \bar{g}_{12} = x_{31}, \bar{g}_{13} = x_{28}, \bar{g}_{14} = x_{33}, \bar{g}_{21} = x_{35}, \bar{g}_{22} = x_{40}, \bar{g}_{23} = x_{39}, \bar{g}_{24} = x_{42}, \bar{g}_{31} = x_{44}, \bar{g}_{32} = x_{49}, \bar{g}_{33} = 0, \bar{g}_{34} = x_{51}, \bar{g}_{41} = x_{53}, \bar{g}_{42} = x_{58}, \bar{g}_{43} = 0, \bar{g}_{44} = x_{60},$

$x_{11} = [c_{11}k_x^2 - dl(\rho_{11})_{11}]dl(\rho_{22})_{11} - m_{11}k_x^2dl(\rho_{12})_{11} + dl(\rho_{12})_{11}^2, x_{12} = -[(c_{13} + c_{44})dl(\rho_{22})_{33} - m_{11}dl(\rho_{12})_{33}]\iota k_x, x_{13} = [\iota m_{11}k_x^3 - \iota k_xdl(\rho_{12})_{11}]dl(\rho_{22})_{33}, x_{14} = [(c_{13} + c_{44})dl(\rho_{22})_{11} - m_{33}dl(\rho_{12})_{11}]\iota k_x, x_{15} = [c_{44}k_x^2 - dl(\rho_{11})_{33}]dl(\rho_{22})_{33} + dl(\rho_{12})_{33}^2, x_{16} = [c_{33}dl(\rho_{22})_{33} -$

$m_{33}dl(\rho_{12})_{33}], x_{17} = m_{11}k_x^2 dl(\rho_{22})_{11} - Rk_x^2 dl(\rho_{12})_{11}, x_{18} = [m_{33}dl(\rho_{22})_{33} - Rdl(\rho_{12})_{33}]\iota k_x, x_{19} = dl(\rho_{22})_{33}\iota Rk_x^3 - \iota k_x dl(\rho_{22})_{11} dl(\rho_{22})_{33}, x_{20} = [(e_{15}+e_{31})dl(\rho_{22})_{11} - \tilde{\zeta}_3 dl(\rho_{22})_{11}]\iota k_x, x_{21} = e_{15}dl(\rho_{22})_{33}k_x^2, x_{22} = [e_{33}dl(\rho_{22})_{33} - \tilde{\zeta}_3 dl(\rho_{12})_{33}], x_{23} = [(\zeta_{15}+\zeta_{31})dl(\rho_{22})_{11} - e_3^* dl(\rho_{22})_{11}]\iota k_x, x_{24} = \zeta_{15}dl(\rho_{22})_{33}k_x^2, x_{25} = [\zeta_{33}dl(\rho_{22})_{33} - e_3^* dl(\rho_{12})_{33}], x_{26} = [x_{12}x_{17} + x_{18}x_{11}], x_{27} = c_{44}dl(\rho_{22})_{11}x_{18}, x_{28} = [x_{13}x_{17} - x_{11}x_{19}], x_{29} = \iota k_x m_{11}x_{17}dl(\rho_{22})_{11} - \iota k_x x_{11}Rdl(\rho_{22})_{11} - c_{44}x_{19}dl(\rho_{22})_{11}, x_{30} = -\iota Rk_x c_{44}dl(\rho_{22})_{11}^2, x_{31} = [x_{11}\tilde{\zeta}_3 - (e_{31}+e_{15})x_{17}]\iota k_x, x_{32} = \iota k_x \tilde{\zeta}_3 c_{44}dl(\rho_{22})_{11}, x_{33} = [x_{11}e_3^* - (\zeta_{31}+\zeta_{15})x_{17}]\iota k_x, x_{34} = \iota k_x e_3^* c_{44}dl(\rho_{22})_{11}, x_{35} = x_{15}x_{17}, x_{37} = [x_{16}x_{17} + x_{18}x_{14}], x_{38} = dl(\rho_{22})_{11}x_{17}dl(\rho_{12})_{33} - dl(\rho_{22})_{33}x_{17}m_{33}k_x^2 - x_{14}x_{19}, x_{39} = -dl(\rho_{22})_{11}[x_{17}m_{33} + \iota Rk_x x_{14}], x_{40} = e_{15}x_{17}k_x^2, x_{41} = e_{33}x_{17} + \iota k_x x_{14}\tilde{\zeta}_3, x_{42} = x_{17}\zeta_{15}k_x^2, x_{43} = \zeta_{33}x_{17} + \iota k_x x_{14}e_3^*, x_{44} = x_{21}x_{17}, x_{46} = [x_{18}x_{20} + x_{17}x_{22}], x_{47} = -[dl(\rho_{22})_{33}x_{17}\zeta_3 k_x^2 + x_{19}x_{20}], x_{48} = -dl(\rho_{22})_{11}[x_{17}\tilde{\zeta}_3 + \iota Rk_x x_{20}], x_{49} = -x_{17}\xi_{11}k_x^2, x_{50} = \iota k_x x_{20}\tilde{\zeta}_3 - \xi_{33}x_{17}, x_{51} = -A_{11}x_{17}k_x^2, x_{52} = \iota k_x x_{20}e_3^* - A_{33}x_{17}, x_{53} = x_{17}x_{24}, x_{55} = [x_{18}x_{23} + x_{25}x_{17}], x_{56} = -[dl(\rho_{22})_{33}e_3^* x_{17}k_x^2 + x_{19}x_{23}], x_{57} = -dl(\rho_{22})_{11}[e_3^* x_{17} + \iota Rk_x x_{23}], x_{58} = -A_{11}x_{17}k_x^2, x_{59} = [\iota k_x x_{23}\tilde{\zeta}_3 - A_{33}x_{17}], x_{60} = -\xi_{11}^* x_{17}k_x^2, x_{61} = [\iota k_x x_{23}e_3^* - \xi_{33}^* x_{17}].$

Appendix D

The elements of matrix $\bar{\Gamma}$ in Eq.(23) are:

$$\bar{\Gamma}_{1j} = [c_{44}(q_j - \iota k_x R_{1j}) - \iota e_{15}k_x R_{4j} - \iota\zeta_{15}k_x R_{5j}]\sin(q_j z + \psi), \bar{\Gamma}_{2j} = [c_{13}k_x - \iota c_{33}R_{1j}q_j + m_{33}(R_{2j}k_x - \iota R_{3j}q_j) - \iota e_{33}R_{4j}q_j - \iota\zeta_{33}R_{5j}q_j]\cos(q_j z + \psi), \bar{\Gamma}_{3j} = [m_{11}k_x - \iota m_{33}R_{1j}q_j + R(R_{2j}k_x - \iota R_{3j}q_j) - \iota\tilde{\zeta}_3 R_{4j}q_j - \iota e_3^* R_{5j}q_j]\cos(q_j z + \psi),$$

$$\bar{\Gamma}_{4j} = R_{4j}\sin(q_j z + \psi),$$

$$\bar{\Gamma}_{5j} = R_{5j}\sin(q_j z + \psi).$$ j=(1, 2,....5)

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