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Soliton excitation in a higher dimensional nonlinear Schrödinger equation

T.A. Ancy kirubha [1,2] and S. Beauno [1,2,*],

*Corresponding author. email: beunaphysics@gmail.com,

[1]Research Scholar, Reg. No.19213282132014 Department of Physics and Research Centre, Women's Christian College, Nagercoil-629 001, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India.

[2]Department of Physics and Research Centre, Women's Christian College, Nagercoil-629 001, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India.

Abstract

In this article, we use quadrupole-quadrupole type interactions to describe the dynamics of the alpha-helical protein chain. To create an equation of motion in order to understand the dynamics. Which analysis the energy transfer mechanism during the transmission. The soliton reveals that the variability is governed by the nonlinear Schrodinger equation's. Under various physical conditions, the centre of mass, mass velocity, and stability were connected to the soliton. The energy of soliton is conserved.

Keywords: Alpha-helical proteins, Soliton, Nonlinear Schrödinger equation.

1 Introduction

One of the important categories of secondary protein shape is the alpha-helix. Because of the hydrogen link formed by the oxygen in carbonyl levels and the hydrogen in amino levels, secondary forms are in the higher order structures. The process that transforms one amino acid's N-H group into the immediate C=O group of the chain's fourth amino acid causes the polypeptide chain of proteins to resemble a right-handed needle. The series "... H-N-C=O... H-N-C=O... H-N-C=O...", where C=O denotes the amide-I bond and the spot lines are denote the hydrogen bonds, assures the main helix. When C=O and N-H, a hydrogen bond is forms. The amide-I vibrations of the atoms in peptide groups enable alpha-helical proteins to play a significant role in the transfer of energy.

According to Davydov [1-5], amide-I vibrational quantum (C=O extended shaking), produced by the energy releasing process during ATP hydrolysis. The interaction between

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the trembling of amino acid molecules in the alpha-helical protein to produce the soliton. A substantial body of literature have been developed on the standard of bio-energy conveyance, which is a rival theory for the process. Many issues relating to the Davydov model have been thoroughly investigated in recent days [6-33]. Discrete chains and continuum models have been used to examine the dynamical properties of Davydov solitons and how they manifest themselves under various initial chain conditions. The majority of these results already have been collected for a one-dimensional system. On the other hand, sincere curiosity about all two and three-dimensional crystalline systems makes it necessary to abstract the lattice standards to the higher order. Analyzing the soliton dynamics in multidimensional lattices in individual is difficult. The integrating model of alpha-helical proteins with dipole-dipole interactions have been recently evaluated by the participating authors [34-35]. However, they have been little research on the solitonic properties and interaction of energy transmission across the alpha-helical protein lattice. We propose a reasonable lattice model of alpha-helical proteins and examine the soliton dynamics which is response to the above-mentioned assumption.

We identify the model Hamiltonian using the double quantized operators of quantum technicians and generate the equations of motion to study the dynamics of Hamiltonian using an appropriate wave function for a fair lattice. The Sine-Cosine function method is used to analyze soliton solutions.

A significant portion of the energy transfer process is also recreated by inhomogeneous alpha-helical protein complexes. Alpha-helical protein inhomogeneity is induced by deficits brought on the presence of causing molecules like drugs in particular sections of the sequence and by the presence of abasic regions like a nonpolar thymine pantomime [36,37]. The impact of inhomogeneity in three-dimensional alpha-helical proteins has not been reported in the scientific literature. However, they have not been documented in the literature. In the current research, we examine the stability features in three dimensions as well as the inhomogeneous impacts on soliton propagation.

The paper's structure is as follows: In Section 2, the Hamiltonian model and equations of motion for a lattice model of the quadrupole-quadrupole interacting alpha-helical proteins are generated. The perturbation method is used in Section 3 to determine the soliton solutions for the associated nonlinear equations. Section 4, calculations are made for a soliton's center of mass, mass, energy, velocity, and stability under various physical conditions. Section 5 presents a picture of the stability of the nonlinear system. A review of the finished work is presented in Section 6.

2 Model Hamiltonian and Equation of Motion

We consider about a homogeneous alpha-helical protein system. Considering the quadrupole-quadrupole interaction of nearby atoms in the same spine and the next chain, the Hamiltonian is written as

$$H = \quad H'_{ex} + H'_{ph} + H'_{ph-ex}, \tag{1}$$

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The exciton Hamiltonian is given by

$$H'_{ex} = \sum_{n,l,m} \{B^{\dagger}_{n,l,m}\{E_0 B_{n,l,m} + E_1 B_{n,l,m} B^{\dagger}_{n,l,m} B_{n,l,m} - J_0(B_{n+1,l,m} + B_{n-1,l,m}) - J'_0(B_{n,l+1,m} + B_{n,l-1,m}) - J''_0(B_{n,l,m+1} + B_{n,l,m-1}) - J_1(B_{n+1,l+1,m+1} + B_{n-1,l-1,m-1}) - J'_1(B_{n+1,l+1,m-1} + B_{n-1,l-1,m+1}) - J''_1(B_{n+1,l-1,m-1} + B_{n-1,l+1,m+1}) - J_2(B_{n+1,l+1,m} + B_{n-1,l-1,m}) - J'_2(B_{n+1,l,m+1} + B_{n-1,l,m-1}) - J''_2(B_{n,l+1,m+1} + B_{n,l-1,m-1}) - J_3(B_{n+1,l,m} B^{\dagger}_{n,l,m} B_{n+1,l,m} + B_{n-1,l,m} B^{\dagger}_{n,l,m} B_{n-1,l,m}) - J'_3(B_{n,l+1,m} B^{\dagger}_{n,l,m} B_{n,l+1,m} + B_{n,l-1,m} B^{\dagger}_{n,l,m} B_{n,l-1,m}) - J''_3(B_{n,l,m+1} B^{\dagger}_{n,l,m} B_{n,l,m+1} + B_{n,l,m-1} B^{\dagger}_{n,l,m} B_{n,l,m-1}) - J_4(B_{n+1,l+1,m+1} B^{\dagger}_{n,l,m} B_{n+1,l+1,m+1} + B_{n-1,l-1,m-1} B^{\dagger}_{n,l,m} B_{n-1,l-1,m-1}) - J'_4(B_{n+1,l+1,m-1} B^{\dagger}_{n,l,m} B_{n+1,l+1,m-1} + B_{n-1,l-1,m+1} B^{\dagger}_{n,l,m} B_{n-1,l-1,m+1}) - J''_4(B_{n+1,l-1,m-1} B^{\dagger}_{n,l,m} B_{n+1,l-1,m-1} + B_{n-1,l+1,m+1} B^{\dagger}_{n,l,m} B_{n-1,l+1,m+1}) - J_5(B_{n+1,l+1,m} B^{\dagger}_{n,l,m} B_{n+1,l+1,m} + B_{n-1,l-1,m} B^{\dagger}_{n,l,m} B_{n-1,l-1,m}) - J'_5(B_{n+1,l,m+1} B^{\dagger}_{n,l,m} B_{n+1,l,m+1} + B_{n-1,l,m-1} B^{\dagger}_{n,l,m} B_{n-1,l,m-1}) - J''_5(B_{n,l+1,m+1} B^{\dagger}_{n,l,m} B_{n,l+1,m+1} + B_{n,l-1,m-1} B^{\dagger}_{n,l,m} B_{n,l-1,m-1})\}\}, \quad (2)$$

Here, E_1 defines the molecules within each unit cell's higher order (quadrupole type) excitations. The terms J_3, J'_3 and J''_3 refer to the quadrupole-quadrupole type coupling along the X, Y and Z axes, respectively between the adjacent unit cells. J_4, J'_4, J''_4, J_5, J'_5 and J''_5 are symbols for the diagonal quadrupole-quadrupole type coupling between the adjacent unit cells. $H'_{ph} = \sum_{n,l,m} \{+\frac{\hat{P}^2_{n,l,m}}{2M} + \frac{K}{2}[(\hat{U}_{n.l,m} - \hat{U}_{n-1.l,m})^2 + (\hat{U}_{n.l,m} - \hat{U}_{n.l-1,m})^2 + (\hat{U}_{n.l,m} - \hat{U}_{n.l,m-1})^2\}$, is the Hamiltonian of the phonon.

Excitons and phonons are coupled together by

$$H'_{ph-ex} = \sum_{n,l,m} \{B^{\dagger}_{n,l,m} B_{n,l,m}[\chi_1(U_{n+1,l,m} - U_{n-1,l,m} + U_{n,l+1,m} - U_{n,l-1,m} + U_{n,l,m+1} - U_{n,l,m-1}) + \chi_2(U_{n+1,l+1,m} - U_{n-1,l-1,m} + U_{n+1,l,m+1} - U_{n-1,l,m-1} + U_{n,l+1,m+1} - U_{n,l+1,m-1})] + 2B^{\dagger}_{n,l,m} B^2_{n,l,m}[B^{\dagger}_{n,l,m} B_{n,l,m} B^{\dagger}_{n,l,m} B_{n,l,m} \chi'_1(U_{n+1,l,m} - U_{n-1,l,m} + U_{n,l+1,m} - U_{n,l-1,m} + U_{n,l,m+1} - U_{n,l,m-1}) + \chi''_2(U_{n+1,l+1,m} - U_{n-1,l-1,m} + U_{n+1,l,m+1} - U_{n-1,l,m-1} + U_{n,l+1,m+1} - U_{n,l+1,m-1})]\}, \quad (3)$$

Here E_1 illustrates the higher order (quadrupole type) excitations of the molecules per unit cell. J_3, J'_3 and J''_3 denote the quadrupole-quadrupole kind coupling along the x,y and z directives respectively.J_4, J'_4, J''_4, J_5, J'_5 and J''_5 illustrate the quadrupole-quadrupole kind coupling along the diagonal. χ'_1 and χ'_2 are the coupling constants for the quadrupole-quadrupole coupling coefficient describing the difference in energy of the amide-I bond generated by the extension of the helix between two bordering unit cells.

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$$i\hbar\frac{\phi_{n,l,m}}{dt} = E_0\phi^*_{n,l,m}\phi^\dagger_{n,l,m}\phi^2_{n,l,m} - J_0(\phi_{n+1,l,m}+\phi_{n-1,l,m}) - J_0(\phi_{n+1,l,m}+\phi_{n-1,l,m}) - J'_0(\phi_{n,l+1,m}+\phi_{n,l-1,m}) - J''_0(\phi_{n,l,m+1}+\phi_{n,l,m-1}) - J_1(\phi_{n+1,l+1,m+1}+\phi_{n-1,l-1,m-1}) - J'_1(\phi_{n+1,l+1,m-1}+\phi_{n-1,l-1,m+1}) - J''_1(\phi_{n+1,l-1,m-1}+\phi_{n-1,l+1,m+1}) - J_2(\phi_{n+1,l+1,m}+\phi_{n-1,l-1,m}) - J'_2(\phi_{n+1,l,m+1}+\phi_{n-1,l,m-1}) - J''_2(\phi_{n,l+1,m+1}+\phi_{n,l-1,m-1}) - J_3(\phi^2_{n+1,l,m}+\phi^2_{n-1,l,m}) - J'_3(\phi^2_{n,l+1,m}+\phi^2_{n,l-1,m}) - J''_3(\phi^2_{n,l,m+1}+\phi^2_{n,l,m-1}) - J_4(\phi^2_{n+1,l+1,m+1}+\phi^2_{n-1,l-1,m-1}) - J'_4(\phi^2_{n+1,l+1,m-1}+\phi^2_{n-1,l-1,m+1}) - J''_4(\phi^2_{n+1,l-1,m-1}+\phi^2_{n-1,l+1,m+1}) - J_5(\phi^2_{n+1,l+1,m}+\phi^2_{n-1,l-1,m}) - J'_5(\phi^2_{n+1,l,m+1}+\phi^2_{n-1,l,m-1}) - J''_5(\phi^2_{n,l+1,m+1}+\phi^2_{n,l-1,m-1}) + \phi_{n,l,m}[\chi_1(U_{n+1,l,m}-U_{n-1,l,m}+U_{n,l+1,m}-U_{n,l-1,m}+U_{n,l,m+1}-U_{n,l,m-1}) + \chi_2(U_{n+1,l+1,m}-U_{n-1,l-1,m}+U_{n+1,l,m+1}-U_{n-1,l,m-1}+U_{n,l+1,m+1}-U_{n,l+1,m-1})] + 2\phi^\dagger_{n,l,m}\phi^2_{n,l,m}[\chi'_1(U_{n+1,l,m}-U_{n-1,l,m}+U_{n,l+1,m}-U_{n,l-1,m}+U_{n,l,m+1}-U_{n,l,m-1}) + \chi''_2(U_{n+1,l+1,m}-U_{n-1,l-1,m}+U_{n+1,l,m+1}-U_{n-1,l,m-1}+U_{n,l+1,m+1}-U_{n,l+1,m-1})], \quad (4)$$

$$M\frac{d^2U_{n,l,m}}{dt} = -K[6U_{n,l,m}-U_{n+1,l+1,m}-U_{n-1,l-1,m}+U_{n+1,l,m+1}-U_{n-1,l,m-1}+U_{n,l+1,m+1}-U_{n,l+1,m-1}] + \chi_1[|\phi_{n+1,l,m}|^2-|\phi_{n-1,l,m}|^2-|\phi_{n,l+1,m}|^2-|\phi_{n,l-1,m}|^2-|\phi_{n,l,m+1}|^2-|\phi_{n,l,m-1}|^2] + \chi_2[|\phi_{n+1,l+1,m}|^2-|\phi_{n-1,l-1,m}|^2-|\phi_{n+1,l,m+1}|^2-|\phi_{n-1,l,m-1}|^2-|\phi_{n,l+1,m+1}|^2-|\phi_{n,l-1,m-1}|^2] + \chi'_1[|\phi_{n+1,l,m}|^4-|\phi_{n-1,l,m}|^4-|\phi_{n,l+1,m}|^4-|\phi_{n,l-1,m}|^4-|\phi_{n,l,m+1}|^4-|\phi_{n,l,m-1}|^4] + \chi''_2[|\phi_{n+1,l+1,m}|^4-|\phi_{n-1,l-1,m}|^4-|\phi_{n+1,l,m+1}|^4-|\phi_{n-1,l,m-1}|^4-|\phi_{n,l+1,m+1}|^4-|\phi_{n,l-1,m-1}|^4]. \quad (5)$$

Eqs. (4) and (5) illustrates the dynamics of three-dimensional alpha-helical proteins in the discrete condition. Utilizing the Taylor expansions

$$\phi_{l\pm1,m,n} = \phi \pm \gamma\phi_x + \frac{1}{2}\gamma^2\phi_{xx} \pm \frac{1}{6}\gamma^3\phi_{xxx} + \frac{1}{24}\gamma^4\phi_{xxxx} \pm \ldots, \quad (6)$$

$$\phi_{l,m\pm1,n} = \phi \pm \delta\phi_y + \frac{1}{2}\delta^2\phi_{yy} \pm \frac{1}{6}\delta^3\phi_{yyy} + \frac{1}{24}\delta^4\phi_{yyyy} \pm \ldots, \quad (7)$$

$$\phi_{l,m,n\pm1} = \phi \pm \sigma\phi_z + \frac{1}{2}\sigma^2\phi_{zz} \pm \frac{1}{6}\sigma^3\phi_{zzz} + \frac{1}{24}\sigma^4\phi_{zzzz} \pm \ldots, \quad (8)$$

$$\phi_{l+1,m+1,n} = \phi + \gamma\phi_x + \delta\phi_y + \frac{1}{2}\gamma^2\phi_{xx} + \frac{1}{2}\delta^2\phi_{yy} + \gamma\delta\phi_{xy} + \frac{1}{6}\gamma^3\phi_{xxx} + \frac{1}{6}\delta^3\phi_{yyy} + \frac{1}{2}\gamma^2\delta\phi_{xxy} + \frac{1}{2}\gamma\delta^2\phi_{xyy} + \frac{1}{24}\gamma^4\phi_{xxxx} + \frac{1}{24}\delta^4\phi_{yyyy}$$

$$+\frac{1}{4}\gamma^2\delta^2\phi_{xxyy}+\frac{1}{6}\gamma^3\delta\phi_{xxxy}+\frac{1}{6}\gamma\delta^3\phi_{xyyy}+\ldots, \tag{9}$$

$$\phi_{l-1,m-1,n}=\phi-\gamma\phi_x-\delta\phi_y+\frac{1}{2}\gamma^2\phi_{xx}+\frac{1}{2}\delta^2\phi_{yy}+\gamma\delta\phi_{xy}-\frac{1}{6}\gamma^3\phi_{xxx}-\frac{1}{6}\delta^3\phi_{yyy}-\frac{1}{2}\gamma^2\delta\phi_{xxy}-\frac{1}{2}\gamma\delta^2\phi_{xyy}+\frac{1}{24}\gamma^4\phi_{xxxx}+\frac{1}{24}\delta^4\phi_{yyyy}+\frac{1}{4}\gamma^2\delta^2\phi_{xxyy}+\frac{1}{6}\gamma^3\delta\phi_{xxxy}+\frac{1}{6}\gamma\delta^3\phi_{xyyy}+\ldots, \tag{10}$$

$$\phi_{l+1,m+1,n+1}=\phi+\gamma\phi_x+\delta\phi_y+\sigma\phi_z+\frac{1}{2}\gamma^2\phi_{xx}+\frac{1}{2}\delta^2\phi_{yy}+\frac{1}{2}\sigma^2\phi_{zz}+\gamma\delta\phi_{xy}+\delta\sigma\phi_{zy}+\gamma\sigma\phi_{xz}+\frac{1}{6}\gamma^3\phi_{xxx}+\frac{1}{6}\delta^3\phi_{yyy}+\frac{1}{6}\sigma^3\phi_{zzz}+\frac{1}{2}\gamma^2\delta\phi_{xxy}+\frac{1}{2}\gamma\delta^2\phi_{xyy}+\frac{1}{2}\delta^2\sigma\phi_{xxz}+\frac{1}{2}\gamma\sigma^2\phi_{xzz}+\frac{1}{2}\delta^2\sigma\phi_{yyz}+\frac{1}{2}\delta\sigma^2\phi_{yzz}+\gamma\delta\sigma\phi_{xyz}+\frac{1}{24}\gamma^4\phi_{xxxx}+\frac{1}{24}\delta^4\phi_{yyyy}+\frac{1}{24}\sigma^4\phi_{zzzz}+\frac{1}{4}\gamma^2\delta^2\phi_{xxyy}+\frac{1}{4}\delta^2\sigma^2\phi_{yyzz}+\frac{1}{4}\gamma^2\sigma^2\phi_{xxzz}+\frac{1}{6}\gamma^3\delta\phi_{xxxy}+\frac{1}{6}\gamma\delta^3\phi_{xyyy}+\frac{1}{6}\delta^3\sigma\phi_{yyyz}+\frac{1}{6}\delta\sigma3\phi_{yzzz}+\frac{1}{6}\gamma^3\sigma\phi_{xxxz}+\frac{1}{6}\gamma\sigma^3\phi_{xzzz}+\frac{1}{2}\gamma^2\delta\sigma\phi_{xxyz}+\frac{1}{2}\gamma\delta^2\sigma\phi_{yyzx}+\frac{1}{2}\gamma\delta\sigma^2\phi_{zzxy}+\ldots, \tag{11}$$

$$\phi_{l-1,m-1,n-1}=\phi-\gamma\phi_x-\delta\phi_y-\sigma\phi_z+\frac{1}{2}\gamma^2\phi_{xx}+\frac{1}{2}\delta^2\phi_{yy}+\frac{1}{2}\sigma^2\phi_{zz}+\gamma\delta\phi_{xy}+\delta\sigma\phi_{zy}+\gamma\sigma\phi_{xz}-\frac{1}{6}\gamma^3\phi_{xxx}-\frac{1}{6}\delta^3\phi_{yyy}-\frac{1}{6}\sigma^3\phi_{zzz}-\frac{1}{2}\gamma^2\delta\phi_{xxy}-\frac{1}{2}\gamma\delta^2\phi_{xyy}-\frac{1}{2}\delta^2\sigma\phi_{xxz}-\frac{1}{2}\gamma\sigma^2\phi_{xzz}-\frac{1}{2}\delta^2\sigma\phi_{yyz}-\frac{1}{2}\delta\sigma^2\phi_{yzz}-\gamma\delta\sigma\phi_{xyz}+\frac{1}{24}\gamma^4\phi_{xxxx}+\frac{1}{24}\delta^4\phi_{yyyy}+\frac{1}{24}\sigma^4\phi_{zzzz}+\frac{1}{4}\gamma^2\delta^2\phi_{xxyy}+\frac{1}{4}\delta^2\sigma^2\phi_{yyzz}+\frac{1}{4}\gamma^2\sigma^2\phi_{xxzz}+\frac{1}{6}\gamma^3\delta\phi_{xxxy}+\frac{1}{6}\gamma\delta^3\phi_{xyyy}+\frac{1}{6}\delta^3\sigma\phi_{yyyz}+\frac{1}{6}\delta\sigma3\phi_{yzzz}+\frac{1}{6}\gamma^3\sigma\phi_{xxxz}+\frac{1}{6}\gamma\sigma^3\phi_{xzzz}+\frac{1}{2}\gamma^2\delta\sigma\phi_{xxyz}+\frac{1}{2}\gamma\delta^2\sigma\phi_{yyzx}+\frac{1}{2}\gamma\delta\sigma^2\phi_{zzxy}+\ldots, \tag{12}$$

Substituting Eqs. (6)-(12) in Eqs. (4) and (5) and considering the lattice parameters γ, δ and σ to be identical, we accept the following equations:

$$i\hbar\phi_t = [E_0-6(J_0+J_1+J_2)]\phi+[2E_1-2(J_0'+J_1'+J_2')]|\phi|^2\phi+\gamma[6\phi(\chi_1+2\chi_2)(A\chi_1+2\chi_2|\phi|^2)+12|\phi|^2\phi(\chi_3+2\chi_4)(A\chi_1+2\chi_2|\phi|^2)]-\gamma^2[(J_0+3J_1+2J_2)(\phi_{xx}+\phi_{yy}+\phi_{zz})+2(3J_1+J_2)(\phi_{xy}+\phi_{xz}+\phi_{yz})+4(J_0'+3J_1'+2J_2')\phi^*(\phi_x^2+\phi_y^2+\phi_z^2)+8(3J_1'+J_2')\phi\phi^*(\phi_{yz}^2+\phi_{xy}^2+\phi_{xz}^2)+4(J_0'+3J_1'+2J_2')\phi\phi^*(\phi_{xx}^2+\phi_{yy}^2+\phi_{zz}^2)+8(J_1'+J_2')\phi^*(\phi_x\phi_y+\phi_y\phi_z)+8(J_2'-J_1')\phi^*$$

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$$(\phi_x\phi_z)] + \gamma^3[(\phi\frac{1}{3}(\chi_1 + 2\chi_2)(2\phi_x\phi^* + \phi\phi_x\phi^* + \phi^*\phi_{xx} + 2\phi_y\phi^* + \phi\phi_y\phi^* + \phi^*\phi_{yy} + 2\phi_z\phi^* + \phi\phi_z\phi^* + \phi^*\phi_{zz})(\chi_1 + 2\chi_2)A_1] + \phi\chi_2(\chi_1 + 2\chi_2)A_2 (\phi_y^*\phi_x + \phi^*\phi_{xy} + \phi_{xy}^*\phi + \phi_x^*\phi_y + \phi_x^*\phi_z + \phi_y^*\phi_x + \phi^*\phi_{xz} + \phi_{xz}^*\phi + \phi_z^*\phi_y + \phi^*\phi_{yz} + \phi_{yz}^*\phi + \phi_y^*\phi_z) + \frac{2}{3}\phi^*\phi^2(\chi_1 + 2\chi_2)A_1(\chi_3 + 2\chi_4)(2\phi_x\phi_x^* + \phi\phi_{xx}^* + \phi_{xx}^*\phi + 2\phi_y\phi_y^* + \phi\phi_{yy}^* + \phi_{yy}^*\phi + 2\phi_z\phi_z^* + \phi\phi_{zz}^* + \phi_{zz}^*\phi) + 4\phi^*\phi^2(\chi_1 + 2\chi_2)A_1\chi_3(\phi_y^*\phi_x + \phi^*\phi_{xy} + \phi_{xy}^*\phi + \phi_x^*\phi_y + \phi_x^*\phi_z + \phi_z^*\phi_x + \phi^*\phi_{xz} + \phi_{xz}^*\phi + \phi_z^*\phi_y + \phi^*\phi_{yz} + \phi_{yz}^*\phi + \phi_y^*\phi_z) - \gamma^4[\frac{1}{12}(J_0 + 3J_1 + 2J_2)(\phi_{xxxx} + \phi_{yyyy} + \phi_{zzzz}) + \frac{1}{3}(3J_1 + J_2)(\phi_{xxxy} + \phi_{xyyy} + \phi_{xxxz} + \phi_{xzzz} + \phi_{yyzx} + \phi_{zzxy}) + \frac{1}{2}(3J_1 + J_2)(\phi_{xxyy} + \phi_{xxzz} + \phi_{yyzz}) + J_1(\phi_{xxyz} + \phi_{yyzx} + \phi_{zzxy})], \tag{13}$$

$$M_{utt} = K\gamma^2[U_{xx} + U_{yy} + U_{zz}] + 2\delta[\chi_1 + 2\chi_2]((|\phi|^2)_x + (|\phi|^2)_y + (|\phi|^2)_z). \tag{14}$$

Presenting the wave variable $\xi = K_1x + K_2y + K_3z - ct$ in Eqs. (13) and (14), and solving Eq. (14), we acquire

$$u_\xi = 2(\chi_1 + 2\chi_2)A|\phi|^2. \tag{15}$$

By utilizing Eq. (15), Eq. (13) turns out to be

$$i\phi_t = a_1\phi - a_2(\phi_{xx} + \phi_{yy} + \phi_{zz}) - a_3(\phi_{xy} + \phi_{xz} + \phi_{yz}) - a_4|\phi^2|\phi - a_5(\phi_{xxxx} + \phi_{yyyy} + \phi_{zzzz}) - a_6(\phi_{xxxy} + \phi_{xyyy} + \phi_{xxxz} + \phi_{xzzz} + \phi_{yyyz} + \phi_{yzzz}) - a_7(\phi_{xxyy} + \phi_{xxzz} + \phi_{yyzz}) - a_8(\phi_{xxyz} + \phi_{yyzx} + \phi_{zzxy}) - a_9|\phi^4|\phi - a_{10}(|\phi_x^2| + |\phi_y^2| + |\phi_z^2|)\phi - a_{11}|\phi^2|(\phi_{xx} + \phi_{yy} + \phi_{zz}) - a_{12}\phi^*(\phi_x^2 + \phi_y^2 + \phi_z^2) - a_{13}\phi(\phi_{xx}^* + \phi_{yy}^* + \phi_{zz}^*) - a_{14}\phi^*(\phi_x\phi_y + \phi_y\phi_z + \phi_x\phi_z) - a_{15}\phi(\phi_x\phi_y^* + \phi_x^*\phi_y + \phi_x^*\phi_z + \phi_z^*\phi_x + \phi_z^*\phi_y + \phi_y^*\phi_z) - a_{16}\phi^2(\phi_{xy}^* + \phi_{xz}^* + \phi_{yz}^*) - a_{17}|\phi^2|(\phi_{xy} + \phi_{xy} + \phi_{yz}) \tag{16}$$

where $a_1 = [\frac{E_0 - 6(J_0 + J_1 + J_2)}{\hbar}]$, $a_2 = [\frac{\gamma(J_0 + 3J_1 + 2J_2)}{\hbar}]$, $a_3 = [\frac{2\gamma(3J_1 + J_2)}{\hbar}]$, $a_4 = [\frac{-2E_1 - 2(J_0' + J_1' + J_2') - 6\gamma(\chi_1 + 2\chi_2)(A\chi_1 + 2\chi_2)}{\hbar}]$, $a_5 = [\frac{\gamma^4(J_0 + 3J_1 + 2J_2)}{12\hbar}]$, $a_6 = [\frac{\gamma^4(3J_1 + J_2)}{3\hbar}]$, $a_7 = [\frac{\gamma^4(3J_1 + J_2)}{2\hbar}]$, $a_8 = [\frac{\gamma^4 J_1}{3\hbar}]$, $a_9 = [\frac{12\gamma(\chi_1' + 2\chi_2')(A_1\chi_1 + 2\chi_2)}{\hbar}]$, $a_{10} = [\frac{-2A_1\gamma^3(\chi_1 + 2\chi_2)^2}{3\hbar}]$, $a_{11} = [\frac{4\gamma^2(J_0' + 3J_1' + 2J_2') - A_1\frac{\gamma}{3}3(\chi_1 + 2\chi_2)^2}{\hbar}]$, $a_{12} = [\frac{4\gamma^2(J_0' + 3J_1' + 2J_2')}{\hbar}]$, $a_{13} = [\frac{-A_1\gamma^3(\chi_1 + 2\chi_2)^2}{3\hbar}]$, $a_{14} = [\frac{8\gamma^2(J_1' + J_2') + 8\gamma^2(J_1' - J_2')}{\hbar}]$, $a_{15} = [\frac{-2\chi_1\gamma^3(\chi_1 + 2\chi_2)A_1}{\hbar}]$, $a_{16} = [\frac{-2\chi_2\gamma^3(\chi_1 + 2\chi_2)A}{\hbar}]$, $a_{17} = [\frac{-2\chi_2\gamma^3(\chi_1 + 2\chi_2)A_1 + 8\gamma^2(3J_1' + J_2')}{\hbar}]$

Eq. (16) defines a (3+1) dimensional Nonlinear Schrödinger equation.

3 .Soliton excitations

To apprehend the soliton excitations, we solve Eq.(16) utilizing the sine-cosine function method. For enforcing the overhead method, we utilize $\phi = u + iv$ in Eq. (16) and split the real and imaginary to obtain.

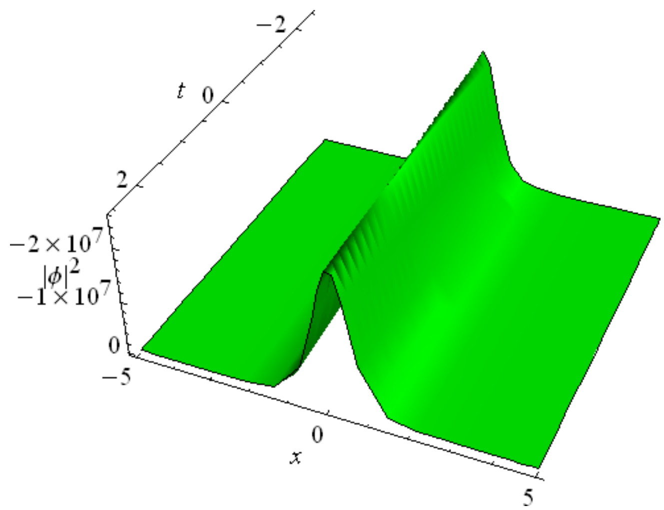

Figure 1: Soliton Excitation

$$a_1 - a_2(u_{xx} + u_{yy} + u_{zz}) - a_3(u_{xy} + u_{xz} + u_{yz}) - a_4(u^3 + uv^2) - a_5(u_{xxxx} + u_{yyyy} + u_{zzzz}) - a_6(u_{xxxy} + u_{xyyy} + u_{xxxz} + u_{xzzz} + u_{yyyz} + u_{yzzz}) - a_7 (u_{xxyy} + u_{xxzz} + u_{yyzz}) - a_8(u_{xxyz} + u_{yyzx} + u_{zzxy}) - a_9(u^5 + 2u^3v + uv^4 - a_{10}(uu_x^2 + uv_x^2 + uu_y^2 + uv_y^2 + uu_z^2 + uv_z^2) - a_{11}(u^2u_{xx} + v^2u_{xx} + u^2u_{yy} + v^2u_{yy} + u^2u_{zz} + v^2u_{zz}) - a_{12}(uu_x^2 - uv_x^2 + uu_y^2 - uv_y^2 + uu_z^2 - uv_z^2) - a_{13}(u^2 u_{xx} + 2uvv_{xx} - v^2u_{xx} + u^2u_{yy} + 2uvv_{yy} - v^2u_{yy} + u^2u_{zz} + 2uvv_{zz} - v^2u_{zz} - a_{14}(uu_xu_y - uv_xv_y + vv_xu_y + vu_xv_y + uu_yu_z - uv_yv_z + vu_yv_z + vv_yv_z + uu_x u_z - uv_xv_z + vu_xu_z + vu_zv_x) + a_{15}(2uu_xu_y + 2uv_yv_x + 2vv_yu_x - 2vv_xu_y + 2u u_xu_z + 2uv_xv_z + 2vu_zv_x - 2vu_xu_z + 2uu_yu_z + 2uv_zv_y + 2vu_yv_z - 2vu_zv_y) - a_{16} (u^2u_{xy} + 2uvv_{xy} - v^2u_{xy} + u^2u_{xz} + 2uvv_{xz} - v^2u_{xz} + u^2u_{yz} + 2uvv_{yz} - v^2 uyz) - a_{17}(u^2u_{xy} + v^2u_{xy} + u^2u_{xz} + v^2u_{xz} + u^2u_{xz} + v^2u_{xz}) = 0, \quad (17)$$

$$a_1 - a_2(v_{xx} + v_{yy} + v_{zz}) - a_3(v_{xy} + v_{xz} + v_{yz}) - a_4(v^3 + vu^2) - a_5(v_{xxxx} + v_{yyyy} + v_{zzzz}) - a_6(v_{xxxy} + v_{xyyy} + v_{xxxz} + v_{xzzz} + v_{yyyz} + v_{yzzz}) - a_7(v_{xxyy} + v_{xxzz} + v_{yyzz}) - a_8(v_{xxyz} + v_{yyzx} + v_{zzxy}) - a_9(v^5 + 2v^3u + vu^4 - a_{10}(vv_x^2 +$$

$$vu_x^2 + vv_y^2 + vu_y^2 + vv_z^2 + vu_z^2) - a_{11}(v^2v_{xx} + u^2v_{xx} + v^2v_{yy} + v^2v_{yy} + v^2v_{zz} + u^2v_{zz}) - a_{12}(vv_x^2 - vu_x^2 + vv_y^2 - vu_y^2 + vv_z^2 - vu_z^2) - a_{13}(v^2v_{xx} + 2vuu_{xx} - u^2 v_{xx} + v^2v_{yy} + 2vuu_{yy} - u^2v_{yy} + v^2v_{zz} + 2vuu_{zz} - u^2v_{zz} - a_{14}(vv_xv_y - vu_xu_y + uu_xv_y + uv_xu_y + vv_yv_z - vu_yu_z + uv_yu_z + uu_yu_z + vv_xv_z - vu_xu_z + uv_xv_z + uv_zu_x) + a_{15}(2vv_xv_y + 2vu_yu_x + 2uu_yv_x - 2uu_xv_y + 2vv_xv_z + 2vu_xu_z + 2u v_zu_x - 2uv_xv_z + 2vv_yv_z + 2vu_zu_y + 2uv_yu_z - 2uv_zu_y) - a_{16}(v^2v_{xy} + 2vuu_{xy} - u^2v_{xy} + v^2v_{xz} + 2vuu_{xz} - u^2v_{xz} + v^2v_{yz} + 2vuu_{yz} - u^2vyz) - a_{17}(v^2v_{xy} + u^2 v_{xy} + v^2v_{xz} + u^2v_{xz} + v^2v_{xz} + v^2v_{xz}) = 0, \tag{18}$$

Utilizing the wave variable $\xi = x + y + z - ct$ in Eq. (17) and (18), we acquire

$$cv_\xi - a_1 + 3a_2u_{\xi\xi} - 3a_3u_{\xi\xi} - a_4(u^3 + uv^2) - 3a_5u_{\xi\xi\xi\xi} - 6a_6u_{\xi\xi\xi\xi} - 3a_7u_{\xi\xi\xi\xi} - 3a_8u_{\xi\xi\xi\xi} - a_9(u^5 + 2u^3v + uv^4) - a_{10}(uu_\xi^2 + uv_\xi^2 + uu_\xi^2 + uv_\xi^2 + uu_\xi^2 + uv_\xi^2) - a_{11}(u^2u_{\xi\xi} + v^2u_{\xi\xi} + u^2u_{\xi\xi} + v^2u_{\xi\xi} + u^2u_{\xi\xi} + v^2u_{\xi\xi}) - a_{12}(uu_\xi^2 - uv_\xi^2 + uu_\xi^2 - uv_\xi^2 + uu_\xi^2 - uv_\xi^2) - a_{13}(u^2u_{\xi\xi} + 2uvv_{\xi\xi} - v^2u_{\xi\xi} + u^2u_{\xi\xi} + 2uvv_{\xi\xi} - v^2u_{\xi\xi} + u^2u_{\xi\xi} + 2uvv_{\xi\xi} - v^2u_{\xi\xi} - a_{14}(uu_\xi u_\xi - uv_\xi v_\xi + vv_\xi u_\xi + vu_\xi v_\xi + uu_\xi u_\xi - uv_\xi v_\xi + vu_\xi v_\xi + vv_\xi v_\xi + uu_\xi u_\xi - uv_\xi v_\xi + vu_\xi u_\xi + vu_\xi v_\xi) + a_{15}(2uu_\xi u_\xi + 2uv_\xi v_\xi + 2vv_\xi u_\xi - 2vv_\xi u_\xi + 2uu_\xi u_\xi + 2uv_\xi v_\xi + 2vu_\xi v_\xi - 2vu_\xi u_\xi + 2uu_\xi u_\xi + 2uv_\xi v_\xi + 2vu_\xi v_\xi - 2vu_\xi v_\xi) - a_{16}(u^2u_{\xi\xi} + 2uvv_{\xi\xi} - v^2u_{\xi\xi} + u^2u_{\xi\xi} + 2uvv_{\xi\xi} - v^2u_{\xi\xi} + u^2u_{\xi\xi} + 2uvv_{\xi\xi} - v^2u\xi\xi) - a_{17}(u^2u_{\xi\xi} + v^2u_{\xi\xi} + u^2u_{\xi\xi} + v^2u_{\xi\xi} + u^2u_{\xi\xi} + v^2u_{\xi\xi}) = 0, \tag{19}$$

$$-cu_\xi a_1 - 3a_2v_{\xi\xi} - 3a_3v_{\xi\xi} - a_4(v^3 + vu^2) - 3a_5v_{\xi\xi\xi\xi} - 6a_6v_{\xi\xi\xi\xi} - 3a_7v_{\xi\xi\xi\xi} - 3a_8v_{\xi\xi\xi\xi} - a_9(v^5 + 2v^3u + vu^4) - a_{10}(vv_\xi^2 + vu_\xi^2 + vv_\xi^2 + vu_\xi^2 + vv_\xi^2 + vu_\xi^2) - a_{11}(v^2v_{\xi\xi} + u^2v_{\xi\xi} + v^2v_{\xi\xi} + v^2v_{\xi\xi} + v^2v_{\xi\xi} + u^2v_{\xi\xi}) - a_{12}(vv_\xi^2 - vu_\xi^2 + vv_\xi^2 - vu_\xi^2 + vv_\xi^2 - vu_\xi^2) - a_{13}(v^2v_{\xi\xi} + 2vuu_{\xi\xi} - u^2v_{\xi\xi} + v^2v_{\xi\xi} + 2vuu_{\xi\xi} - u^2v_{\xi\xi} + v^2v_{\xi\xi} + 2vuu_{\xi\xi} - u^2v_{\xi\xi} - a_{14}(vv_\xi v_\xi - vu_\xi u_\xi + uu_\xi v_\xi + uv_\xi u_\xi + vv_\xi v_\xi - vu_\xi u_\xi + uv_\xi u_\xi + uu_\xi u_\xi + vv_\xi v_\xi - vu_\xi u_\xi + uv_\xi v_\xi + uv_\xi u_\xi) + a_{15}(2vv_\xi v_\xi + 2vu_\xi u_\xi + 2uu_\xi v_\xi - 2uu_\xi v_\xi + 2vv_\xi v_\xi + 2vu_\xi u_\xi + 2uv_\xi u_\xi - 2uv_\xi v_\xi + 2vv_\xi v_\xi + 2vu_\xi u_\xi + 2uv_\xi u_\xi - 2uv_\xi u_\xi) - a_{16}(v^2v_{\xi\xi} + 2vuu_{\xi\xi} - u^2v_{\xi\xi} + v^2v_{\xi\xi} + 2vuu_{\xi\xi} - u^2v_{\xi\xi} + v^2v_{\xi\xi} + 2vuu_{\xi\xi} - u^2v\xi\xi) - a_{17}(v^2v_{\xi\xi} + u^2v_{\xi\xi} + v^2v_{\xi\xi} + u^2v_{\xi\xi} + v^2v_{\xi\xi} + v^2v_{\xi\xi}) = 0, \tag{20}$$

where $a_1 = [\frac{E_0 - 6(J_0 + J_1 + J_2)}{\hbar}]$, $a_2 = [\frac{\gamma(J_0 + 3J_1 + 2J_2)}{\hbar}]$, $a_3 = [\frac{2\gamma(3J_1 + J_2)}{\hbar}]$, $a_4 = [\frac{-2E_1 - 2(J_0' + J_1' + J_2') - 6\gamma(\chi_1 + 2\chi_2)(A\chi_1 + 2\chi_2)}{\hbar}]$, $a_5 = [\frac{\gamma^4(J_0 + 3J_1 + 2J_2)}{12\hbar}]$, $a_6 = [\frac{\gamma^4(3J_1 + J_2)}{3\hbar}]$, $a_7 = [\frac{\gamma^4(3J_1 + J_2)}{2\hbar}]$, $a_8 = [\frac{\gamma^4 J_1}{3\hbar}]$, $a_9 = [\frac{12\gamma(\chi_1' + 2\chi_2')(A_1\chi_1 + 2\chi_2)}{\hbar}]$, $a_{10} = [\frac{-2A_1\gamma^3(\chi_1 + 2\chi_2)^2}{3\hbar}]$, $a_{11} = [\frac{4\gamma^2(J_0' + 3J_1' + 2J_2') - A_1\frac{\gamma}{3}3(\chi_1 + 2\chi_2)^2}{\hbar}]$, $a_{12} = [\frac{4\gamma^2(J_0' + 3J_1' + 2J_2')}{\hbar}]$, $a_{13} = [\frac{-A_1\gamma^3(\chi_1 + 2\chi_2)^2}{3\hbar}]$, $a_{14} =$

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$[\frac{8\gamma^2(J_1'+J_2')+8\gamma^2(J_1'-J_2')}{\hbar}]$, $a_{15} = [\frac{-2\chi_1\gamma^3(\chi_1+2\chi_2)A_1}{\hbar}]$, $a_{16} = [\frac{-2\chi_2\gamma^3(\chi_1+2\chi_2)A}{\hbar}]$, $a_{17} = [\frac{-2\chi_2\gamma^3(\chi_1+2\chi_2)A_1+8\gamma^2(3J_1'+J_2')}{\hbar}]$

Directly we consider that Eqs. (19) and (20) reveal solutions as follows:

$$u(x,y,z,t) = \lambda_1 cos^{\beta 1}(\mu\xi), v(x,y,z,t) = \lambda_2 cos^{\beta 2}(\mu\xi) \quad (21)$$

Where λ_1 and λ_2 are constant parameters. To find β_1 and β_2, we offset the linear higher order derivative period with the nonlinear expression of Eqs. (19) and (20), and ultimately, we acquire $\beta_1 = \beta_2 = -1$. Using Eq. (21) with $\beta_1 = \beta_2 = -1$ in Eqs. (19) and (20), we reach a system of algebraic equations. To solve the design of algebraic utilizing MATHEMATICA, we acquire

$$\lambda = \sqrt{\frac{-a_4-(3a_{11}+3a_{13}+3a_{16}+3a_{17}+3a_{10}+3a_{14}+4.5a_{15})}{2a_9}}\mu, \quad (22)$$

$$\mu = \sqrt{\frac{-5(a_5+2a_6+a_7+a_8)}{(a_2+a_3)}} \quad (23)$$

Therefore, the solutions of Eq. (16) become

$$u(x,y,z,t) = \lambda_1 \times sech[\mu(x+y+z-ct)], \quad (24)$$

$$v(x,y,z,t) = \lambda_2 \times sech[\mu(x+y+z-ct)]. \quad (25)$$

Fig. (1) exposes that the higher dimensional alpha-helical protein chain helps the soliton-like excitations. The (3+1) dimensional alpha-helical protein system is shown to support soliton-like excitations in Fig. 2A. Fig. (2A) shows the soliton interaction at time (a) t=-4, (b) t=0, and (c) t=4 . By constructing curves for various values of J_0, it is possible to understand the effects of quadrupole-quadrupole coupling in dynamics. The parametric values are $(a)J_0 = 1.3$ $(b)J_0 = 1.4$ and $(c)J_0' = 1.5$. Fig. (3B) denotes the curves for various values of J_1 with $(a)J_1 = J_2 = 0.2$, $(b)J_1 = J_2 = 3.1$ and $(c)J_1 = J_2 = 4.1$. Figures (3A)and(3B) shows that the amplitude of the soliton increases to some extent when J_0, J_1 and J_2 values are slightly increased from their actual values.

4 Center of mass of soliton

Figs. (4(A,B)) depict the interpretation of the soliton's center of mass as a time variable. We plot the soliton's centre of mass in relation to the physical parameters in order to hold a superior physical performance of the activity J_0, J_0', J_0'', J_1, J_1', J_1'', J_2, J_2', J_2'',J_3, J_3', J_3'', J_4, J_4', J_4'', J_5, J_5', J_5'', E_0 and χ. Fig. (4A) indicates the function of the center mass of the soliton for various values of J_0 bearing $J_0' = J_0''$. Likewise $J_1 = J_1' = J_1'' = 0.2$, $J_2 = J_2' = J_2'' = 3.1$, $E_0 = 1.02$ and $\chi = 1.04$. The transition in the centre of mass position as J_1 values vary with $J_1' = J_1''$ is shown in Fig. (4B). Figs. (4A) and (4B) shows that the protein system that nearly maintains the same position for the soliton's center of mass. This shows that in this parameter range the fundamental solitons through the protein lattices are nonlinearly unchanging.

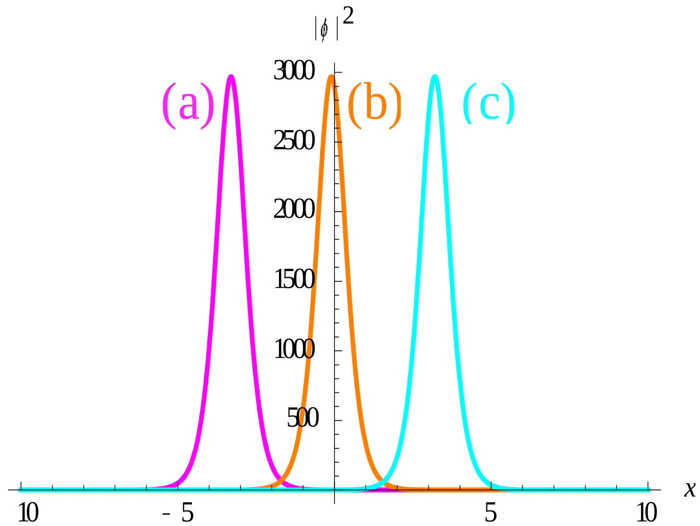
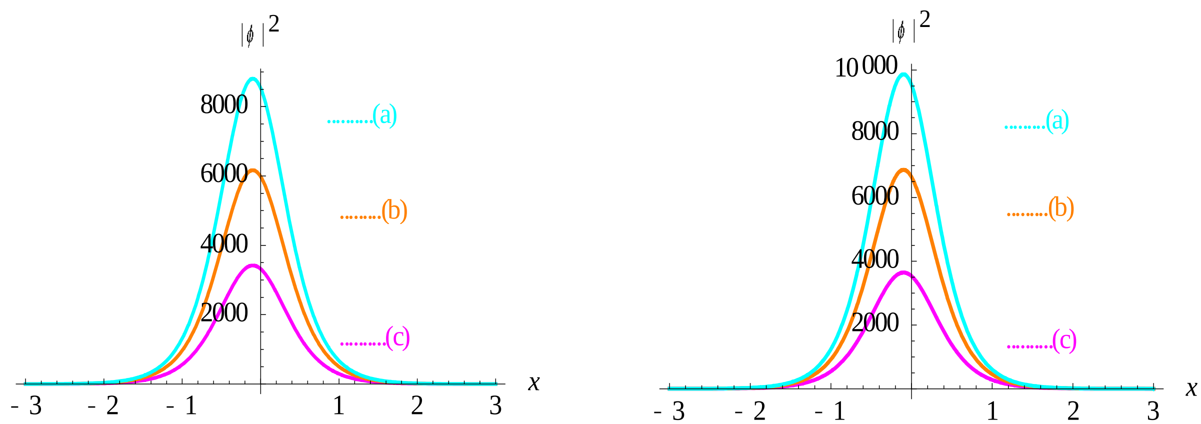


Figure 2: (A) Soliton interaction at time (a) t=-4, (b) t=0, (c) t=4



(A) (B)

Figure 3: Density profiles of soliton for $(A)[(a)J_0 = 0.2, (b)J_0' = 0.4, (c)J_0'' = 0.6]$ and $(B)[(a)J_0 = -2.3, (b)J_0' = -3.3, (c)J_0'' = -3.9]$

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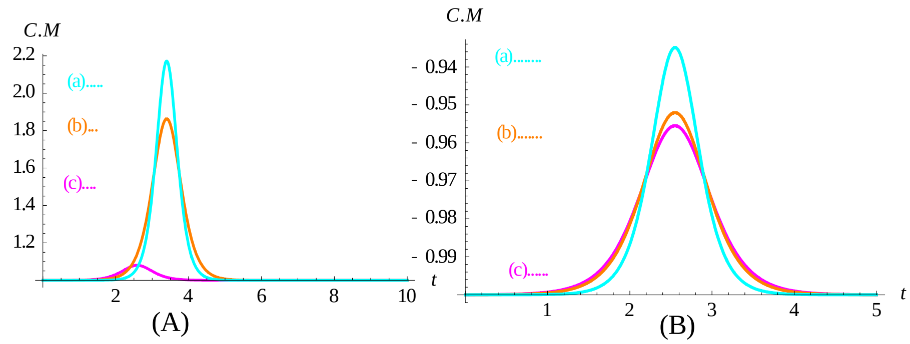

Figure 4:Time variation of center of mass of the soliton for. $(A)[(a)J_0 = -2.3, (b)J_0' = -4.3, (c)J_0'' = -9.3]$ and $(B)[(a)J_0 = -10.3, (b)J_0' = -11.3, (c)J_0'' = -12.3]$

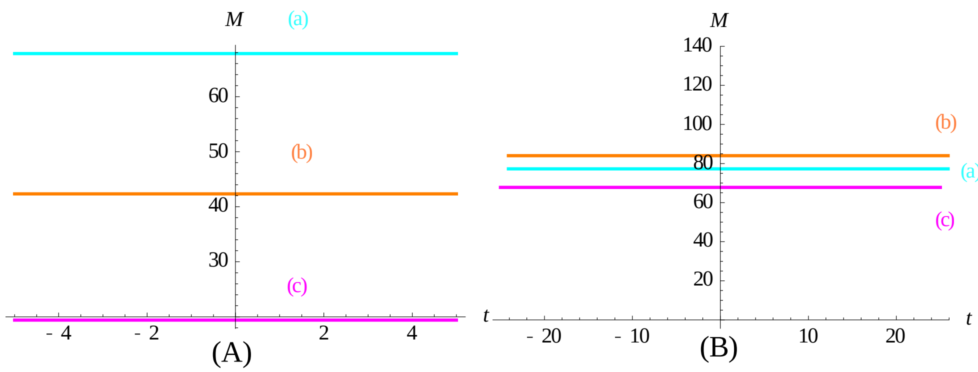

Figure 5: Mass of the solition

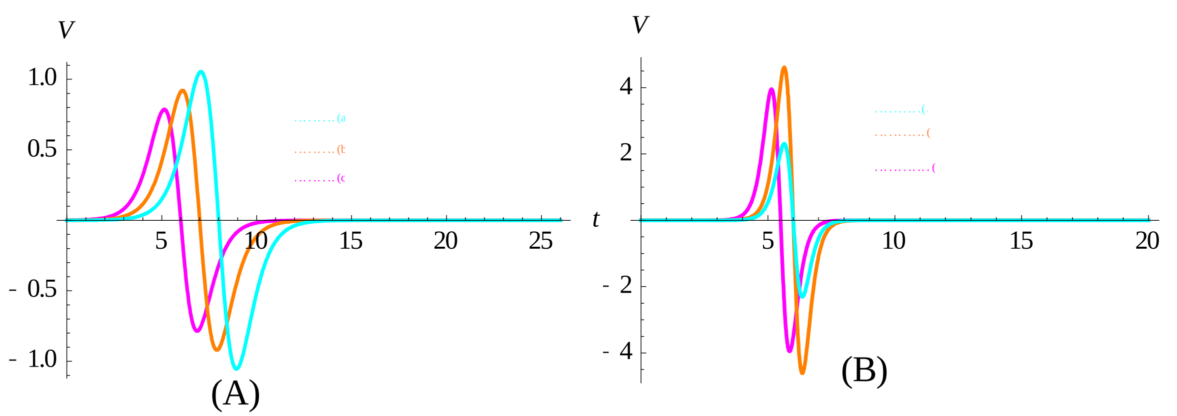

Figure 6: Velocity of soliton in protein lattice

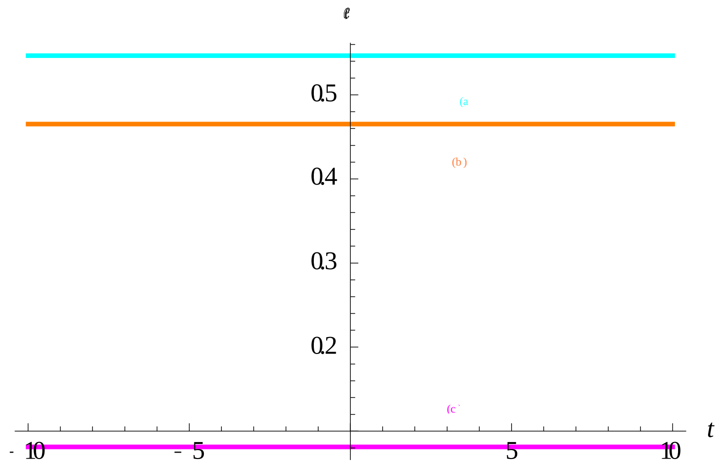

Figure 7: Energy of soliton

5 Soliton mass

The interpretation of the soliton's mass for different values are shown in Fig. 5(A, B, and C) as J_0, J_1, J_2 and χ, respectively. According to their predictions, the soliton mass is constant under all conditions.

6 Velocity

The soliton's velocity has been displayed in Figs. (6A) and (6B) for various values of J_0, J_1 and J_2. The constitution reveals that the hopping integrals cause a quick shift in velocity, which helps in controlling the soliton, before displaying the uniform velocity, they additionally shows an oscillating interpretation of the soliton's velocity.

7 Soliton energy

The energy of solitonic excitations in the alpha-helical protein chain are illustrated in Fig. 7 ((a) Potential energy, (b) Kinetic energy, and (c) Total energy). The parametric options are $J_0 = J_0' = J_0'' = -1.3$, $J_1 = J_1' = J_1'' = 0.2$, $J_2 = J_2' = J_2'' = 3.1$, $E_0 = 1.02$ and $\chi = 1.04$. This illustration shows how the soliton's energy is conserved.

8 Linear stability analysis

Starting with the NLS equation, we can complete the linear stability analysis (16) Assumed is a planar wave of shape with constant amplitude.

$$\phi(x, y, z, t) = U_0 exp[i(q_1 x + q_2 y + q_3 z - \omega t)] \tag{26}$$

where ω is the frequency, U_0 is the amplitude, and q_1, q_2, q_3, q_4, q_5 and q_6 are the wave numbers. Substituting Eq. (33) in Eq. (16), we discover the relationship that is amplitude-dependent.

$$\omega = a_1 + a_2(k_1^2 + k_2^2 + k_3^2) + a_3(k_1k_2 + k_1k_3 + k_2k_3) - a_4U_0^2 - a_5(k_1^4 + k_2^4 + k_3^4) - a_6(k_1^3k_2 + k_1k_2^3 + k_1^3k_3 + k_1k_3^3 + k_2^3k_3 + k_2k_3^3) - a_7(k_1^2k_2^2 + k_1^2$$

$$k_3^2 + k_2^2k_3^2) - a_8(k_1^2k_2k_3 + k_1k_2^2k_3 + k_1k_2k_3^2) - a_9U_0^4 - a_{10}U_0^3(k_1 + k_2 + k_3) + a_{11}U_0^2(k_1^2 + k_2^2 + k_3^2) - a_{12}(k_1^2 + k_2^2 + k_3^2) + a_{13}U_0(k_1^2 + k_2^2 + k_3^2) - a_{14}(k_1k_2 + k_1k_3 + k_2k_3) - 2a_{15}(k_1k_2 + k_1k_3 + k_2k_3) + a_{16}(k_1k_2 + k_1k_3 + k_2k_3) - a_{17}U_0^2(k_1k_2 + k_1k_3 + k_2k_3) \quad (27)$$

known as the dispersion relation. Now we analyze linear stability of Eq. (16) by considering the perturbed plane wave solutions of the form

$$\phi(x,y,z,t) = (U_0 + \epsilon\phi_1)exp[i(q_1x + q_2y + q_3z - \omega t) + \epsilon\phi_2(x,y,z,t)] \quad (28)$$

where ω is the small parameter and

$$\phi_1(x,y,z,t) = aexp[i\beta(x,y,z,t)], \quad (29)$$

$$\phi_2(x,y,z,t) = bexp[i\beta(x,y,z,t)] \quad (30)$$

Utilizing $\beta(x,y,z,t) = Qx + Qy + Qz - \Omega t$, the dispersion relationship between the frequency and wave number Ω provided by

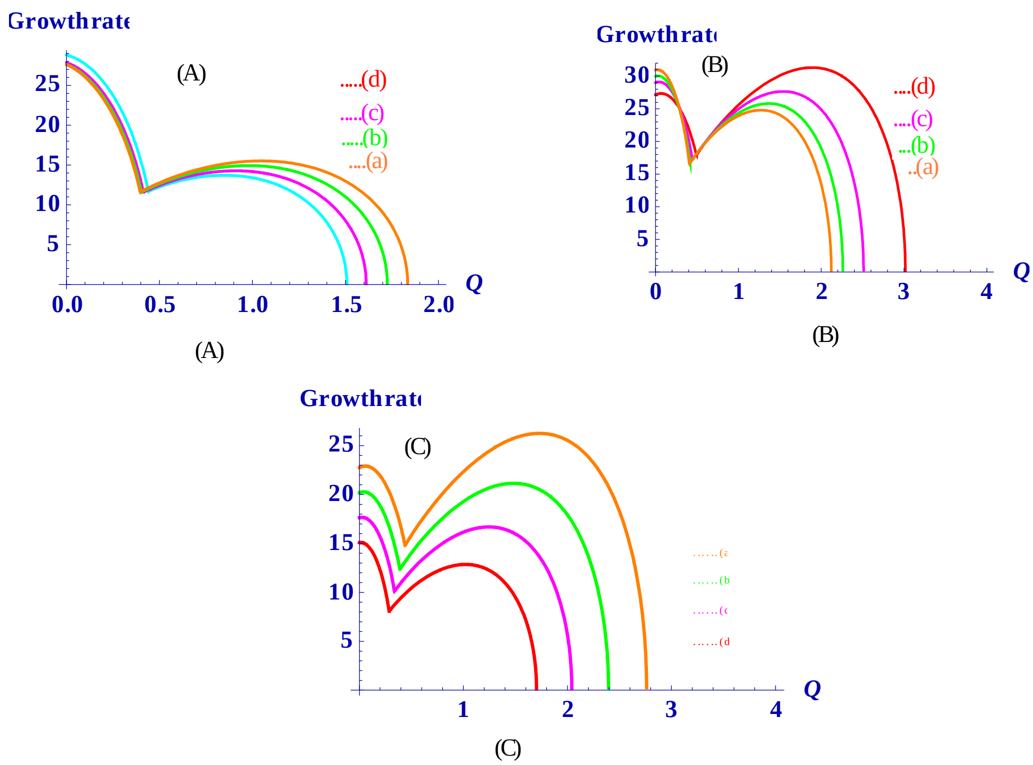

Figure 8: Modulational Instability based on growth rate and wave number Q for different

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values of (A) q_1, (B) q_2 and (C) q_3

$$\Omega^2 U_0 + \Omega(RU_0 + S) + RS = 0 \tag{31}$$

We derive the dispersion relation from the quadratic equation (31).

$$\Omega = -(RU_0 + S) \pm \sqrt{\frac{(RU_0+S)^2 - 4U_0RS}{2U_0}} \tag{32}$$

Here, Ω can be used to determine the stability of a nonlinear alpha-helical protein chain. From the relation (32), it is proven that if $(RU_0 + S) > RS, \Omega$ becomes complex and in this instance, the perturbation rises exponentially over time. Consequently, MI is shown by the excited alpha-helical protein system, supporting the formation of soliton.

9 Growth Rate Vs Wave Numbers

The growth rate curve is shown in Fig. (8A) by holding q_2 and q_3 and changing q_1. The parameters are $q_2 = 2.3, q_3 = 2.3, U_0 = 0.7, J_0 = J_0' = J_0'' = -1.3, J_1 = J_1' = J_1'' = 0.2, J_2 = J_2' = J_2'' = 3.1, J_3 = J_3' = J_3'' = 0.3, J_4 = J_4' = J_4'' = 0.2,$ and $J_5 = J_5' = J_5'' = 0.3$ with (a) $q1 = -1.1$, (b)$q1 = -1.6$, (c) $q1 = -1.9$ and (d) $q1 = -2.2$. The graph displays how the growth rate depends on the constant values of q_1, q_2 and q_3 for $J_0, J_0', J_0'', J_2, J_2'$ and J_2''. The growth rate and band width shrivels, and maximum gain reduce as q_1 increase, indicating that plane wave instability is increased and soliton stability is enriched.

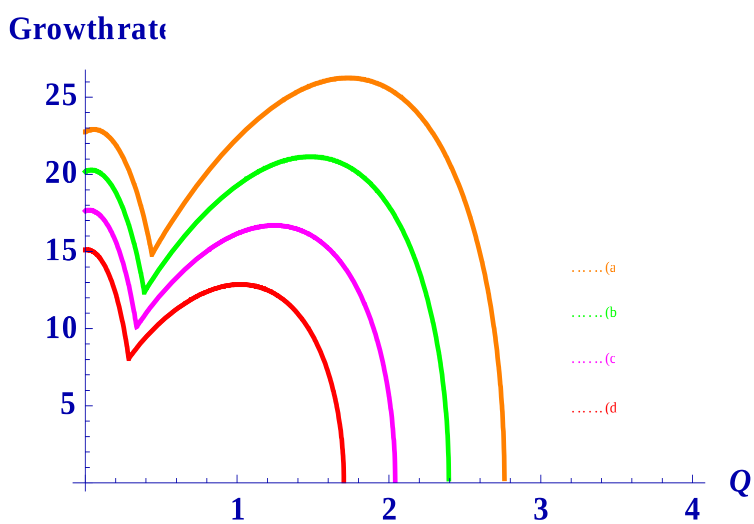

Figure 9: Growth rate Vs wave number Q for different values of amplitude U_0

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10 Growth Rate Vs Wave Numbers

The growth rate curve is shown in Fig. (8A) by holding q_2 and q_3 and changing q_1. The parameters are $q_2 = 2.3, q_3 = 2.3, U_0 = 0.7, J_0 = J_0' = J_0'' = -1.3, J_1 = J_1' = J_1'' = 0.2, J_2 = J_2' = J_2'' = 3.1, J_3 = J_3' = J_3'' = 0.3, J_4 = J_4' = J_4'' = 0.2,$ and $J_5 = J_5' = J_5'' = 0.3$ with (a) $q1 = -1.1$, (b)$q1 = -1.6$, (c) $q1 = -1.9$ and (d) $q1 = -2.2$. The graph displays how the growth rate depends on the constant values of q_1, q_2 and q_3 for $J_0, J_0', J_0'', J_2, J_2'$ and J_2''. The growth rate and band width shrivels, and maximum gain reduce as q_1 increase, indicating that plane wave instability is increased and soliton stability is enriched.

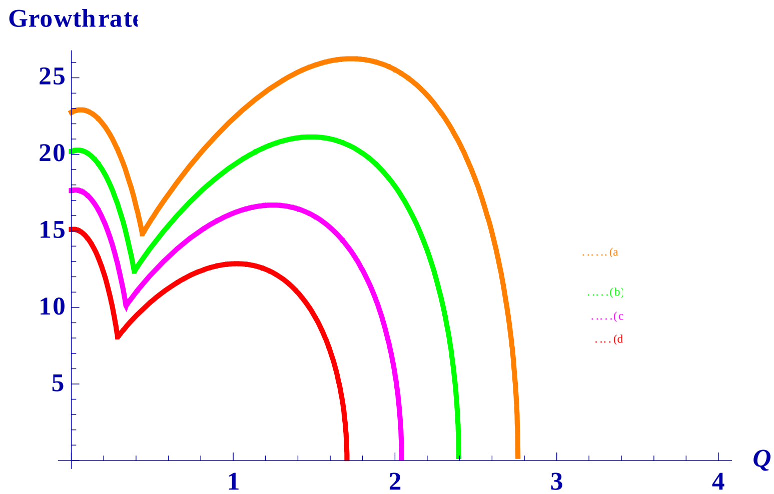

Figure 10: Gain spectrum for wave number Q by increasing J_0

The Fig. (8B) shows the growth rate curve by holding q_1 and q_3 and changing q_2. The parameters are $q_1 = -0.4, q_3 = 1.3, U_0 = 1, J_0 = J_0' = J_0'' = -1.4, J_1 = J_1' = J_1'' = 0.4, J_2 = J_2' = J_2'' = 3.1, J_3 = J_3' = J_3'' = 0.3, J_4 = J_4' = J_4'' = 0.2,$ and $J_5 = J_5' = J_5'' = 0.3$ with (a) $q_2 = 1.5$, (b) $q_2 = 1.7$, (c) $q_2 = 1.8$ and (d)$q_2 = 1.9$. From Fig. (8B) it is evident that increasing bandwidth and maximum gain correspond to lower significances of q_2. As q_2 increases, the growth rate and the bandwidth decline, and the maximum gain lowers. Hence in both cases, reducing the values of q_1, q_2 and q_3 will increase the MI growth, and hence the stability is upgraded.

The Fig. (8C) explains the growth rate curve by fixing q_1 and q_2 and changing q_3. The parameters are $q_1 = -0.4$, $q_2 = 1.5, U_0 = 1, J_0 = J_0' = J_0'' = -1.4, J_1 = J_1' = J_1'' = 0.4, J_2 = J_2' = J_2'' = 3.1, J_3 = J_3' = J_3'' = 0.3, J_4 = J_4' = J_4'' = 0.2,$ and $J_5 = J_5' = J_5'' = 0.3$ with (a) $q_3 = 1.4$, (b) $q_3 = 1.5$, (c) $q_3 = 1.6$ and (d)$q_3 = 1.7$. Fig. (8C), it is evident that increasing bandwidth and maximum gain correspond to lower significances of q_3. As q_3 increases, the growth rate and the bandwidth decline, and the maximum gain lowers. Hence in both cases, reducing the values of q_1, q_2 and q_3 will increase the MI growth and hence the stability.

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11 Growth Rate Vs Perturbation Amplitude

Fig. (9) illustrates the growth rate curve by modifying U_0. The parameters are $q_1 = -0.4,, q_2 = 1.3, q_3 = 1.3, J_0 = J_0' = J_0'' = -1.4, J_1 = J_1' = J_1'' = 0.4, J_2 = J_2' = J_2'' = 3.1, J_3 = J_3' = J_3'' = 0.3, J_4 = J_4' = J_4'' = 0.2$, and $J_5 = J_5' = J_5'' = 0.3$ with (a)$U_0 = 0.6$, (b)$U_0 = 0.7$, (c) $U_0 = 0.8$ and (d)$U_0 = 0.9$. The curve verifies how the growth rate turns on the perturbation amplitude U_0 for stable $q_1, q_2, q_3, J_0, J_0', J_0'', J_2, J_2'$ and J_2''. As U_0 increases, the growth rate increases, but the bandwidth is fixed in the exact position so that the maximum gain declines and as a effect, the stability increases.

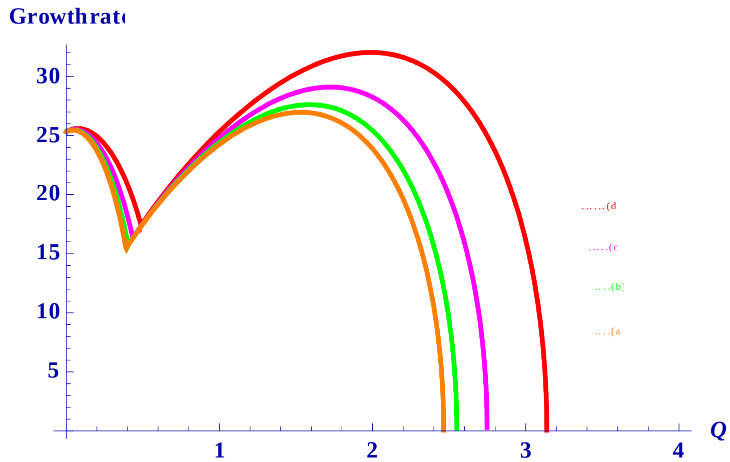

Figure 11: Gain spectrum for wave number Q by increasing J_2

12 Variation of Growth Rate With Interaction Parameters

It is verified that the MI region turns not only on the wave number of the alpha-helical protein system but also the interaction parameters $J_0, J_0', J_0'', J_2, J_2'$ and J_2''. Fig. (10) depicts the growth rate curve for different values of J_0, accepting $J_0 = J_0' = J_0''$. The parameters are $E_0 = 1.02, E_1 = 0.02, \chi = 0.13, q_1 = -0.4,, q_2 = 1.3, q_3 = 1.3, U_0 = 1$ and $J_2 = J_2' = J_2'' = 3.1$ with (a) $J_0 = J_0' = J_0'' = -0.4$, (b)$J_0 = J_0' = J_0'' = -0.7$, (c)$J_0 = J_0' = J_0'' = -0.9$ and (d) $J_0 = J_0' = J_0'' = -1.0$. By lowering J_0, the growth rate decreases but the band width rises to the maximum.

The growth rate curve is shown in Fig. (11) for various values of J_2. The growth rate rises as J_2 increases and the bandwidth stays in the exact position, and as an outcome, the gain gains. Due to the impact of interaction parameters, the MI obtains repressed, and the distinguishing growth rate of this reaction function is increased.

13 Conclusion

In this study, we use the exciton, phonon and phonon-exciton modes to develop a model Hamiltonian incorporating higher order molecular excitations. It is discovered that a (3+1) dimensional NLS type equation governs the equation of motion of dynamics that

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were developed. To create the soliton solution, we first apply the perturbation technique. Plotting appropriate curves in three dimensions allows for a detailed discussion of the soliton profile variations. Soliton's centres of mass, interaction energies, masses, and velocities are examined. Graphical explanations of the results are provided. Also, we discover the need for modulational instability of the (3+1) dimensional alpha-helical protein model with higher order molecular excitations. We also examine the modulational instability usual dependence on the perturbation wave number and system parameters for the growing rate of the instability. The outcomes are displayed graphically.

14 Appendix

Where,

$$R = U_0b\Omega - 2a_2q_1U_0bQ - 3a_2U_0bQ^2 - 2a_2q_2U_0bQ - 2a_2q_3U_0bQ - 2a_3q_1U_0bQ - 3a_3U_0bQ^2 - 4a_3q_2U_0bQ - 2a_3q_1U_0bQ + 4a_5q_1^3U_0bQ + 6a_5q_1^3U_0bQ^2 + 4a_5q_1U_0bQ^2 + 3a_5U_0bQ^4 + 4a_5q_2^3U_0bQ + 6a_5q_2^3U_0bQ^2 + 4a_5q_2U_0bQ^2 + 4a_5q_3^3U_0bQ + 6a_5q_3^3U_0bQ^2 + 4a_5q_3U_0bQ^2 + 2a_6q_1^3U_0bQ + 6a_6q_1^3U_0bQ^2 + 6a_6q_2^3U_0bQ^2 + 6a_6q_3^3U_0bQ^2 + 3a_6q_1^2q_2U_0bQ + 3a_6q_2^2q_1U_0bQ + 3a_6q_1^2q_3U_0bQ + 3a_6q_1^2q_3U_0bQ + 3a_6q_3^2q_2U_0bQ + 8a_6q_1U_0bQ^3 + 6a_6q_2U_0bQ^3 + 8a_6q_3U_0bQ^3 + 6a_6q_1q_2U_0bQ^2 + 6a_6q_1q_3U_0bQ^2 + 6a_6q_3q_2U_0bQ^2 + a_7q_1^2q_2U_0bQ + 4a_7q_2^2q_1U_0bQ + a_7q_2^2q_3U_0bQ + 2a_7q_3^2q_1U_0bQ + a_7q_2^2q_3U_0bQ + 2a_7q_3^2q_2U_0bQ + 2a_7q_1^2U_0bQ^2 + 2a_7q_2^2U_0bQ^2 + 2a_7q_3^2U_0bQ^2 + 4a_7q_1U_0bQ^3 + 2a_7q_2U_0bQ^3 + 2a_7q_3U_0bQ^3 + 4a_7q_1q_2U_0bQ^2 + 4a_7q_1q_3U_0bQ^2 + 4a_7q_3q_2U_0bQ^2 + 3a_7U_0bQ^4 + 4a_7q_3U_0bQ^3 + a_8q_1^2q_2q_3U_0bQ + a_8q_2^2q_1q_3U_0bQ + a_8q_3^2q_2q_1U_0bQ + a_8q_1^2U_0bQ^2 + a_8q_2^2U_0bQ^2 + a_8q_3^2U_0bQ^2 + a_8q_1^2q_3U_0bQ + a_8q_2^2q_1U_0bQ + a_8q_1^2q_3U_0bQ + a_8q_3^2q_2U_0bQ + 4a_8q_1U_0bQ^3 + 6a_8q_2U_0bQ^3 + 4a_8q_3U_0bQ^3 + 6a_8q_1q_3U_0bQ^2 + 6a_8q_1q_2U_0bQ^2 + 6a_8q_3q_2U_0bQ^2 + 6a_8q_1q_2q_3U_0bQ + 3a_8U_0bQ^4 + 2a_{10}q_1U_0^3bQ + 2a_{10}q_2U_0^3bQ + 2a_{10}q_3U_0^3bQ - 2a_{11}q_1U_0^3bQ - 2a_{11}q_2U_0^3bQ - 2a_{11}q_3U_0^3bQ - 3a_{11}U_0^3bQ^2 - 2a_{12}q_1U_0^3bQ - 2a_{12}q_2U_0^3bQ - 2a_{12}q_3U_0^3bQ - 2a_{13}q_1U_0^2bQ - 2a_{13}q_2U_0^2bQ - 2a_{13}q_3U_0^2bQ + 3a_{13}U_0^3bQ^2 - 2a_{14}q_1U_0^3bQ - 2a_{14}q_2U_0^3bQ - 2a_{14}q_3U_0^3bQ + 4a_{15}q_1U_0^3bQ + 4a_{15}q_2U_0^3bQ + 4a_{15}q_3U_0^3bQ - 2a_{16}q_1U_0^3bQ - 2a_{16}q_2U_0^3bQ - 2a_{16}q_3U_0^3bQ + 3a_{16}U_0^3bQ^2 - 2a_{17}q_1U_0^3bQ - 2a_{17}q_2U_0^3bQ - 2a_{17}q_3U_0^3bQ + 3a_{17}U_0^3bQ^2 \quad (33)$$

$$S = a\Omega - 3a_2aQ^2 - 3a_1q_1aQ - 3a_2q_2aQ - 3a_2q_3aQ - 3a_3aQ^2 - 2a_3q_1aQ - 3a_3q_2aQ - 3a_3q_3aQ - 3a_3aQ + 2a_4aU_0^2 + 3a_4aQ^3 + 5a_5q_1aQ^3 + 5a_5q_2aQ^3 + 5a_5q_3aQ^3 + 6a_5q_1^2aQ^2 + 6a_5q_2^2aQ^2 + 6a_5q_3^2aQ^2 + 5a_5q_1^3aQ + 5a_5q_2^3aQ + 5a_5q_3^3aQ + 6a_6aQ^4 - 6a_6q_1aQ^3 - 6a_6q_2aQ^3 - 6a_6q_3aQ^3 + 8a_6q_1q_2aQ^2 + 8a_6q_1q_3aQ^2 + 8a_6q_3q_2aQ^2 + 8a_6q_1^2aQ^2 + 8a_6q_2^2aQ^2 + 8a_6q_3^2aQ^2 + 4a_6q_1^2q_2aQ + 4a_6q_2^2q_1aQ + 4a_6q_1^2q_3aQ + 4a_6q_3^2q_1aQ + 4a_6q_2^2q_3aQ + 4a_6$$

$$q_3^2q_2aQ + 2a_6q_1^3aQ + 2a_6q_2^3aQ + 2a_6q_3^3aQ + 3a_7aQ^4 + 6a_7q_1aQ^3 + 5a_7 q_2aQ^3 + 4a_7q_3aQ^3 + 2a_7q_1^2aQ^2 + 2a_7q_2^2aQ^2 + 2a_7q_3^2aQ^2 + 4a_7q_1q_2aQ^2 + 4a_7q_1q_3aQ^2 + 4a_7q_3q_2aQ^2 + 3a_7q_2^2q_1aQ + 2a_7q_1^2q_2aQ + 3a_7q_3^2q_1aQ + 2a_7q_1^2q_3aQ + 3a_7q_3^2q_2aQ + 2a_7q_2^2q_3aQ + 3a_8aQ^4 + 5a_8q_1aQ^3 + 5a_8q_2 aQ^3 + 5a_8q_3aQ^3 + 6a_8q_2q_3aQ^2 + 6a_8q_1q_3aQ^2 + 6a_8q_1q_3aQ^2 + 7a_8q_1q_2 aQ^2 + 9a_8q_1q_2q_3aQ + a_8q_1^2aQ^2 + a_8q_2^2aQ^2 + a_8q_3^2aQ^2 + a_8q_1^2q_3aQ + a_8 q_1^2q_2aQ + a_8q_2^2q_1aQ + a_8q_2^2q_3aQ + a_8q_3^2q_2aQ + a_8q_3^2q_1aQ + 4a_9aU_0^4 + 2 a_{10}q_1^2U_0^2a + 2a_{10}q_2^2U_0^2a + 2a_{10}q_3^2U_0^2a - 3a_{11}U_0^2aQ^2 - 3a_{11}q_1U_0^2aQ - 3 a_{11}q_2U_0^2aQ - 3a_{11}q_3U_0^2aQ - a_{11}q_1^2U_0^2a - a_{11}q_2^2U_0^2a - a_{11}q_3^2U_0^2a - 2a_{12}q_1 U_0^2aQ - 2a_{12}q_2U_0^2aQ - 2a_{12}q_3U_0^2aQ - 2a_{12}q_1^2U_0^2a - 2a_{12}q_2^2U_0^2a - 2a_{12}q_3^2 U_0^2a - 3a_{13}U_0^2aQ^2 - 3a_{13}q_2U_0^2aQ + 3a_{13}q_3U_0^2aQ - a_{13}q_3^2U_0^2a - 2a_{14}q_1U_0^2 aQ - 2a_{14}q_2U_0^2aQ - 2a_{14}q_3U_0^2aQ - 2a_{14}q_2q_3U_0^2a - 2a_{14}q_2q_1U_0^2a - 2a_{14}q_1 q_3U_0^2a + 6a_{15}q_1q_2U_0^2a + 2a_{15}q_1q_3U_0^2a + 4a_{15}q_2U_0^2aQ + 4a_{15}q_3U_0^2aQ - 3 a_{16}U_0^2aQ^2 + 2a_{16}q_2U_0^2aQ + 2a_{16}q_1U_0^2aQ + 2a_{16}q_3U_0^2aQ + 2a_{16}q_3U_0^2aQ - 3a_{17}U_0^2aQ^2 - 2a_{17}q_2U_0^2aQ - 2a_{17}q_1U_0^2aQ - 2a_{17}q_3U_0^2aQ. \tag{34}$$

15 References

[1] Davydov A S and Kisluka N I 1973 Phys. Stat. Sol. b 59 465

[2] Davydov A S and Kisluka N I 1976 Sov. Phys. JETP 44 571

[3] Davydov A S 1979 Phys. Scripta 20 387

[4] Davydov A S 1980 Sov. Phys. JETP 51 397

[5] Davydov A S 1985 Solitons in molecular Systems (Reidel, Dordrecht)

[6] Scott A C 1982 Phys. Rev. A 26 578

[7] Scott A C 1983 Phys. Rev. A 27 2767

[8] Bartnik E A, Tuszynski J A, D. Sept 1995 Phys. Rev. A 204 (3-4) 263

[9] Biswas K C, Gillespie J, Majid F, Edwards M E, Biswas A 2009 Advanced Studies in Biology 1 1-8

[10] Bartnik L, Eremko A, Piette B, Zakrzewski W 1995 Phys. Rev. E 70031914

[11] Brown D Ivic Z 1989 Phys. Rev. B 40 14

[12] Careri G, Compatangelo E, Christiansen P L, Halding J, Skovgaard O 1987 Phys. Scripta 35 1 64

[13] Caspi S and Ben-Jacob E 2000 Phys. Lett. A 272 (1-2) 124

[14] Christiansen P L, Scott A C 1991 Springer-Verlag 56 877

[15] Daniel M and Latha M M 1999 Physica A 252 92

[16] Davydov A S 1991 Kluwer Academic Publishers

[17] Edler J, Pfister R, Pouthier V, Falvo C and Hamm P 2004 Phys. Rev.Lett. 93 106405

[18] Falvo C and Pouthier V 2005 J. Chem. Phys. 123 184709

[19] Förner W 1993 Physica D

[20] Förner W 1991 Phys. Rev. A 44 2694

[21] Ivic Z, Kostic D, Przulj Z, Kapor D 1997 Journal of Physics: Condensed Matter 9(2) 413

[22] Kohn W D, Mant C T, Hodges R S 1997 The Journal of Biological Chemistry 272 (5) 2583

[23] Lawrence A F, Mcdaniel J C, Chang D B, Birge R R 1987 Journal of Biophysical Society 51 (5) 785

[24] Simo E and Caputo J G 2008 Chinese Journal of Physics 46 (2) 201

[25] Tan B and Boyd J P 1998 Phys. Lett. A 240 (5) 282

[26] Zekovic S and Ivic Z 1999 Bioelectrochemistry and Bioenergetics 48 (2)

[27] Daniel M and Latha M M 1997 Physica A 240 526

ISSN 2063-5346

[28] Scott A C 1984 Phys. Scr. 29 279

[29] V. A. Kuprievich and Z. G. Kudritskaya 1982Preprint ITP -Institute for Theoretical Physics Kiev 82 64-E

[30] Davydov A S, Eremko A A and Segienko A I 1978 Ukr. Fiz. Zh 23 983

[31] Eilbeck J C, Lomdahl P S and Scott A C 1984 Phys. Rev. B 30 4703

[32] Cruzeiro 2009 J. Biol. Phys. 35(1) 43

[33] Veni S S and Latha M M 2012 Phys. Scr. 86 025003

[34] Wazwaz A M 2004 The Appl. Math. Computation 159 559

[35] Wazwaz A M 2004 Math. Comput. Modelling 40 499

[36] M. M. Latha and G. Merlin, Phys. Lett. A 376, 938 (2012).

[37] J. Ladik and J. Cizek, Quantum. Chem. 26, 955 (1984).

ISSN 2063-5346