



# FUZZY NEUTROSOPHIC DENSE SETS AND BAIRE SPACES

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## Abstract:

This paper introduces the concept of fuz. neu. den. sets in fuz. neu. top. sp.s. Also, we obtain several charact. of fuz. neu. den. sets in fuz. neu. Baire sp.s are established. Connection. between fuz. neu. den. sets and fuz. neu. Baire sp., fuz. neu. den. Baire sp.s are studied. Several properties are also discussed with illustration and suitable examples.

## Keywords:

Fuz. neu. den. set, Fuz. neu. Baire sp., Fuz. neu. den. Baire sp..

## 1. Introduction:

The concept of fuz. sets were first initiate by Zadeh. L. A [11] in 1965, in which he proposed that the traditional notion of set, in which an element is either a member or not a member, should be generalized to allow for degree of membership. This gives rise to the expansion of fuz. set theory, which has since been applied to a large assortment of fields, involve control theory, image processing, and artificial intelligence.

Neu. sets were first initiate by Florentin Smarandache [7] in 1955, which is a overview of the concepts of fuz. sets and intuitionistic fuz. sets. Neu. sets allow for consideration of indeterminacy, neutrality, and inconsistency in a given sp.. In 2017, Veereswari [10] initiate fuz. neu. top. sp.s, which combines the ideas of fuz. sets and neu. sets to introduce a more general framework for studying top. sp.s.

Since then, researches have been working on developing the theory of fuz. neu. top. sp.s and its application to diverse fields such as soft computing, artificial intelligence, decision making, pattern recognition, and image processing.

## 2. Preliminaries:

Throughout this present paper,  $X$  denotes the fuzzy neutrosophic topological spaces. Let  $A_N$  be a fuzzy neutrosophic set on  $X$ . The fuzzy neutrosophic interior and closure of  $A_N$  is denoted as  $fn(A_N)^+$ ,  $fn(A_N)^-$  respectively. A fuzzy neutrosophic set  $A_N$  is defined to be fuzzy neutrosophic open set ( $fnOS$ ) if  $A_N \leq fn(((A_N)^-)^+)^-$ . The complement of a fuzzy neutrosophic open set is called fuzzy neutrosophic closed set ( $fnCS$ ).

### **Definition 2.1 [2]:**

A fuzzy neutrosophic set  $A$  on the universe of discourse  $X$  is defined as  $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$ ,  $x \in X$  where  $T, I, F: X \rightarrow [0,1]$  and  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ .

With the condition  $0 \leq T_{A^*}(x) + I_{A^*}(x) + F_{A^*}(x) \leq 2$ .

### **Definition 2.2 [2]:**

A fuzzy neutrosophic set  $A$  is a subset of a fuzzy neutrosophic set  $B$  (i.e.,)  $A \subseteq B$  for all  $x$  if  $T_A(x) \leq T_B(x)$ ,  $I_A(x) \leq I_B(x)$ ,  $F_A(x) \geq F_B(x)$ .

### **Definition 2.3 [2]:**

Let  $X$  be a non-empty set, and  $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$ ,  $B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$  be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle$$

$$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \rangle$$

**Definition 2.4 [2]:**

The difference between two fuzzy neutrosophic sets  $A$  and  $B$  is defined as  $A \setminus B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x)) \rangle$

**Definition 2.5 [2]:**

A fuzzy neutrosophic set  $A$  over the universe  $X$  is said to be null or empty fuzzy neutrosophic set if  $T_A(x) = 0$ ,  $I_A(x) = 0$ ,  $F_A(x) = 1$  for all  $x \in X$ . It is denoted by  $0_N$ .

**Definition 2.6 [2]:**

A fuzzy neutrosophic set  $A$  over the universe  $X$  is said to be absolute (universe) fuzzy neutrosophic set if  $T_A(x) = 1$ ,  $I_A(x) = 1$ ,  $F_A(x) = 0$  for all  $x \in X$ . It is denoted by  $1_N$ .

**Definition 2.7 [2]:**

The complement of a fuzzy neutrosophic set  $A$  is denoted by  $A^c$  and is defined as

$A^c = \langle x, T_{A^c}(x), I_{A^c}(x), F_{A^c}(x) \rangle$  where  $T_{A^c}(x) = F_A(x)$ ,  $I_{A^c}(x) = 1 - I_A(x)$ ,  $F_{A^c}(x) = T_A(x)$

The complement of fuzzy neutrosophic set  $A$  can also be defined as  $A^c = 1_N - A$ .

**Definition 2.8 [1]:**

A fuzzy neutrosophic topology on a non-empty set  $X$  is a  $r$  of fuzzy neutrosophic sets in  $X$

(i)  $0_N, 1_N \in r$

(ii)  $A_1 \cap A_2 \in r$  for any  $A_1, A_2 \in r$

(iii)  $\cup A_i \in r$  for any arbitrary family  $\{A_i: i \in J\} \in r$

Satisfying the following axioms.

In this case the pair  $(X, r)$  is called fuzzy neutrosophic topological space and any Fuzzy neutrosophic set in  $r$  is known as fuzzy neutrosophic open set in  $X$ .

**Definition 2.9 [1]:**

The complement  $A^c$  of a fuzzy neutrosophic set  $A$  in a fuzzy neutrosophic topological space  $(X, r)$  is called fuzzy neutrosophic closed set in  $X$ .

**Definition 2.10 [1]:**

Let  $(X, r_N)$  be a fuzzy neutrosophic topological space and  $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$  be a fuzzy neutrosophic set in  $X$ . Then the closure and interior of  $A$  are defined by

$$\text{int}(A) = \cup \{G: G \text{ is a fuzzy neutrosophic open set in } X \text{ and } G \subseteq A\}$$

$$\text{cl}(A) = \cap \{G: G \text{ is a fuzzy neutrosophic closed set in } X \text{ and } A \subseteq G\}$$

**Definition 2.11 [4]:**

A fuzzy neutrosophic set  $A_N$  in a fuzzy neutrosophic topological space  $(X, r_N)$  is called a fuzzy neutrosophic  $F_\sigma$  – set if  $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$ , where  $A_{N_i} \in r_N$  for  $i \in I$ .

**Definition 2.12 [4]:**

A fuzzy neutrosophic set  $A_N$  in a fuzzy neutrosophic topological space  $(X, r_N)$  is called a fuzzy neutrosophic  $G_\delta$  – set in  $(X, r_N)$  if  $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$ , where  $A_{N_i} \in r_N$  for  $i \in I$ .

**Definition 2.13 [4]:**

A fuzzy neutrosophic set  $A_N$  in a fuzzy neutrosophic topological space  $(X, r_N)$  is called a fuzzy neutrosophic semi-open if  $A_N \leq \text{fn}((A_N)^+)^-$ . The complement of  $A_N$  in  $(X, r_N)$  is called a fuzzy neutrosophic semi-closed set in  $(X, r_N)$ .

**Definition 2.14 [4]:**

A fuzzy neutrosophic set  $A_N$  in a fuzzy neutrosophic topological space  $(X, r_N)$  is called a fuzzy neutrosophic dense if there exist no fuzzy neutrosophic closed set  $B_N$  in  $(X, r_N)$  such that  $A_N \subset B_N \subset 1_X$ . That is,  $fn(A_N)^- = 1_N$ .

**Definition 2.15 [4]:**

A fuzzy neutrosophic set  $A_N$  in a fuzzy neutrosophic topological space  $(X, r_N)$  is called a fuzzy neutrosophic nowhere dense set if there exist no non-zero fuzzy neutrosophic open set  $B_N$  in  $(X, r_N)$  such that  $B_N \subset fn(A_N)^-$ . That is,  $fn(((A_N)^-)^+) = 0_N$ .

**Definition 2.16 [4]:**

Let  $(X, r_N)$  be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set  $A_N$  in  $(X, r_N)$  is called fuzzy neutrosophic one category set if  $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$ , where  $A_{N_i}$ 's are fuzzy neutrosophic nowhere dense sets in  $(X, r_N)$ . Any other fuzzy neutrosophic set in  $(X, r_N)$  is said to be of fuzzy neutrosophic two category.

**Definition 2.17 [4]:**

A fuzzy neutrosophic topological space  $(X, r_N)$  is called fuzzy neutrosophic one category space if the fuzzy neutrosophic set  $1_X$  is a fuzzy neutrosophic one category set in  $(X, r_N)$ . That is  $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$ , where

$A_{N_i}$ 's are fuzzy neutrosophic nowhere dense sets in  $(X, r_N)$ . Otherwise  $(X, r_N)$  will be called a fuzzy neutrosophic two category space.

**Definition 2.18 [4]:**

Let  $A_N$  be a fuzzy neutrosophic first category set in  $(X, r_N)$ . Then  $\overline{A}$  is called fuzzy neutrosophic residual set in  $(X, r_N)$ .

### 3. On Fuzzy Neutrosophic Dense Sets and Baire Spaces

**Definition 3.1:**

Let  $(M, r_N)$  be a fuz. neu. top. sp.. A fuz. neu. set  $A_N$  in  $(M, r_N)$  is called fuz. neu. Baire den. set if  $B_N$  is a fuz.  $G_\delta$  - set in  $(M, r_N)$ ,  $A_N \cap B_N$  is a fuz. neu. two caty. set in  $(M, r_N)$ .

**Eg; 3.1:**

Let  $M = \{a, b, c\}$  and consider the family  $r_N = \{0_N, 1_N, A_N, B_N, C_N, E_N, G_N\}$  where

$$A_N = \{\langle a, 0.3, 0.3, 0.5 \rangle, \langle b, 0.6, 0.6, 0.5 \rangle, \langle c, 0.6, 0.6, 0.5 \rangle\}$$

$$B_N = \{\langle a, 0.6, 0.6, 0.6 \rangle, \langle b, 0.3, 0.3, 0.3 \rangle, \langle c, 0.6, 0.6, 0.6 \rangle\}$$

$$C_N = \{\langle a, 0.3, 0.3, 0.4 \rangle, \langle b, 0.7, 0.7, 0.4 \rangle, \langle c, 0.3, 0.3, 0.4 \rangle\}$$

$$E_N = \{\langle a, 0.3, 0.3, 0.3 \rangle, \langle b, 0.3, 0.3, 0.3 \rangle, \langle c, 0.7, 0.6, 0.6 \rangle\}$$

$$G_N = \{\langle a, 0.4, 0.3, 0.3 \rangle, \langle b, 0.3, 0.2, 0.4 \rangle, \langle c, 0.6, 0.6, 0.5 \rangle\}$$

Thus  $(M, r_N)$  is a fuz. neu. top. sp.s. Now,  $\overline{A \overline{B}}$  and  $C_N$ , are fuz. neu. nowh. den. sets in  $(M, r_N)$ . Now,  $\overline{A \overline{B}}$  and  $C_N$ , are fuz. neu. one caty. sets in  $(M, r_N)$ . Now, the fuz. neu. sets  $E_N \& A_N, G_N \& B_N$  are not fuz. neu. one caty. sets in  $(M, r_N)$  and hence fuz. neu. two caty. sets in  $(M, r_N)$ . Thus  $E_N \& G_N$  are fuz. neu. Baire den. set in  $(M, r_N)$ .

**Pro. 3.1:**

If  $A_N$  is a fuz. neu. Baire den. set in a fuz. neu. top. sp.  $(M, r_N)$ , then for a fuz.  $G_\delta$  - set  $B_N$  in  $(M, r_N)$ ,  $A_N \wedge B_N = \bigvee_{i=1}^{\infty} (C_N)_i$ , where  $(C_N)_i$ 's are fuz. neu. nowhere den. set in  $(M, r_N)$ .

**Proof:**

Let  $A_N$  be a fuz. neu. Baire den. set in a fuz. neu. top. sp.  $(M, r_N)$ , then for a fuz.  $G_\delta$  - set in  $B_N$  in  $(M, r_N)$ ,  $A_N \wedge B_N$  is a fuz. neu. two caty. set in  $(M, r_N)$  and hence  $A_N \wedge B_N$  is not a fuz. neu. one caty. set in  $(M, r_N)$ . Thus  $A_N \wedge B_N = \bigvee_{i=1}^{\infty} (C_N)_i$ , where  $(C_N)_i$ 's are fuz. neu. nowhere den. set in  $(M, r_N)$ .

**Pro. 3.2:**

If  $A_N$  is a fuz. neu. Baire den. set in a fuz. neu. top. sp.  $(M, r_N)$ , then for a fuz. neu.  $G_\delta$  - set  $B_N$  in  $(M, r_N)$ ,  $1 - (A_N \wedge B_N) = \bigwedge_{i=1}^{\infty} E_N$ , where  $f_n (E_{N_i})^- = 1_N$ .



**Proof:**

Let  $A_N$  be a fuz. neu. Baire den. set in a fuz. neu. top. sp.  $(M, r_N)$ . Then by Pro. (3.1),  $A_N \wedge B_N = \bigvee_{i=1}^{\infty} (C_N)_i$ , where  $(C_N)_i$ 's are fuz. neu. nowhere den. set in  $(M, r_N)$ . Then  $1 - (A_N \wedge B_N) = 1 - \bigvee_{i=1}^{\infty} (C_N)_i = \bigwedge_{i=1}^{\infty} (1 - C_N)_i$  in  $(M, r_N)$ . Since,  $(C_N)_i$  is a fuz. neu. nowhere den. set in  $(M, r_N)$ ,  $fn((C_N)_i^-) = 0$ . Since  $fn(C_{N_i}^+) \leq fn((C_{N_i}^-)^+)$  in  $(M, r_N)$ ,  $fn(C_{N_i}^+) = 0$  and hence  $1 - fn(C_{N_i}^+) = 1$  in  $(M, r_N)$ . This  $\Rightarrow fn(1 - C_N)_i^- = 1$ . Thus  $1 - (A_N \wedge B_N) = \bigwedge_{i=1}^{\infty} (1 - C_N)_i$ , where  $(1 - C_N)_i$ 's are fuz. neu. nowhere den. set in  $(M, r_N)$ . Let  $E_{N_i} = 1 - C_{N_i}$ . Then let  $1 - (A_N \wedge B_N) = \bigwedge_{i=1}^{\infty} (E_N)_i$ , where  $(E_N)_i$ 's are fuz. neu. nowhere den. set in  $(M, r_N)$ .

**Pro. 3.3:**

If  $A_N$  is a fuz. neu. Baire den. set in a fuz. neu. top. sp.  $(M, r_N)$ , then there exist a fuz.  $G_{\delta}$  - set  $B_N$  in  $(X, r_N)$ , fuz. neu. two caty. set  $C_N$  s.t  $C_N \leq A_N$  in  $(M, r_N)$ .

**Proof:**

Let  $A_N$  be a fuz. neu. Baire den. set in  $(M, r_N)$ , then for a fuz.  $G_{\delta}$  - set in  $B_N$  in  $(M, r_N)$ ,  $A_N \wedge B_N$  is a fuz. two caty. set in  $(M, r_N)$ . Clearly  $A_N \wedge$

$B_N \leq A_N$ . Let  $A_N \wedge B_N = C_N$ . Thus  $C_N$  is a fuz. neu. two caty. set in  $(M, r_N)$  s.t  $C_N \leq A_N$  in  $(M, r_N)$ .

**Pro. 3.4:**

If  $A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ , then each fuz. neu. two caty. set is a fuz. neu. den. set in a fuz. neu. topological  $(M, r_N)$ , then  $A_N$  is a fuz. neu. den. set in  $(M, r_N)$ .

**Proof:**

Let  $A_N$  be a fuz. neu. Baire den. set in  $(M, r_N)$ , then by Pro. (3.3),  $\exists$  a fuz. neu. two caty. set  $C_N \leq A_N$  in  $(M, r_N)$  and hence  $fn(C_N)^- = fn(A_N)^+$ , in  $(M, r_N)$ . By hypothesis,  $fn(C_N)^- = 1$  in  $(M, r_N)$ . Then,  $1 \leq fn(A_N)^-$ . That is,  $fn(A_N)^- = 1$  in  $(M, r_N)$ . Hence  $A_N$  is a fuz. neu. den. set in  $(M, r_N)$ .

**Definition 3.2:**

A fuz. neu. top. sp.  $(M, r_N)$  is called a fuz. neu. submaximal sp. if for each fuz. neu. set  $A_N$  in  $(M, r_N)$  s.t  $fn(A_N)^- = 1$ ,  $A_N \in r_N$  in  $(M, r_N)$ .

**Definition 3.3:**

A fuz. neu. top. sp.  $(M, r_N)$  is called a fuz. neu.  $P$  – sp. if countable intersection of fuz. neu. op. sets in  $(M, r_N)$  is fuz. neu. open. That is, every non zero fuz. neu.  $G_\delta$  – set in  $(M, r_N)$  is a fuz. neu. op. set in  $(M, r_N)$ .

**Definition 3.4:**

If the fuz. neu. top. sp.  $(M, r_N)$  is a fuz. neu. Baire sp. then no non-zero fuz. neu. op. set is a fuz. neu. one caty. set in  $(M, r_N)$ .

**Definition 3.5:**

If  $A_N$  is a fuz. neu. nowhere den. set in a fuz. neu. submaximal sp.  $(M, r_N)$ , then  $A_N$  is a fuz. neu. cld. set in  $(M, r_N)$ .

**Definition 3.6:**

If  $A_N$  is a fuz. neu.  $G_\delta$  – set in a fuz. neu. Baire and fuz. neu.  $P$  – sp.  $(M, r_N)$ , then  $A_N$  is a fuz. neu. two caty. set in  $(M, r_N)$ .

**Pro. 3.5:**

If each fuz. neu.  $G_\delta$  – set is a fuz. neu. Baire den. set in a fuz. neu. submaximal sp.  $(M, r_N)$ , then  $(M, r_N)$  is a fuz. neu.  $P$  – sp..

**Proof:**

Let  $A_N$  be a fuz. neu.  $G_\delta$  – set in a fuz. neu. submaximal sp.  $(M, r_N)$ . Then by the hypothesis,  $A_N \cap B_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ . Since  $(M, r_N)$  is a fuz. neu. submaximal sp., the fuz. neu. Baire den. set  $A_N \cap B_N$  in  $(M, r_N)$ , is a fuz. neu. op. set in  $(M, r_N)$ . That is, every fuz. neu.  $G_\delta$  – set in  $(M, r_N)$  is a fuz. neu. op. set in  $(M, r_N)$ . Therefore  $(M, r_N)$  is a fuz. neu.  $P$  – sp..

**Pro. 3.6:**

If  $A_N$  is a fuz. neu. Baire den. set in a fuz. neu. submaximal sp.  $(M, r_N)$ , then for fuz. neu. op. set  $B_N$  in  $(M, r_N)$ ,  $A_N \wedge B_N$  is not a fuz. neu.  $F_\sigma$  – set in  $(M, r_N)$ .

**Proof:**

Let  $A_N$  be a fuz. neu. Baire den. set in  $(M, r_N)$ . Then by Pro. (3.1), for a fuz. neu. op. set  $B_N$  in  $(M, r_N)$ ,  $A_N \wedge B_N = \bigvee_{i=1}^{\infty} (C_N)_i$ , where  $(C_N)_i$ 's are fuz. neu. nowh. den. sets in  $(M, r_N)$ . Since  $(M, r_N)$  is a fuz. neu. submaximal sp., by definition (3.5), the fuz. neu. nowh. den. sets  $(C_N)_i$ 's are fuz. neu. closed sets in  $(M, r_N)$ . Then  $\bigvee_{i=1}^{\infty} (C_N)_i$  is a fuz. neu.  $F_\sigma$  – set in  $(M, r_N)$ . Thus  $A_N \wedge B_N = \bigvee_{i=1}^{\infty} (C_N)_i$ ,  $\Rightarrow A_N \wedge B_N$  is not a fuz. neu.  $F_\sigma$  – set in  $(M, r_N)$ .

**Pro. 3.7:**

If  $A_N$  is a fuz. neu. set in a fuz. neu. Baire and fuz. neu.  $P$  – sp.  $(M, r_N)$  s.t  $A_N \wedge B_N$  is a fuz. neu.  $G_\delta$  – set, for a fuz. neu. op. set  $B_N$  in  $(M, r_N)$ , then  $A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ .

**Proof:**

Let  $A_N$  be a fuz. neu. set in  $(M, r_N)$  s.t  $A_N \wedge B_N$  is a fuz. neu.  $G_\delta$  – set, for a fuz. neu. op. set  $B_N$  in  $(M, r_N)$ . Since  $(M, r_N)$  is a fuz. neu.  $P$  – sp.,

the fuz. neu.  $G_\delta$  – set  $A_N \wedge B_N$  is a fuz. neu. op. set  $B_N$  in  $(M, r_N)$ . Also since  $(M, r_N)$  is a fuz. neu. Baire sp. by definition (3.4), the non-zero fuz. neu. op. set  $A_N \wedge B_N$  is not a fuz. neu. one caty. set in  $(M, r_N)$  and hence  $A_N \wedge B_N$  is a fuz. neu. two caty. set in  $(M, r_N)$ . This  $\Rightarrow A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ .

**Pro. 3.8:**

If  $A_N$  is a fuz. neu.  $G_\delta$  – set in a fuz. neu. Baire and fuz. neu.  $P$  – sp.  $(M, r_N)$  s.t  $A_N \leq B_N$ , for fuz. neu. op. set  $B_N$  in  $(M, r_N)$ , then  $A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ .

**Proof:**

Let  $A_N$  be a fuz. neu.  $G_\delta$  – set s.t  $A_N \leq B_N$ , for a fuz. neu. op. set  $B_N$  in  $(M, r_N)$ . Since  $(M, r_N)$  is a fuz. neu. Baire and fuz. neu.  $P$  – sp. by definition (3.6), the fuz. neu.  $G_\delta$  – set is a fuz. neu. two caty. set in  $(M, r_N)$ . Now  $A_N \wedge B_N = A_N \Rightarrow A_N \wedge B_N$  is a fuz. neu. two caty. set in  $(M, r_N)$  and hence  $A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ .

**Pro. 3.9:**

If  $A_N$  is a fuz. neu.  $F_\sigma$  – set in a fuz. neu. Baire and fuz. neu.  $P$  – sp.  $(M, r_N)$  s.t  $A_N \geq B_N$ , for fuz. neu. cld. set  $B_N$  in  $(M, r_N)$ , then  $1 - A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ .

**Proof:**

Let  $A_N$  be a fuz. neu.  $F_\sigma$  - set s.t  $A_N \geq B_N$ , for a fuz. neu. cld. set  $B_N$  in  $(M, r_N)$ . Now  $1 - A_N$  is a fuz. neu.  $G_\delta$  - set s.t  $1 - A_N \leq 1 - B_N$ , where  $1 - B_N$  is a fuz. neu. op. set in  $(M, r_N)$ . Then by Pro. (3.8),  $1 - A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ .

**Pro. 3.10:**

If  $A_N$  is a fuz. neu. op. set in a fuz. neu. Baire sp.  $(M, r_N)$ , then  $A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ .

**Proof:**

Let  $A_N$  be a fuz. neu. op. set in  $(M, r_N)$ . Then for a fuz. neu. op. set  $B_N$  in  $(M, r_N)$ ,  $A_N \wedge B_N$  is a fuz. neu. op. set in  $(M, r_N)$ . Since  $(M, r_N)$  is a fuz. neu. Baire sp. by definition (3.4),  $A_N \wedge B_N$  is not a fuz. neu. one caty. set in  $(M, r_N)$  and hence  $A_N \wedge B_N$  is a fuz. neu. two caty. set in  $(M, r_N)$ . Thus  $A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ .

**Pro. 3.11:**

If  $A_N$  is a fuz. neu.  $G_\delta$  - set in a fuz. neu. Baire and fuz. neu.  $P$  - sp.  $(M, r_N)$ , then  $A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ .

**Proof:**

Let  $A_N$  be a fuz. neu.  $G_\delta$  - set in  $(M, r_N)$ . Since  $(M, r_N)$  is a fuz. neu.  $P$  - sp., the fuz. neu.  $G_\delta$  - set  $A_N$  is a fuz. neu. op. set in  $(M, r_N)$ . Since

$(M, r_N)$  is a fuz. neu. Baire sp., by Pro. (3.10),  $A_N$  is a fuz. neu. Baire den. set in  $(M, r_N)$ .

### Shortcut keywords:

S.No	KEYWORDS	SHORTCUTS
1	fuzzy	fuz.
2	neutrosophic	neu.
3	category	caty.
4	such that	s.t
5	closed set	cld. set
6	open set	op. set
7	Nowhere dense sets	Nowh.den.sets
8	implies that	$\Rightarrow$
9	Proposition	Pro.
10	Submaximal space	Submax. sp.
11	$(X, r_n)$	$(M, r_n)$
12	topological space	top.sp.
13	implies that	$\Rightarrow$
14	dense sets	den. sets
15	Example	Eg;
16	residual	resd.
17	universe	U of X
18	characterizations	charact.
19	relations	connection.
20	introduced	initate
21	Lofti A Zadeh	Zadeh. L. A

**Conclusion:**

In this paper, the concept of fuz. neu. den. sets in fuz. neu. top. sp.s and its properties are discussed. Also, we obtained several charact. of fuz. neu. den. sets in fuz. neu. Baire sp.s. Connection. between fuz. neu. den. sets and fuz. neu. Baire sp., fuz. neu. den. Baire sp.s are studied. Some of its charact. and examples of fuz. neu. Baire sp.s are established. This shall be extended in the future research studies.

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