AN EXTENSION OF CWA-VIKOR METHOD FROM VIKOR METHOD UNDER HESITANT FUZZY SETS

Rafi Raza[1], Ahmad Termimi Ab Ghani[2]*, Lazim Abdullah[3]

Abstract
The VIKOR method depends on a compromise programming technique-derived aggregating function that symbolizes "closeness to the ideal. This approach can be useful when a decision maker is unable or unsure of how to describe their choice at the beginning of system design. Furthermore, due to the hesitant fuzzy set's effectiveness and strength in capturing ambiguity and uncertainty, it has come under increasing scrutiny. The criteria weight average VIKOR method (CWA-VIKOR) is expanded in this work to account for uncertain fuzzy situations, and criteria weight average (CWA) introduced a new idea weight, W_{av}, in place of v and find out central compromise solution. We suggest a CWA-VIKOR method based on these novel techniques, and a real-world example is given to demonstrate how well our method works when dealing with multi-criteria decision-making issues involving criteria weight average and hesitant fuzzy preference data.

Keyword: MCDM, VIKOR method under hesitant fuzzy data, Decision Making, Hesitant fuzzy sets, CWA.

[1, 2*, 3]Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Malaysia, E-mail:- p5089@pps.umt.edu.my[1], termimi@umt.edu.my[2], lazim_m@umt.edu.my[3]

*__Corresponding author: -__ Ahmad Termimi Ab Ghani
[*]Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Malaysia, E-mail: termimi@umt.edu.my

DOI: - 10.48047/ecb/2023.12.si10.00434

Introduction

Decisions-making has become an essential part of everyday life, particularly in the fields of business, family, employment, medicine, engineering, the social sciences, economics, education, marketing, and so forth. Making the best decisions is therefore essential for a stress-free and healthy way of life [7]. Finding an answer to problems after weighing various options based on competing criteria is the process of decision-making. Due to people's increasing desire to select the best option, mathematicians have expanded their field of study by developing a variety of multi-criteria decision-making (MCDM) techniques, including VIKOR [1], TOPSIS [2], AHP [3], and PROMETHEE for the MCDM conundrum, the compromise solutions was introduced by Yu [5] and Zeleny [6], and which aids decision-makers in finding a solution that will work. A compromise solution's basic concept is that it aspires to be the furthest thing from the ideal solution. Here, the term "compromise" refers to a settlement reached through reciprocal concession.

Establishing the VIKOR technique in the presence of HFSs and a criteria weight average is the primary goal of this article. A MCDM method for complex structures called VIKOR was first introduced by Opricovic [1] in 1998 with the intention of dealing with crisp information. Subsequently, it was extended to rationalize a wider range of informational settings. Finding a compromise solution to the MCDM problem that fulfill the criteria of maximum group utility and minimal individual regret among opponents is the primary goal of this method. The two defining essential elements of a feasible compromise solution, Opricovic used the following l_p measure and an aggregation function: l_p metric as:

$$l_p^i = \left\{ \sum_{j=1}^{n} [W_j (F_{j^*} - F_{ij} / F_{j^*} - F_{i^-})]^p \right\}^{\frac{1}{p}}$$

Where $F_{j^*} = \max_i(F_{ij})$ and $F_{i^-} = \min_i(F_{ij})$

Traditional MCDM methods use crisp information to evaluate the alternatives and attributes in common unpredictable and uncertain situations, despite the unrealistic assumption that a decision-maker has exact and rigorous examples of the judgmental preferences situations. Because priorities are a very vague and ambiguous aspect of human character [4]. As a result, a decision-maker should not use precise information to assess the tendency of his issue. Fuzzy membership values are selected from the range [0; 1] when performing decision-making in the FS domain; Bellmen and Zadeh [11] first applied fuzzy numbers (FNs) for the MCDM process. One of the best tools for making decisions is the fuzzy set theory, which Zadeh [8] first developed. Many writers have since used the FS theory to illustrate real-world situations. The fuzzy VIKOR (F-VIKOR) technique was proposed by Wang and Chang [12] to capture multi-criteria decision-making (MCDM). Chang [13] classified Taiwanese hospitals based on their amenities and services using the F-VIKOR method. Other F-VIKOR method applications to address MAGDM issues with supplier selection were put forth [14, 15, 16]. The conventional F-VIKOR method of managing the Mlava river's water resources was modified by Opricovic [17]. An expansion of the VIKOR method for trapezoidal FNs was introduced by Ju et al. [18]. In connection to green supply chain management, Rostamzadeh et al. [19] highlighted another application of the F-VIKOR technique in the creation of triangular FNs. Wang et al. [a0] created the F-VIKOR method using triangular FNs as linguistic variables to determine the best software company. B. Uyukozkan [21] started an expanded interpretation of F-VIKOR for the ranking of web-based learning methods. To evaluate the Saudi Arabian power networks for investment, Taylan et al. [22] used various decision-making methods, including F-VIKOR, fuzzy AHP, as well as fuzzy TOPSIS. All of these VIKOR method variations could only handle the data in a way that benefited the object in question in accordance with FS's capabilities. As a result, these techniques are ineffective when there is a high level of discontent.

The easiest statistical measure to calculate is the arithmetic average. For a small set of data, the arithmetic mean can be quickly calculated in your mind or on paper. Arithmetic average is also more convenient to use as input to subsequent analyses and calculations due to its simple formula and universally understood meaning. Arithmetic average is much more likely to be known by your coworkers in a larger team than geometric average or mode. In this study, we assess the average criteria and use the VIKOR method rather than the hesitant fuzzy and v. Extending the VIKOR technique to criteria weight average under HFSs, the VIKOR technique can be written as the CWA-VIKOR method.

Fuzzy set theory was developed by Zadeh [8]. It can deal with ambiguous and imprecise information. Whenever working with insufficient and fuzzy data when several or so more sources of uncertainty are present simultaneously, the classic fuzzy set has a number of drawbacks [23]. First, Torra and Narukawa [9, 10] defined the hesitant fuzzy set (HFS), providing new research areas for the study of decision-making in uncertain circumstances in the future. This generalized type of fuzzy set founded on aforementioned extensional forms of fuzzy sets. The HFS illustrates fuzziness by presenting all possible values when calculating an element's membership degree in a certain set, despite the fact that it does not offer an accurate membership function.

The benefit of employing HFS is clear. The benefits of HFS in decision-making can be demonstrated by two different types of instances. On the one hand, employing HFS makes it quite similar to how humans think. It should be noted that when deciding whether an element corresponds to a particular set, the decision-maker relies on single or interval values that should include and convey the given information when eliciting imprecise information using the aforementioned extended forms. However, some instances, the problem-solving decision-makers. They may have a range of potential values; therefore they cannot supply a single phrase or an expression. Interval value of convey their choices or evaluations when they are considering many potential values simultaneously. As a result, the HFS, whose membership includes, expressed by a range of numbers, when used, can perfectly solve this problem, while the extensions listed above are not valid.

A different kind of instance occurs frequently in our daily lives. Because of the rising due to the complexity of today's socioeconomic environments, solitary people are becoming less and less when assessing the taken into account items, a decision maker must take into account all pertinent parts of a situation. As mentioned by Yu [24], A group of people who want to strengthen their overall negotiating power may decide to form a union or a corporation with themselves as the shareholders. Typically have a few differences of opinion. The differences stem from the disparity in their subjective assessments of the emerging decision-making issues. Because the decision-makers may get varying viewpoints on the options being considered due to their distinct knowledge bases or advantages, it might be challenging to arrive at a consistent result because they cannot simply persuade one another. However, some factors can help. Because possible values it is more powerful, the HFS is suited to solve this problem. After that, any additional extended fuzzy sets. As an illustration, assume the decision-making body or constitution is requested to provide the levels at which a substitute decision-makers prefer to represent one alternative over another using values between 0 and 1. Consequently, it is more effective and appropriate to use HFS to describe the uncertain evaluation information. The article's remaining sections are organized as follows. The FS, HFSs, average and criteria weight average are defined in Section 2 along with a quick overview of some fundamental ideas that are used throughout the article. We present the VIKOR approach of HFSs for MCDM issues in Section 3. We describe the creation of the CWA-VIKOR approach in Section 4. In Section 5, an example is given to explain the proposed strategy. The paper's principal results are then briefly discussed.

Preliminaries

Definition [8]. Suppose X be the universal set. FS in X is defined by membership function $A \subseteq X$, $\mu_A(x): X \to [0,1]$ as given below:

$A = \{(x, \mu_A(x)),\ x \in X\},$

Where $\mu_A(x)$ represent the degree of membership of in A and every pair $(x, \mu_A(x))$ is singleton.

Definition [25, 26]. Suppose X be the universe of discourse, then a HFSs as H on X is defined by function $\rho_H(x)$ that X returns to subset of [0,1]. Mathematically it can be written as:\begin{center}

$$H = \{\langle x, \rho_H(x)\rangle \mid x \in X\},$$

Where $\rho_H(x)$ is describing as a set of membership degree for an element under a subset of [0,1], indicating the membership degree of an element $x \in X$.

Example 1. Let $X = \{a_1, a_2, a_3\}$ be the universe of discourse $\rho_H(a_1) = \{0.1, 0.9, 0.6\}$, $\rho_H(a_2) = \{0.6, 0.7\}$, $\rho_H(a_3) = \{0.7, 0.3\}$ then HFSs can be written as:

$$H = \{\langle a_1, \{0.1, 0.9, 0.6\}\rangle, \langle a_2, \{0.6, 0.7\}\rangle, \langle a_3, \{0.7, 0.3\}\rangle\}$$

Definition [25]. The ratio of the sum of the values in a given group to all the values in the set is the mean value, which is called average. Basically, it is the average of the numbers represented by the $\bar{x}$ character. The sign μ is another way to represent it.

$$\mu = \frac{x_1 + x_2 + x_3 + ..., x_n}{n}$$

Definition: The ration of the sum of criteria values of weight to the total number of criteria weight is known as CWA and can be written as given below:

$$W_{av} = \frac{w_1 + w_2 + w_3 + ..., w_n}{n}$$

Extended VIKOR method on HFSs

The Opricovic [17] adopted the F-VIKOR method is to solve problems in an uncertain way where the criteria and weight characterize fuzzy sets [6].

Step 1; Determine PIS and NIS:

$$A^* = \left\{h_1^*, h_2^*, h_3^*, ..., h_n^*\right\},$$

Where $h_i^* = \bigcup_i^m h_{ij}$

$$A^- = \left\{h_1^-, h_2^-, h_3^-, ..., h_n^-\right\},$$

Where $h_i^- = \bigcap_i^m h_{ij}$, $j = 1,2,3, \dots, m$

Step 2: Compute S_i and R_i as below

$$S_i = \sum_{j=1}^{n} W_j \| h_j^* - h_{ij} \| / \| h_j^* - h_j^- \|$$

Calculate regret measure

$$R_i = \max_j (S_i) = W_j \| h_j^* - h_{ij} \| / \| h_j^* - h_j^- \|$$

Where $i = 1,2,3, \dots, m$.

Step 3: Evaluate Q_i as given below

$Q_i = V \frac{S_i - S^-}{S^* - S^-} + (1-V) \frac{R_i - R^-}{R^* - R^-}$ And V is introduced weight,

Where $S^* = \min_i (S_i)$, $S^- = \max_i (S_i)$, $R^- = \max_i (R_i)$, $R^* = \min_i (R_i)$ where $i = 1,2,3, \dots, m$.

Step 4: Classify the alternatives, categorizing through S, R and Q values, from largest to smallest. The outcome is in three grades.

Step 5: Provide the alternative (A_1), which is graded top by smallest values by Q, as a compromise solution if the two factors persist as given below:

C1. Acceptable advantage:

$Q(A_2) - Q(A_1) \geq DQ$ Where (A_2) is the alternative with 2^{nd} position in the grading list by Q $DQ = \frac{1}{m-1}$ where m denotes the possible alternatives.

C2."Acceptable stability in decision making":

An alternative (A_1) should be also possess the highest position from S or R. In a decision-making procedure, such as voting by majority rule $V \succ 0.5$ or by consensus $V \approx 0.5$ or with veto $V \prec 0.5$, this compromise solution is stable.

As suggest the following compromise solution is if one of the requirements is not fulfilled.

- Alternative (A_1) and (A_2) if only C2 is not satisfied, or
- Alternatives (A_i) where $i = 1,2,3, \dots, m$. if C1 is not satisfied; (A_m) is determine by the relation $Q(A_m) - Q(A_1) \leq DQ$ by maximum m

An extension of CWA-VIKOR method from VIKOR method under hesitant fuzzy sets

CWA mean criteria weight averaging and evaluated value is used in the HFSs information VIKOR method. Since the values of the criteria weight average should more appropriately be considered as hesitant fuzzy elements, benefit criteria are elements. Because of this, in the current research, we extend the CWA-VIKOR under HFSs approach to solve the MCDM problem the following structure is used for hesitant fuzzy elements:

Step 1: Determine PIS and NIS:

$$A^* = \left\{h_1^*, h_2^*, h_3^*, ..., h_n^*\right\},$$

Where $h_i^* = \bigcup_i^m h_{ij}$

$$A^- = \left\{h_1^-, h_2^-, h_3^-, ..., h_n^-\right\},$$

Where $h_i^- = \bigcap_i^m h_{ij}$, $j = 1,2,3, \dots, m$

Step 2: Compute S_i and R_i as below

$$S_i = \sum_{j=1}^{n} W_j \| h_j^* - h_{ij} \| / \| h_j^* - h_j^- \|$$

Calculate regret measure

$$R_i = \max_j (S_i) = W_j \| h_j^* - h_{ij} \| / \| h_j^* - h_j^- \|$$

Where $i = 1,2,3,\ldots,m$.

Step 3: Find out W_{av} by using Definition 2.4.

Step 4: Evaluate Q_i as given below

$Q_i = W_{av} \frac{S_i - S^-}{S^* - S^-} + (1 - W_{av}) \frac{R_i - R^-}{R^* - R^-}$ Where W_{av} is the criteria weight average, and $S^* = \min_i (S_i)$, $S^- = \max_i (S_i)$, $R^- = \max_i (R_i)$, $R^* = \min_i (R_i)$ where $i = 1,2,3,\ldots,m$.

Step 5: Classify the alternatives, categorizing through S, R and Q values, from largest to smallest. The outcome is in three grades.

Step 6: Provide the alternative (A_1^1), which is graded top by smallest values by Q, as a compromise solution if the two factors persist as given below:

C1: Acceptable advantage:

$Q(A_2^2) - Q(A_1^1) \geq DQ$ Where (A_2) is the alternative with 2^{nd} position in the grading list by Q $DQ = \frac{1}{m-1}$ where m denotes the possible alternatives.

C2:"Acceptable stability in decision making":

An alternative (A_1) should be also possess the highest position from S or R.

As suggest the following compromise solution is if one of the requirements is not fulfilled.

- Alternative (A_1) and (A_2) if only C2 is not satisfied, or
- Alternatives (A_i) where $i = 1,2,3,\ldots,m$ if C1 is not satisfied; (A_m) is determine by the relation $Q(A_m^2) - Q(A_1^1) \leq DQ$ by maximum m.

Numerical example

Suppose we select sites A_1, A_2, A_3, and A_4 for fish farming on the basis of criteria with weight $W = (0.21, 0.312, 0.4, 0.068)$.

κ_1 : Water Quality

κ_2 : Climate

κ_3 : Water supply

κ_4 : Hydrological characteristics

Table 1 Hesitant fuzzy decision matrix

Alternatives	κ_1	κ_2	κ_3	κ_4
A_1	(0.1,0.3,0.6)	(0.1,0.5,0.7)	(0.1,0.2,0.5,0.6,0.8)	(0.2,0.3,0.4,0.6,0.7)
A_2	(0.1,0.3,0.6,0.8)	(0.1,0.2,0.3,0.4)	(0.2,0.3,0.5,0.8)	(0.4,0.5,0.7,0.8)
A_3	(0.2,0.4,0.5,0.6)	(0.1,0.3,0.5)	(0.2,0.4,0.6,0.7)	(0.1,0.4,0.5,0.6)
A_4	(0.2,0.4,0.5)	(0.1,0.3,0.7)	(0.4,0.5,0.6)	(0.1,0.8)

Step 1: Determine PIS and NIS:

A^* = [0.8, 0.7, 0.8, 0.8]

A^- = [0.1, 0.1, 0.1, 0.1],

Step 2: Compute S_i and R_i as below

$$S_i = \sum_{j=1}^{n} W_j \| h_j^* - h_{ij} \| / \| h_j^* - h_j^- \|$$

S_1 =0.5243, S_2 =0.5634, S_3 =0.5497, S_4 =0513.

Calculate regret measure

$$R_i = \max_j (S_i) = W_j \| h_j^* - h_{ij} \| / \| h_j^* - h_j^- \|$$

R_1 =0.2108, R_2 =0.234, R_3 =0.208, R_4 =0.1757

Step 3: Find out $W_{av} = 0.25$ using Definition 2.4.

Step 4: Evaluate Q_i as given below where $i =$ 1,2,3,4.

$$Q_i = W_{av} \frac{S_i - S^-}{S^* - S^-} + (1 - W_{av}) \frac{R_i - R^-}{R^* - R^-}$$

Where

$S^* = 0.513$, $S^- = 0.5634$, $R^* = 0.1757$ $R^- = 0.234$

Q_1=0.4923, Q_2=0.000, Q_3=0.3978, Q_4=1.000.

Step 5: Classify the alternatives, categorizing through S, R, and Q values, from largest to smallest. The outcome is in three grades as listed in Table 2.

Step 6: As the alternative (A_2^1), which is graded top by smallest values by Q, as a central compromise solution if the two factors persist as given below:

C1. Acceptable advantage:

$Q(A_3^2) - Q(A_2^1) \geq \frac{1}{3}$ is satisfied so (A_2) is the central compromise solution.

C2:"Acceptable stability in decision making":
An alternative (A_2) also possesses the highest position from S or R.

Table 2: Ranking the alternatives

Alternatives	S	R	Q	Ranking
A_1	0.5243	0.2108	0.4923	3
A_2	0.5634	0.234	0.000	1
A_3	0.5497	0.208	0.3978	2
A_4	0.513	0.1757	1.000	4

Conclusions

It is straightforward and convenient to use the arithmetic average as a measure of central tendency. But in order to make the most of it and avoid having any negative effects on your research and decision-making, you should be aware of the circumstances in which it falls short and the instances in which other tools are more effective. In this study, we find out average of criteria weight W_{av} instead of V in the VIKOR method under hesitant fuzzy information and formed an CWA-VIKOR model to solve multi-criteria problems with differing and non-commensurable criteria, specifically taking into account the complicated subjective character of the decision maker. The compromise solution in CWA-VIKOR method is known as central compromise solution because it find out central compromise solution. This method is particularly helpful because it provides a weight average of the criteria instead of the variation value of V under hesitant fuzzy sets.

References

1. Opricovic, S. (1998). Multicriteria optimization of civil engineering systems. Faculty of civil engineering, Belgrade, 2(1), 5-21.
2. Hwang, C. L., Yoon, K., Hwang, C. L., \& Yoon, K. (1981). Methods for multiple attribute decision making. Multiple attribute decision making: methods and applications a state-of-the-art survey, 58-191.
3. Saaty, T. L. (1986). Axiomatic foundation of the analytic hierarchy process. Management science, 32(7), 841-855.
4. Akram, M., Muhiuddin, G., \& Santos-García, G. (2022). An enhanced VIKOR method for multi-criteria group decision-making with complex Fermatean fuzzy sets. Math. Biosci. Eng, 19(7), 7201-7231.
5. Yu, P. L. (1973). A class of solutions for group decision problems. Management science, 19(8), 936-946.
6. Zeleny, M., \& Cochrane, J. L. (1982). Multiple criteria decision making McGraw-Hill New York, 34.
7. Hariri, A., Domingues, P., \& Sampaio, P. (2023). Integration of multi-criteria decision-making approaches adapted for quality function deployment: an analytical literature review and future research agenda. International Journal of Quality \& Reliability Management.
8. L. A. Zadeh, Fuzzy sets, “Information and Control”, Vol. 8, pp.338- 356, 1965.
9. Torra, V., \& Narukawa, Y. (2009, August). On hesitant fuzzy sets and decision. In 2009 IEEE international conference on fuzzy systems (pp. 1378-1382). IEEE.
10. Torra, V. (2010). Hesitant fuzzy sets. International journal of intelligent systems, 25(6), 529-539.
11. R. E. Bellman, L. A. Zadeh, Decision-making in a fuzzy environment, Manage. Sci., 17(1970),141–164. https://doi.org/10.1287/mnsc.17.4.B141.
12. T. C. Wang, T. H. Chang, Fuzzy VIKOR as a resolution for multicriteria group decision-making, in: The 11th International Conference

on Industrial Engineering and Engineering Management, (2005), 352–356.
13. Chang, T. H. (2014). Fuzzy VIKOR method: A case study of the hospital service evaluation in Taiwan. Information Sciences, 271, 196-212.
14. Mishra, S., Samantra, C., Datta, S., \& Mahapatra, S. S. (2012). Multi-attribute group decision-making (MAGDM) for supplier selection using fuzzy linguistic modelling integrated with VIKOR method. International Journal of Services and Operations Management, 12(1), 67-89.
15. Sanayei, A., Mousavi, S. F., \& Yazdankhah, A. (2010). Group decision making process for supplier selection with VIKOR under fuzzy environment. Expert systems with applications, 37(1), 24-30.
16. Shemshadi, H. Shirazi, M. Toreihi, M. J. Tarokh, A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting, Expert Syst. Appl., 38(10)(2011), 12160–12167. https://doi.org/10.1016/j.eswa.2011.03.027
17. Opricovic, S. (2011). Fuzzy VIKOR with an application to water resources planning. Expert Systems with Applications, 38(10), 12983-12990.
18. Ju, Y., \& Wang, A. (2013). Extension of VIKOR method for multi-criteria group decision making problem with linguistic information. Applied Mathematical Modelling, 37(5), 3112-3125.
19. Rostamzadeh, R., Govindan, K., Esmaeili, A., \& Sabaghi, M. (2015). Application of fuzzy VIKOR for evaluation of green supply chain management practices. Ecological Indicators, 49, 188-203.
20. Wang, T. C., Liang, J. L., \& Ho, C. Y. (2006, October). Multi-criteria decision analysis by using fuzzy VIKOR. In 2006 International conference on service systems and service management (Vol. 2, pp. 901-906). IEEE.
21. Büyüközkan, G., Ruan, D., \& Feyzioğlu, O. (2007). Evaluating e-learning web site quality in a fuzzy environment. International Journal of Intelligent Systems, 22(5), 567-586.
22. Taylan, O., Alamoudi, R., Kabli, M., AlJifri, A., Ramzi, F., \& Herrera-Viedma, E. (2020). Assessment of energy systems using extended fuzzy AHP, fuzzy VIKOR, and TOPSIS approaches to manage non-cooperative opinions. Sustainability, 12(7), 2745.
23. Rodriguez, R. M., Martinez, L., \& Herrera, F. (2011). Hesitant fuzzy linguistic term sets for decision making. IEEE Transactions on fuzzy systems, 20(1), 109-119.
24. Yu, P. L. (1973). A class of solutions for group decision problems. Management science, 19(8), 936-946.
25. Umamaheswari, A., \& Kumari, P. (2014). Fuzzy TOPSIS and Fuzzy VIKOR methods using the Triangular Fuzzy Hesitant Sets. International Journal of Computer Science Engineering and Information Technology Research, 4, 15-24.
26. Wang, C., Zhou, X., Tu, H., \& Tao, S. (2017). Some geometric aggregation operators based on picture fuzzy sets and their application in multiple attribute decision making. Ital. J. Pure Appl. Math, 37, 477-492.