STUDY OF THE HEAT SOURCE PROBLEM IN A CYLINDRICAL BODY WITH CAPUTO-FABRIZIO DERIVATIVES

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Abstract
In the framework of the fractional-order Caputo-Fabrizio differential, the mathematical modelling is established for a thick cylindrical body under the influence of a heat source. The Robin conditions for the limits of heat exchange on the curved outer surface are set to zero, while additional cross-sectional heating has been applied to the top and bottom surfaces of the body. The governing equation for the mathematical modelling is solved analytically and evaluated computationally using the integral transformation technique. The results obtained from the thermal variations are presented graphically and illustrated numerically, assuming the material properties of aluminium metal.

Keywords: Caputo-Fabrizio derivative, thermal variation, integral transform, cylindrical body.

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Introduction

In the last decade, many studies have proved that fractional calculus has successfully described the physical process that occurs at the microscopic level. Also, mathematical modelling based on fractional calculus is suitable because it predicts a retarded response, not the instant that happens while using a traditional model. Caputo and Mainardi [1, 2, 3 4] analysed memory mechanisms in linear dissipation models or elastic solids and showed experimental results. A thermal theory of a fractional kind that incorporates memory influence is used in the study of thermal elastic problems. Povstenko [7, 8] first published research on fractional-order thermoelasticity. In a non-axisymmetric problem of solving the infinite cylindrical structure, Povstenko [9, 10] derived an appropriate diffuse waveform equation's response by employing the time order of the fractional derivative.

A novel definition of fractional order derivative without a single kernel was provided by Caputo and Fabrizio [11]. Improved fractional derivatives that are compatible and utilise a non-singular and non-local kernel have been put forward by Abdon and Dumitru [12]. Shaikh et al. Provided a nonlinear differential model that included the Caputo-Fabrizio statement in [13] and demonstrated its existence and uniqueness criterion. They also evaluated its solution by applying the method of the iterative Laplace transform. The improved definition, which includes the Caputo-Fabrizio fractional differential equation operator combined with the nonlinear fraction heat theory, has been described by Yepez and Gomez [14]. Amal et al. [15] proposed an analytical study that involves differential operators of the Caputo-Fabrizio type with a nonsingular kernel and trigonometric and exponential functions. Making use of the fundamental transformation, Maiti et al. [16] investigated the blood flow concept with a fraction time-order derivation under thermal radiation. Also, they find the blood flow velocity, temperature, and concentration. Thakare et al. [17] examined the impact of inhomogeneity on a two-dimensional thick hollow cylinder under fractional order derivatives by the integral transform method. They also expressed the numerical results for both homogeneous and nonhomogeneous cases. Elhagary [18,19] studied the fractional-order infinite medium problem of spherical cavities with heating. Gaikwad and Bhandwalkar [20] adopted a direct approach to finding a heating response in generalised two-temperature thermoelastic problems with the influence of the differential derivative of the Caputo-Fabrizio kind. Lamba and Deshmukh [21] determined the hygrothermoelastic response in a finitely extended hollow cylindrical region analytically and discussed the impacts of the thermal variation in the circular cylindrical region for composite material. Verma et al. [22] studied memory impact in a hollow body and successfully examined temperature and moisture effects on diffusion wave theory According to the time-fractional order theory and the convective heating exchange boundary condition, Lamba [23] examined the manner in which a thermosensitive hollow cylinder would respond. Also, Verghese and Khalsa [26] and Gahane et al. [27] contributed their work by considering cylindrical body with heat source.

Recently, Khalil et al. [24] described the viscothermoelastic phenomenon and thermodynamics by modelling mathematical models using derivatives of the Caputo-Fabrizio type. Abouelregal et al. [6] assumed the heat equation with thermal relaxation time and the Caputo-Fabrizio differential operator, as well as the semi-infinite space model exposed to changing heat sources. Further, they apply numerical Zakian's algorithm to solve the governing equation and illustrate the effect of fractional and magnetic fields graphically.

Heat Equation with Caputo and Fabrizio Differential Operator

Consider a cylindrical body (geometry of the problem is as represented in Fig. 1) with a source of heating taking up the physical space $\{a \le r \le b, -h \le z \le h\}$. All other properties are taken into account as constant, and the cylindrical body's material characteristics are homogenized and isotropic in nature The stresses are to be computed in a robin-type heat transfer process with an internal thermal source and according to boundary conditions.

The differential equation is satisfied by the below equation for the operator of Caputo Fabrizio kind:

$$\left[\theta''_{rr} + \frac{1}{r}\theta'_{r} + \theta''_{zz}\right] + k\,\Theta = k\,\frac{\partial^{\alpha}\theta}{\partial t^{\alpha}}; \quad \alpha \in (0,1) \quad (1)$$

Here, prime in the above equations denotes the derivative w.r.t. The suffix variable.

Here Θ is the internal source function, λ is the thermal conductivity of the material, $\kappa = \lambda / \rho C$, ρ that is its density, and C is its heat capacity, which is supposed to be constant.

Also, $\frac{\partial^\alpha \theta}{\partial t^\alpha}$ denotes Caputo and Fabrizio differential operator specified as

$$\frac{\partial^\alpha \theta}{\partial t^\alpha} = \frac{1}{1-\alpha}\int_0^t \theta'(t)\exp\left(-\alpha\frac{(t-\tau)}{1-\alpha}\right)d\tau, \qquad 0 \le \alpha \le 1 \quad (2)$$

For the purpose of ease, let's contemplate.

$$\Theta = \frac{\delta(r-r_0)\delta(z-z_0)}{2\pi r_0}e^{-\omega t}, \; a \le r_0 \le b, \; -h \le z_0 \le h, \; \omega > 0 \quad (3)$$

Boundary Restrictions for Initial and Convective Thermal Transfer

The initial and ending conditions for the aforementioned heating equations are as follows [26, 27]:

$$\left[\theta(t=0)\right] = 0 \quad (4)$$

$$\left[\theta(r=a) + k_1\frac{\partial\theta(r=a)}{\partial r}\right] = 0 \quad (5)$$

$$\left[\theta(r=b) + k_2\frac{\partial\theta(r=b)}{\partial r}\right] = 0 \quad (6)$$

$$\left[\theta(z=h) + k_3\frac{\partial\theta(z=h)}{\partial z}\right] = \delta(r-r_0)e^{-\omega t} \quad (7)$$

$$\left[\theta(z=-h) + k_4\frac{\partial\theta(z=-h)}{\partial z}\right] = \delta(r-r_0)e^{-\omega t} \quad (8)$$

$\delta(r-r_0)$ is the Dirac Delta operation with $a \le r_0 \le b$; $\omega > 0$ is the value of constant; $\exp(-\omega t)\delta(r-r_0)$ is the extra sectional heat present on its surface at $z = \pm h$; k_i ; $i = 1,2,3,4$ are radiation factors respectively.

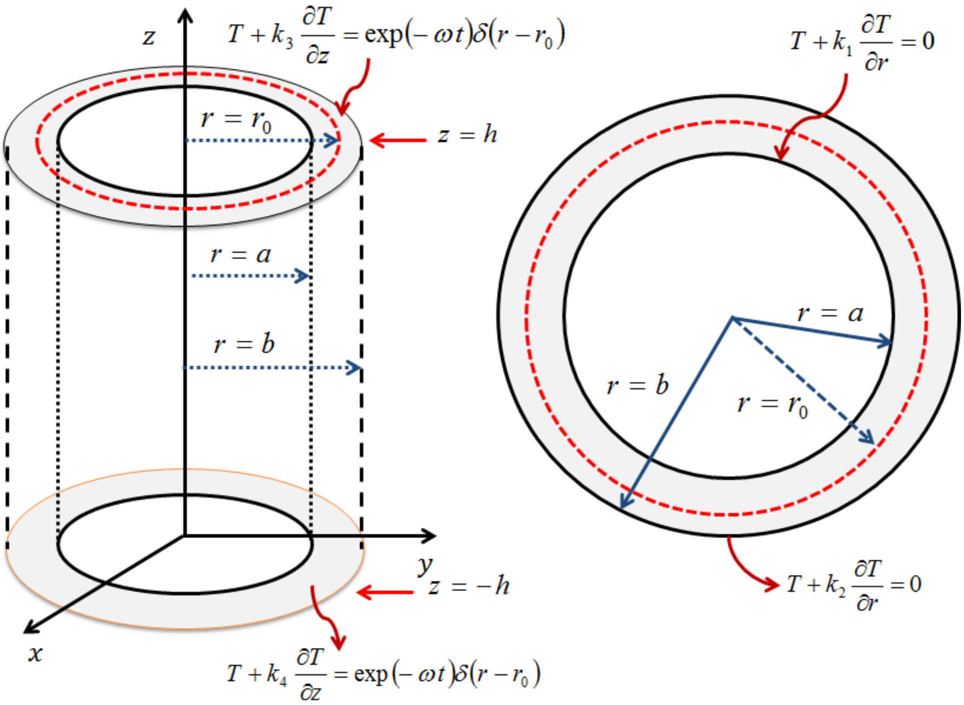

Fig.1: Geometry of the thermoelastic problem of thick hollow cylinder

Displacement Function and Stress relationship

In any two-dimensional axisymmetric thermoelastic difficulty in nature, Navier's formulas excluding structural forces are given as.

$$(1-2\upsilon)\left(\nabla^2 u_r - \frac{1}{r}u_r\right) - 2(1+\upsilon)\alpha_t\theta'_r + e'_r = 0, \quad (9)$$

$$(1-2\upsilon)\nabla^2 u_z - 2(1+\upsilon)\alpha_t\theta'_z - e'_z = 0 \quad (10)$$

Where, respectively, u_z, u_r, and e, represent the axial, radial, and dilation portions.

Goodier's thermoelastic displacing potential $\phi(r,z,t)$ and Michell's function M serve as representations of the displacement function in the cylindrical coordinate system.

$$u_r = \varphi'_r - M''_{rz}, \quad (11)$$

$$u_z = \varphi'_z + 2(1-\upsilon)\nabla^2 M - M''_{zz} \quad (12)$$

Where the equation for Goodier's thermoelastic potential must hold

$$\nabla^2\phi = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\theta \quad (13)$$

The Michell's function, M must respond to the equation

$$\nabla^2(\nabla^2 M) = 0 \quad (14)$$

The elements of the stresses are depicted as follows:

$$\sigma_{rr} = 2G\left\{\left(\varphi''_{rr} - \nabla^2\varphi\right) + \left(\upsilon\nabla^2 M - M''_{rr}\right)'_z\right\} \quad (15)$$

$$\sigma_{\theta\theta} = 2Gr\left\{\left(\varphi'_r - r\nabla^2\varphi\right) + \left(r\upsilon\nabla^2 M - M'_r\right)'_z\right\} \quad (16)$$

$$\sigma_{zz} = 2G\left\{\left(\varphi''_{zz} - \nabla^2\varphi\right) + \left((2-\upsilon)\nabla^2 M - M''_{zz}\right)'_z\right\} \quad (17)$$

$$\sigma_{rz} = 2G\left\{\varphi''_{rz} + \left((1-\upsilon)\nabla^2 M - M''_{zz}\right)'_r\right\} \quad (18)$$

where G and υ are the shear modulus and Poisson's ratio, respectively.

The boundary conditions on the traction-free surface stress functions are

$$\sigma_{zz}\big|_{z=\pm h} = \sigma_{rz}\big|_{z=\pm h} = 0 \quad (19)$$

The area under study is mathematically formulated by the equations (1) through (19).

Solution of Temperature Distribution

When the equations (1) to (8) are subjected to the finite integral Marchi-Zgrablich transform described in [5], one obtains

$$\kappa\left[-\mu_n^2\bar{\theta} + \bar{\theta}''_{zz}\right] + \frac{1}{2\pi r_0}\delta(z-z_0)\, r_0\, L_0(k_1,k_2,\mu_n r_0)e^{-\omega t} = \frac{\partial^\alpha\bar{\theta}}{\partial t^\alpha} \quad (20)$$

$$\left[\bar{\theta}(t=0)\right] = 0 \quad (21)$$

$$\left[\bar{\theta}(z=h) + k_3\frac{\partial\bar{\theta}(z=h)}{\partial z}\right] = e^{-\omega t} r_0\, L_0(k_1,k_2,\mu_n r_0) \quad (22)$$

$$\left[\bar{\theta}(z=-h) + k_4\frac{\partial\bar{\theta}(z=-h)}{\partial z}\right] = e^{-\omega t} r_0\, L_0(k_1,k_2,\mu_n r_0) \quad (23)$$

Where $\bar{\theta}$ represents the transformed function of θ, n the transform parameter, and μ_n the characteristic equation's positive roots.

$$J_0(k_2,\mu b)\, Y_0(k_1,\mu a) = J_0(k_1,\mu a)\, Y_0(k_2,\mu b)$$

Next when equations (20), (22) and (23), together with the finite Marchi-Fasulo transformation given in [6], combine to produce, one obtains

$$\frac{\partial^\alpha\bar{\theta}^*}{\partial t^\alpha} + \kappa(\Lambda_{n,m})\bar{\theta}^* = H(\mu_n, a_m)\exp(-\omega t) \quad (24)$$

$$\left[\bar{\theta}^*(t=0)\right] = 0 \quad (25)$$

Where $\Lambda_{n,m} = \mu_n^2 + a_m^2$

And

$$H(\mu_n, a_m) = \left\{\frac{P_m(h)\kappa}{k_3} - \frac{P_m(-h)\kappa}{k_4} + \frac{P_m(z_0)}{2\pi r_0}\right\} r_0\, L_0(k_1,k_2,\mu_n r_0) \quad (26)$$

Where m is the transform parameter and $\bar{\theta}^*$ stands for the Marchi-Fasulo integral transform of $\bar{\theta}$.

Next, applying integral Laplace transform and taking their corresponding inverse above equation (24) on using (25), becomes

$$\bar{\theta}^*(n,m,t) = \frac{H(\mu_n, a_m)}{\kappa(\Lambda_{n,m}) - \omega}\left[\frac{l_1[l_2-\omega]}{[l_3-\omega l_4]}e^{-\omega t} + \frac{l_1[l_2 l_4 - l_3]}{l_4[\omega l_4 - l_3]}e^{-\frac{l_3}{l_4}t}\right] \quad (27)$$

When the integral Marchi-Fasulo transform and the integral Marchi-Zgrablich transform are inverted and applied to the equation (27), one gets

$$\theta(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\Omega_{n,m}}{\lambda_m}\left[\frac{l_1[l_2-\omega]}{[l_3-\omega l_4]}e^{-\omega t} + \frac{l_1[l_2 l_4 - l_3]}{l_4[\omega l_4 - l_3]}e^{-\frac{l_3}{l_4}t}\right]\right\} P_m(z) L_0(k_1,k_2,\mu_n r) \quad (28)$$

Where $\Omega_{n,m} = \frac{H(\mu_n, a_m)}{\kappa(\Lambda_{n,m}) - \omega}$, $(1-\alpha) = l_1$, $\frac{\alpha}{(1-\alpha)} = l_2$,

$\kappa\Lambda_{m,n}\,\alpha = l_3$ and $(1 + \kappa\Lambda_{m,n}\, l_1) = l_4$

The temperature of the cylindrical body is represented by the function in equation (28) at all times and in every point.

Thermoelastic Solution

Referring the equations (13) and (28), the solution for Goodier's thermoelastic displacement potential ϕ is shown as

$$\varphi=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-P_m(z)\Omega_{n,m}}{\lambda_m(\Lambda_{n,m})}\left[\frac{l_1[l_2-\omega]}{[l_3-\omega l_4]}e^{-\omega t}+\frac{l_1[l_2 l_4-l_3]}{l_4[\omega l_4-l_3]}e^{-\frac{l_3}{l_4}t}\right]\right\}$$

$$\times L_0(k_1,k_2,\mu_n r) \qquad (29)$$

Similar to this, it is believed that the Michell's function solution M satisfies equation (14)'s governed condition as

$$M=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\Omega_{n,m}}{\lambda_m(\Lambda_{n,m})}\left[\frac{l_1[l_2-\omega]}{[l_3-\omega l_4]}e^{-\omega t}+\frac{l_1[l_2 l_4-l_3]}{l_4[\omega l_4-l_3]}e^{-\frac{l_3}{l_4}t}\right]\right\}$$

$$\times[B_{nm}\sinh(\mu_n z)+C_{nm}z\cosh(\mu_n z)]L_0(k_1,k_2,\mu_n r) \quad (30)$$

Using (29) and (30) in equations (11) and (12), one obtains

$$u_r=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\Omega_{n,m}}{\lambda_m(\Lambda_{n,m})}\left[\frac{l_1[l_2-\omega]}{[l_3-\omega l_4]}e^{-\omega t}+\frac{l_1[l_2 l_4-l_3]}{l_4[\omega l_4-l_3]}e^{-\frac{l_3}{l_4}t}\right]\right\}$$

$$\times\left[\{P_m(z)-[(B_{nm}\mu_n+C_{nm})\cosh(\mu_n z)+C_{nm}z\sinh(\mu_n z)\}\right]$$

$$\times L_0'(k_1,k_2,\mu_n r) \qquad (31)$$

$$u_z=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\Omega_{n,m}}{\lambda_m(\Lambda_{n,m})}\left[\frac{l_1[l_2-\omega]}{[l_3-\omega l_4]}e^{-\omega t}+\frac{l_1[l_2 l_4-l_3]}{l_4[\omega l_4-l_3]}e^{-\frac{l_3}{l_4}t}\right]\right\}$$

$$\times\{\,[-a_m(Q_m\sin(a_m z)+W_m\cos(a_m z))$$

$$-\mu_n^2(-1+2\upsilon)(B_{nm}\sinh(\mu_n z)+C_{nm}\,z\cosh(\mu_n z))]\,\}$$

$$-2(-1+2\upsilon)C_{nm}\sinh(\mu_n z)\mu_n]\,S_0(k_1,k_2,\mu_n r)$$

$$+\mu_n(2(1-\upsilon))\,[B_{nm}\sinh(\mu_n z)+C_{nm}\,z\cosh(\mu_n z)][\mu_n L_0''(k_1,k_2,\mu_n r)$$

$$+\frac{1}{r}L_0'(k_1,k_2,\mu_n r)\,\} \qquad (32)$$

Now using the equations (29) and (30) in equations (15) to (18), one obtains the expression for stress components as

$$\sigma_{rr}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\Omega_{n,m}}{\lambda_m(\Lambda_{n,m})}\left[\frac{l_1[l_2-\omega]}{[l_3-\omega l_4]}e^{-\omega t}+\frac{l_1[l_2 l_4-l_3]}{l_4[\omega l_4-l_3]}e^{-\frac{l_3}{l_4}t}\right]\right\}$$

$$\times\{-\frac{P_m(z)}{r}[L_0'(k_1,k_2,\mu_n r)-r\,a_m^2\,L_0(k_1,k_2,\mu_n r)]$$

$$+\mu_n^2(\upsilon-1)\,[\mu_n\left(B_{nm}\cosh(\mu_n z)+C_{nm}(z\sinh(\mu_n z)+\cosh(\mu_n z)\right)\,L_0''(k_1,k_2,\mu_n r)$$

$$+\mu_n\,\upsilon\,[\,B_{nm}\cosh(\mu_n z)\mu_n+C_{nm}\,(z\sinh(\mu_n z)+\cosh(\mu_n z))]$$

$$\times\frac{1}{r}[L_0'(k_1,k_2,\mu_n r)+r\mu_n L_0(k_1,k_2,\mu_n r)]$$

$$+2\upsilon\,C_{nm}\,\mu_n^2\cosh(\mu_n z)\,L_0(k_1,k_2,\mu_n r)]\} \qquad (33)$$

$$\sigma_{\theta\theta}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\Omega_{n,m}}{\lambda_m(\Lambda_{n,m})}\left[\frac{l_1[l_2-\omega]}{[l_3-\omega l_4]}e^{-\omega t}+\frac{l_1[l_2 l_4-l_3]}{l_4[\omega l_4-l_3]}e^{-\frac{l_3}{l_4}t}\right]\right\}$$
$$\times\{-P_m(z)[\mu_m^2 L_0''(k_1,k_2,\mu_n r)-a_m^2 L_0(k_1,k_2,\mu_n r)]$$
$$+\frac{\mu_n(\upsilon-1)}{r}\ [(\mu_n B_{nm}+C_{nm})\cosh(\mu_n z)+\mu_n C_{nm} z\sinh(\mu_n z)]\ L_0'(k_1,k_2,\mu_n r)$$
$$+\mu_n^2\upsilon\,[(\mu_n B_{nm}+C_{nm})\cosh(\mu_n z)+\mu_n C_{nm} z\sinh(\mu_n z)]$$
$$[L_0''(k_1,k_2,\mu_n r)+L_0(k_1,k_2,\mu_n r)]$$
$$+2\upsilon C_{nm}\,\mu_n^2\cosh(\mu_n z)\,L_0(k_1,k_2,\mu_n r)]\} \qquad (34)$$

$$\sigma_{zz}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\Omega_{n,m}}{\lambda_m(\Lambda_{n,m})}\left[\frac{l_1[l_2-\omega]}{[l_3-\omega l_4]}e^{-\omega t}+\frac{l_1[l_2 l_4-l_3]}{l_4[\omega l_4-l_3]}e^{-\frac{l_3}{l_4}t}\right]\right\}$$
$$\times\left\{-\frac{\mu_n P_m(z)}{r}\left[r\,\mu_n L_0''(k_1,k_2,\mu_n r)+L_0(k_1,k_2,\mu_n r)\right]\right.$$
$$+\mu_n^2\ [B_{nm}\cosh(\mu_n z)+C_{nm}\,z\sinh(\mu_n z)](2-\upsilon)$$
$$[\mu_n\,L_0''(k_1,k_2,\mu_n r)+r^{-1}L_0(k_1,k_2,\mu_n r)]$$
$$+(1-\upsilon)\,\mu_n\,L_0(k_1,k_2,\mu_n r)$$
$$+\mu_n\,C_{nm}\cosh(\mu_n z)\frac{1}{r}[(2-\upsilon)[r\mu_n\,L_0''(k_1,k_2,\mu_n r)+L_0(k_1,k_2,\mu_n r)]$$
$$\left.+(1-\upsilon)\,\mu_n^2\,L_0(k_1,k_2,\mu_n r)]\ \right\} \qquad (35)$$

$$\sigma_{rz}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-\Omega_{n,m}}{\lambda_m(\Lambda_{n,m})}\left[\frac{l_1[l_2-\omega]}{[l_3-\omega l_4]}e^{-\omega t}+\frac{l_1[l_2 l_4-l_3]}{l_4[\omega l_4-l_3]}e^{-\frac{l_3}{l_4}t}\right]\right\}$$
$$\times\{\ [-\mu_n a_m(Q_m\sin(a_m z)+W_m\cos(a_m z))]L_0'(k_1,k_2,\mu_n r)$$
$$+[(B_{nm}\sinh(\mu_n z)+C_{nm}\,z\cosh(\mu_n z))\,\mu_n^2\,[\upsilon\mu_n+\frac{(1-\upsilon)}{r}]$$
$$-2\upsilon\mu_n^2 C_{nm}\ \sinh(\mu_n z)]\ L_0'(k_1,k_2,\mu_n r)$$
$$+(1-\upsilon)[B_{nm}\sinh(\mu_n z)+C_{nm}\,z\cosh(\mu_n z)]$$
$$\times\frac{1}{r^2}[r^2\mu_n^3\,L_0'''(k_1,k_2,\mu_n r)-L_0(k_1,k_2,\mu_n r)]\} \qquad (36)$$

Where unknown B_{nm} and C_{nm} can be easily evaluated by using boundary condition (19) to the equation (35) and (36).

Limiting case

As a limiting case, set $\alpha=1$
Then equation (1) can rewritten as

$$\kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial z^2}\right]+\Theta=\frac{\partial\theta}{\partial t} \qquad (37)$$

Which is nothing but a classical equation of heat transfer in cylindrical coordinates under the influence of a thermal source.
Following the integral transformation method stated above, one can easily find out the complete

solution of equation (37) in terms of temperature and stresses, respectively by putting $\alpha = 1$ in equation (28), (31) to (36).

Numerical computation

A cylinder is supposed to have a thickness ranging from $z = -1$ to $z = 1$ and a radius ranging from $r = 1$ to $r = 2$ for the purposes of numerical computation.

Further, it is considered that thick hollow cylinder is made up of aluminium metal whose material properties are as follows:

Thermal diffusivity, κ (cm^2/sec) = 0.86; the coefficient factor of thermal expansion, α_t (cm/cm-^{0}C)= 25.5 × 10^{-6};Poisson ratio, υ=0.281;Shear modulus=2.7 × 10^{11}, G (dynes/cm^2) ;Modulus of Elasticity, E (dynes/cm^2)= 6.9 × 10^{11}.

Also, fixing $k_1 = 0.86$ $k_2 = 1$, r_0 = 1.5, z_0 = 0.1 and $\omega = 0.5$.

Graphical representation

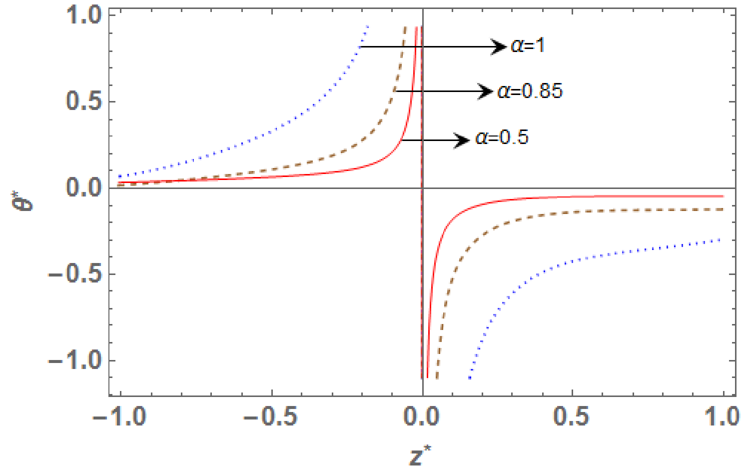

Fig 2: Variation of dimensionless temperature function along dimensionless thickness for various values of fractional variables

Fig. **2** shows the variation of the dimensionless temperature behaviour along the dimensionless thickness direction for a hollow cylinder with a linear relationship between a heat source and temperature. Additional sectional heating impact was also applied to the lower and upper surfaces of the cylinder. Due to sectional heating, it is clear from the graphical analysis that the temperature is something other than zero at the bottom and top surfaces. Throughout the thickness, it also increases, reaches its maximum value in the middle, and then begins to decrease towards the top for various fractional parameters. Further, it is seen that temperature distribution depends on the different values of fractional, and for higher values, a large variation in the curve is noted.

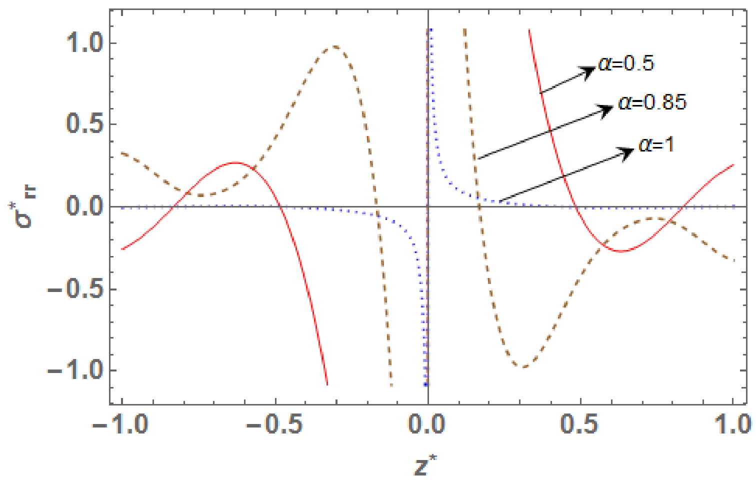

Fig 3: Variation of radial stress along dimensionless thickness for various values of fractional variables

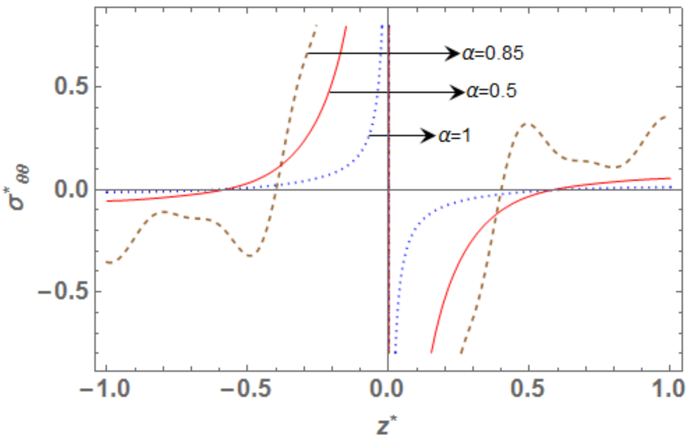

Fig 4: Variation of tangential stress along dimensionless thickness for various values of fractional variables

Figs. 3 and **4** represent the deviations of dimensionless radial and tangential stress around the dimensional thickness direction for various values of fractional variables. In both cases, the maximum impact of the stress function occurs in the middle of the cylinder. Further, the non-zero stress reflection is due to additional sectional heating at the bottom and top surfaces of the assumed cylinder. Also, radial and tangential stress is found to be tensile at the bottom and top surfaces and seems compressive in the middle of the cylindrical region.

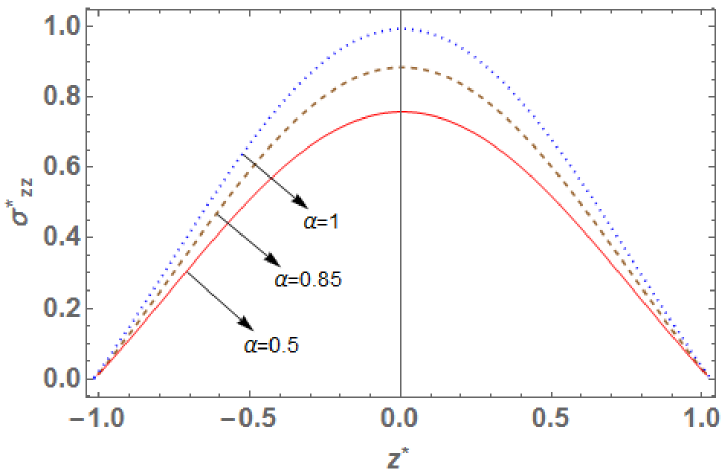

Fig 5: Variation of axial stress along dimensionless thickness for various values of fractional variables

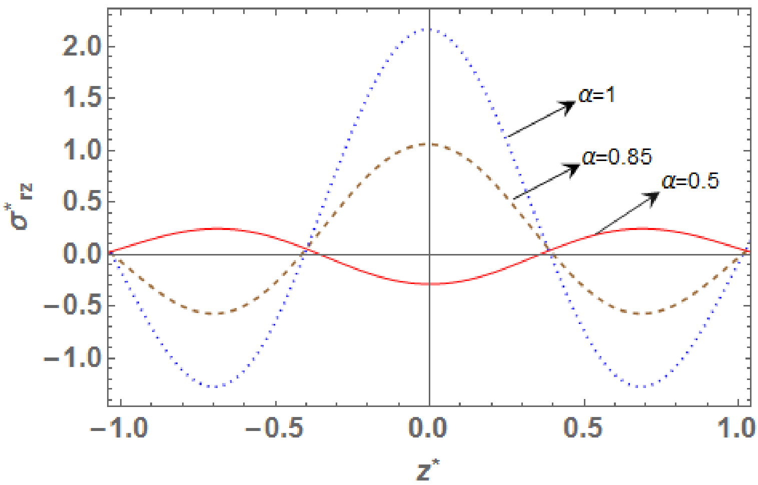

Fig 6: Variation of shear stress along dimensionless thickness for various values of fractional variables

Figs. 5 and 6 represent the distribution of dimensionless axial and shear stress around the thickness direction for a thick hollow cylinder and for various values of fractional variables. Initially, at the thick cylinder's upper and bottom surfaces, the effect of the stress function is zero, which matches the prescribed mathematical boundary condition defined in equation (22). Further, the maximum stress response is found in the middle of the cylinder due to the influence of the heat source. Additionally, it is discovered that the variance in stresses in both figures is directly correlated with the various values of fractional derivatives. The variation in stresses in both depicts is also found to be strongly connected to the different fractional order derivative values.

The series solution converges

Let's examine at the manner in which the series solution converges.

$$A_n = e\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m}\left[\frac{l_1\left[l_2-\omega\right]}{\left[l_3-\omega l_4\right]}e^{-\omega t}+\frac{l_1\left[l_2\,l_4-l_3\right]}{l_4\left[\omega l_4-l_3\right]}e^{-\frac{l_3}{l_4}t}\right]\right\}\times P_m(z)\;S_0(k_1,k_2,\mu_n r)$$

Then

$$\theta(r,z,t)=\sum_{n=1}^{\infty}A_n$$

We have

$$\lim_{n\to\infty}\frac{A_{n+1}}{A_n}<1$$

This implies $\theta(r,z,t)$ converges for all r >1.

Conclusion

The thick hollow cylinder with a Caputo-Fabrizio differential equations operator under temporal thermoelastic modelling is investigated successfully and the effects of its stress and temperature are analyzed. The integral transformation method was used to carry out an analytical investigation into the equations of temperature distribution, analysis of displacement function, and stress variation with additional sectional heat at the lower and upper surfaces of the cylinder. By utilizing the material properties of aluminium metal, all the obtained results are investigated numerically and plotted graphically for the thick cylindrical model. From the graphical representation, it is observed that vary in temperature distribution and thermal stresses interpolate the conventional heat conduction equation. I.e. Wave equation for $\alpha = 0.5$ and $\alpha = 0.85$ which implies the impact of the memory effect as shown in numerical plotting. This obtained result is in good agreement with Povstenko [12]. The solution of the theory of thermoelastic diffusion is obtained for $\alpha = 1$, and its variation along thickness under assumed initial and boundary conditions matches standard plotting. Thus, the above study of a thermoelastic thick cylindrical body with a heat source and convective boundary in the context of the Caputo-Fabrizio differential operator may be utilized for the design of new structural materials. Such material applies to many physical problems and involves microscopic processing. Also, the above study is very useful for the researcher working with the modelling of fractional calculus by considering thermoelasticity.

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