THE FAULT TOLERANT GEODETIC NUMBER OF TOTAL AND MIDDLE NUMBER OF A GRAPH

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Abstract

The total graph $T(G)$ of a graph G is a graph such that the vertex set of $T(G)$ corresponds to the vertices and edges of G and two vertices are adjacent in $T(G)$ if and only if their corresponding u_1 element are either adjacent or incident in G. The middle graph of connected graph G denoted by $M(G)$ is the graph whose vertex set is $V(G) \cup E(G)$ where two vertices are adjacent if they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it. In this article, we studied the fault tolerant geodetic number of total and middle graph of a graph.

Keywords: Total graph, middle graph, geodetic number, fault tolerant geodetic number.

AMS Subject Classification: 05C12.

1 Introduction

By a graph $G = (V, E)$, we mean a finite, undirected connected graph without loops or multiple edges. The order and size of G are denoted by n and m respectively. For basic graph theoretic terminology, we refer to [5, 8]. Two vertices u and v of said to be adjacent in G if $uv \in E(G)$. The neighbourhood $N(v)$ of the vertex v in G is the set of vertices adjacent to v. The degree of the vertex v is $deg(v) = |N(v)|$. If $e = \{u, v\}$ is an edge of a graph G with $deg(u) = 1$ and $deg(v) > 1$, then we call e an end edge, u a leaf and v a support vertex. For any connected graph G, a vertex $v \in V(G)$

is called a cut vertex of G if $V(G) - v$ is disconnected. The subgraph induced by set S of vertices of a graph G is denoted by $\langle S\rangle$ with $V(\langle S\rangle) = S$ and $E(\langle S\rangle) = \{uv \in E(G) : u, v \in S\}$. A vertex v is called an extreme vertex of G if $\langle N(v)\rangle$ is complete.

A vertex x is an internal vertex of an $u - v$ path P if x is a vertex of P and $x \neq u, v$. An edge e of G is an internal edge of an $u - v$ path P if e is an edge of P with both of its ends or in P. The distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G. An $u - v$ path of length $d(u, v)$ is called an $u - v$ geodesic. A vertex x is said to lie on an $u - v$ geodesic P if x isa vertex of P including the vertices u and v. For a vertex v of G, the eccentricity $e(v)$ is the distance between v and a vertex farthest from v. The closed interval $I[u, v]$ consists of u, v and all vertices lying on some $u - v$ geodesic of G. For a non-empty set $S \subseteq V(G)$, the set $I[S] = \bigcup_{u,v\in S} I[u, v]$ is the closure of S. A set $S \subseteq V(G)$ is called a geodetic set if $I[S] = V(G)$. Thus every vertex of G is contained in a geodesic joining some pair of vertices in S. The minimum cardinality of a geodetic set of G is called the geodetic number of G and is denoted by $g(G)$. For references on geodetic parameters in graphs see [1-4, 6, 7, 9-14, 16, 17]. Let S be a geodetic set of G and W be the set of extreme vertices of G. Then S is said to be a fault tolerant geodetic set of G, if $S - \{v\}$ is also a geodetic set of G for every $v \in S \setminus W$. The minimum cardinality of a fault tolerant geodetic set is called fault tolerant geodetic number and is denoted by $g_{ft}(G)$. The minimum fault tolerant geodetic dominating set of G is denoted by g_{ft}-set of G. These concepts were studied in [15].The following theorem is used in the sequel.

Theorem 1.1. [6] Each extreme vertex of a connected graph G belongs to every fault tolerant geodetic set of G.

2 The Fault Tolerant Geodetic Total Number of a Graph

Definition 2.1. The total graph $T(G)$ of a graph G is a graph such that the vertex set of $T(G)$ corresponds to the vertices and edges of G and two vertices are adjacent in $T(G)$ if and only if their corresponding u_1 element are either adjacent or incident in G.

Definition 2.2. The middle graph of connected graph G denoted by $M(G)$ is the graph whose vertex set is $V(G) \cup E(G)$ where two vertices are adjacent if

(i) They are adjacent edges of G, or

(ii) One is a vertex of G and the other is an edge incident with it.

Theorem 2.3. Let G be the total graph of the path P_n $(n \geq 4)$. Then $g_{ft}(G) = 4$.

Proof. Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $E(P_n) = \{u_1, u_2, \dots, u_{n-1}\}$. Then $V(G) = V(P_n) \cup E(P_n)$. Therefore $|V(G)| = 2n - 1$. $E(G) = \{v_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{u_i v_i, u_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{u_i u_i; 1 \leq i \leq n-2\}$. Therefore $|E(G)| = 4n - 2$. Let $Z = \{v_1, v_n\}$ be the set of all extreme vertices of G. By Theorem 1.1, Z is a subset of every fault tolerant geodetic set of G. Since $I[Z] \neq V(G)$, Z is not a fault tolerant geodetic set of G. It is easily observed that every minimum fault tolerant geodetic set of G contains exactly two vertices of $E(P_n)$ and so $g_{ft}(G) \geq 4$. Let $S = Z \cup \{u_1, u_{n-1}\}$. Then S is a fault tolerant geodetic set of G so that $g_{ft}(G) = 4$.

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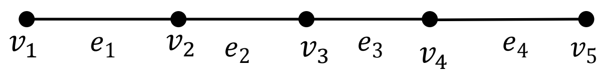
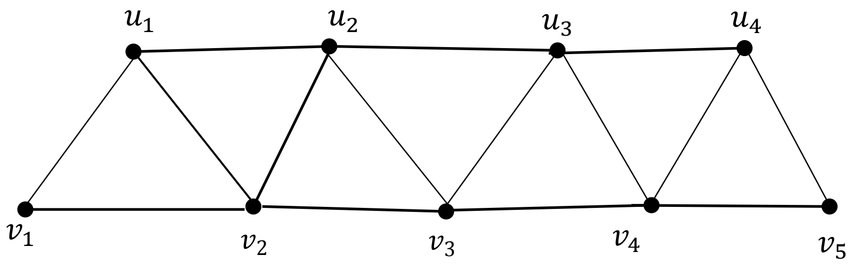


P_5



$T(P_5)$

Theorem 2.4. Let G be the total graph of the graph C_n $(n \geq 4)$. Then

$$g_{ft}(G) = \begin{cases} 2n & \text{if } n \in \{4,5\} \\ 8 & \text{if } n \text{ is even and } n \geq 6 \\ 12 & \text{if } n \text{ is odd and } n \geq 7 \end{cases}$$

Proof. Let $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and $E(C_n) = \{u_1, u_2, \dots, u_{n-1}\}$. Then $V(G) = V(C_n) \cup E(C_n)$. Therefore $|V(G)| = 2n$. $E(G) = \{v_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{u_i v_i, u_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{u_i u_{i+1}; 1 \leq i \leq n-2\}$. Therefore $|E(G)| = 8n - 2$.

For $n = 4$ or 5, $S = V(G)$ is the unique g_{ft}-set of G so that $g_{ft}(G) = |V(G)| = 2n$. Let S be a g_{ft}-set of G. We have the following cases.

Case (i) Let n be a even. Let $n = 2k$ $(k \geq 3)$. Then S contains four pair of antipodal vertices from $V(G)$ and so $g_{ft}(G) \geq 8$. Let $S' = \{v_1, v_2, v_{k+1}, v_{2k}\} \cup \{u_1, u_2, u_{k+1}, u_{2k}\}$. Then S' is a g_{ft}-set of G so that $g_{ft}(G) = 8$.

Case (ii) Let n be odd. Let $n = 2k + 1$ $(k \geq 3)$. It is easily observed that S contains four pairs of antipodal vertices of $V(G)$ and so $g_{ft}(G) \geq 12$. Let $S = \{v_1, v_2, v_{k+1}, v_{k+2}, v_{2k-1}, v_{2k}\} \cup u_1, u_2, u_{k+1}, u_{k+2}, u_{2k-1}, u_{2k}\}$. Then S is a g_{ft}-set of G so that $g_{ft}(G) = 12$.

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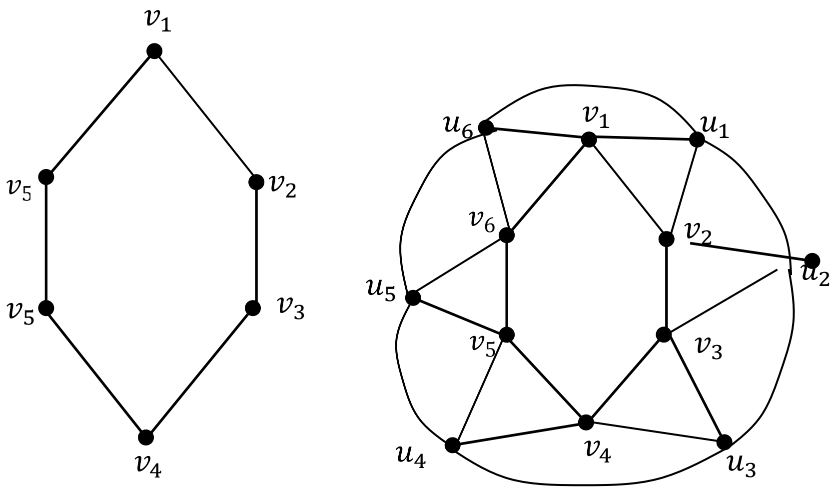
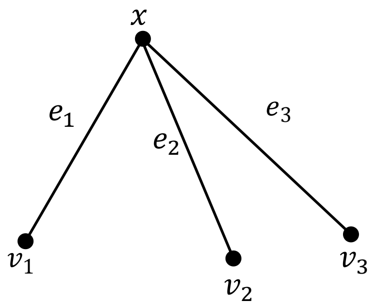


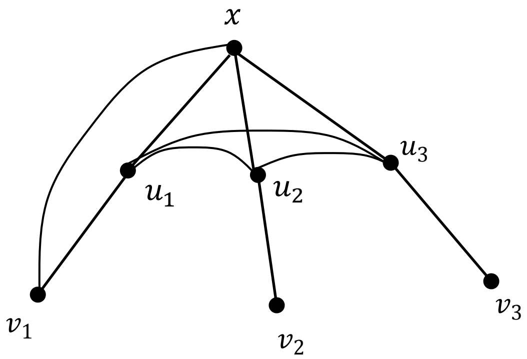
$G = C_6$ $\qquad$ $T(C_6)$

Theorem 2.5. Let G be the total graph of the star $K_{1,n-1}$ $(n \geq 4)$. Then $g_{ft}(G) = 2n - 2$.

Proof. Let $Z = \{v_1, v_2, \dots, v_{n-1}\}$ be the set of all end vertices of G. Then by Theorem 1.1, Z is a subset of every fault tolerant geodetic set of G and so $g_{ft}(G) \geq n - 1$. Since Z is a fault tolerant geodetic set of G, $g_{ft}(G) \geq n$. Let $S = Z \cup \{u_1, u_2, \dots, u_{n-1}\}$. Then S is a fault tolerant geodetic set of G so that $g_{ft}(G) \leq 2n - 2$. We prove that $g_{ft}(G) = 2n - 2$. On the contrary suppose that $g_{ft}(G) \leq 2n - 3$. Then there exists a g_{ft}-set S' of G set that $|S'| \leq 2n - 3$. Let $u \in S'$such that $u \notin S$. Then $S' - \{u\}$ is not a fault tolerant geodetic set of G, which is a contradiction. Therefore $g_{ft}(G) = 2n - 2$. ■



$G = K_{1,3}$



$T(K_{1,3})$

Theorem 2.6. Let G be the middle graph of the path P_n $(n \geq 4)$. Then $g_{ft}(G) = n$.

Proof. Let $S = V(P_n)$. Then S is the set of all extreme vertices of G. By the definition of fault tolerant geodetic set of G, $g_{ft}(G) = n$.

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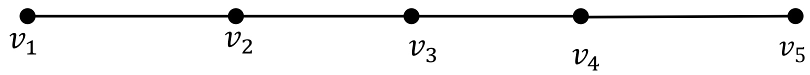
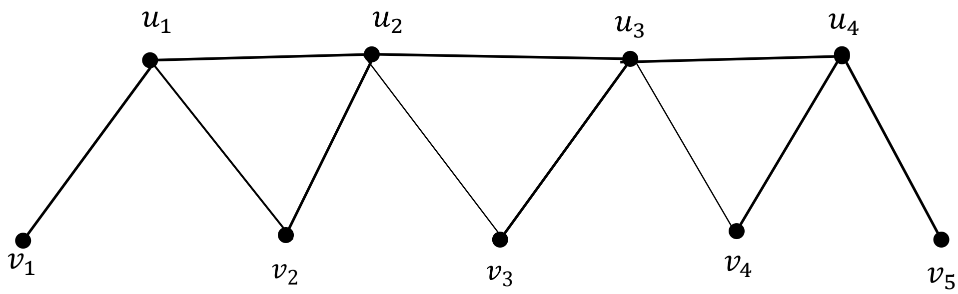


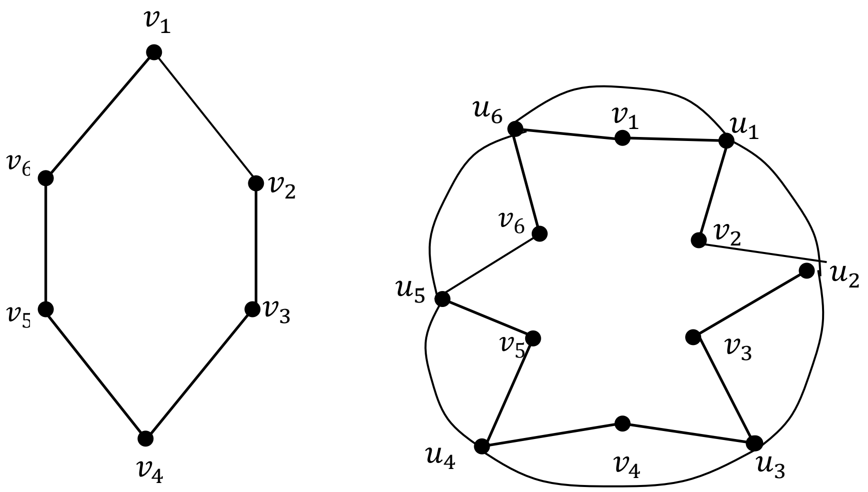
P_5



$M(P_5)$

Theorem 2.7. Let G be the middle graph of the path C_n $(n \geq 8)$. Then $g_{ft}(G) = n$.

Proof. Let $S = \{x, v_1, v_2, \ldots, v_{n-1}\}$. Then S is the set of all extreme vertices of G. By the definition of fault tolerant geodetic set of G, $g_{ft}(G) = n$. ■



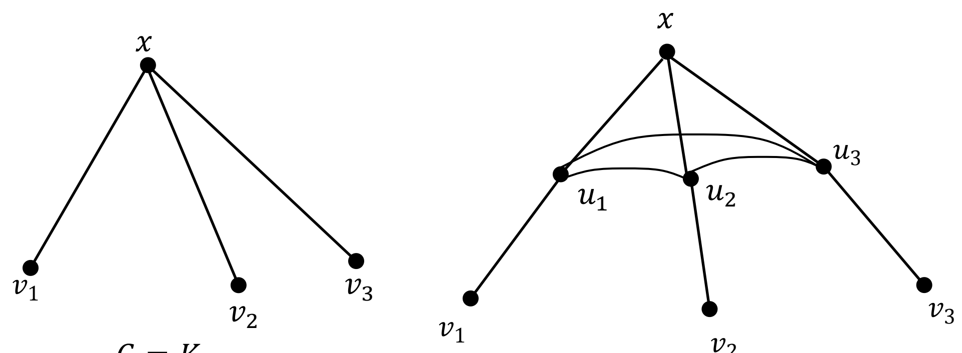
$G = C_6$ $M(C_6)$

77

Theorem 2.8. Let G be the middle graph of the star $K_{1,n-1}$ $(n \geq 4)$. Then $g_{ft}(G) = n$.

Proof. Let $S = V(C_n)$. Then S is the set of all extreme vertices of G. By the definition

of fault tolerant geodetic set, $g_{ft}(G) = n$. ■



$G = K_{1,3}$ $M(K_{1,3})$

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