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A NOVEL METHOD FOR THE FUZZY ASSIGNMENT PROBLEM USING SBD TECHNIQUE

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Abstract

This paper develops a novel strategy to Fuzzy Assignment Problems (FAP) based on arithmetic progression and ranking methods for fuzzy triangular numbers and fuzzy trapezoidal numbers. Additionally, in this work, the FAP is defuzzified into crusty values using the Statistical Beta Distribution approach, and an ideal solution is found using the Hungarian method. The suggested strategy is demonstrated with the use of a numerical example.

Keywords : Fuzzy Assignment Problem, Trapezoidal Fuzzy Number, Trapezoidal FAP, Defuzzification.

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1. Introduction

In terms of optimization problems a Linear Programming delinquent can be thought of as a specific circumstance of an assignment problem. The allocation problem's core objective is to evenly distribute resources *r (*usually workers) to tasks t *(*usually jobs) on an individual basis taking this into account that the total assignment costs and time is minimized. It should be noted that all work must be carried out by exactly one worker and that every worker must be assigned to a single job.

Over the last 50 years, many algorithms have been proposed [2]-[4], [9], [11]-[13] to solve allocation problems. In recent times fuzzy allocation problems have received more importance in research field.

In 1980, Yager [17] applied centroid-index for ranking fuzzy numbers. In the very next year, he headed another article on the integral of level sets for ordering fuzzy subsets. Heilpern proposed the idea of ordering fuzzy numbers in 1992 utilising the ideas of expected interval and expected value [10].

Using the centroid-index ranking method, Cheng [5] proposed a new method for determining a fuzzy number's centroid-point using centroid index ranking method. In [8] the author suggested two fresh approaches to address the transportation problems based on interval data. Yao and Wu designed a ranking arrangement for fuzzy numbers using a decomposition opinion and crisp ranking structure on R [18].

Moreover, S.J. Chen and S.M. Chen came up with a simpler and more efficient way of sorting trapezoidal fuzzy numbers that relies on the centre of gravity. [6]. Later in 2005, Yong and Qi [19] arrived on a new concept of utilizing the centroid-index ranking system and employed TOPSIS to order the trapezoidal fuzzy numbers. In [1] the notion of ranking fuzzy numbers using distance minimization has been presented by Asady and Zendehnam. Also Chen and Wang [7] used α-cuts for ordering the fuzzy numbers. In 2010 Fuzzy ranking system has been employed by Mukherjee, S., and Basu, K [14], for solving Fuzzy Cost assignment problems. A. Rahmaniet. al. presented an innovative method for ranking fuzzy numbers and defuzzifying them using the statistical beta distribution [15]. In their explanation of the Row Minimum Allocation Method of Assignment Problems under Crisp and Triangular Fuzzy Numbers, Venkatachalapathy, M., et al. [16] provided the Optimum Job Allocation for Design of Machines where, the defuzzification of a fuzzy assignment model using a significant beta distribution is analysed, and also Geethalakshmi et. al., [21][22] discussed the fuzzy based support system in real-world applications .

2. Preliminaries

2.1 Fuzzy Set

For a crisp set C⊆ X the characteristic function μ_C gives each member of X a value of 0 or 1. For the Fuzzy subset $\tilde{C} \subseteq$ X, the characteristic function $\mu_{\tilde{C}}$ is defined so that the value given to the universal set X falls inside the desired range i.e. $\mu_{\tilde{C}}$: X → [0, 1], where the assigned value indicates the membership grade of the element in the crisp set A. $\mu_{\tilde{C}}(x)$is called the membership function and the $\tilde{C}$ = {(x, $\mu_{\tilde{C}}(x)$) : x ∈ X} is called a fuzzy set.

2.2 Fuzzy number

Definition 1: [see 15] A fuzzy set in the form of $\tilde{C}$: R [0, 1] that meets the requirements given below is stated to as a fuzzy number:

(1) $\tilde{c}$ is upper semicontinuous,

(2)Beyond the range [l, u]$\tilde{c}$ (x) is zero,

(3) Real numbers m_1, m_2 exist such that $l \le m_1 \le m_2 \le u$

And moreover

(4) $\tilde{c}(x)$is increasing on [l, m_1]

(5) $\tilde{c}(x)$is decreasing on [m_2, u]

(6) $\tilde{c}(x)=1$ on if $m_1 \le x \le m_2$

(i) The fuzzy number $\tilde{c}$ (x) = (l, m, u) is known as the triangular fuzzy number and is defined accordingly, if $m_1 = m_2 = m$.:

$$\tilde{c}(x) = \begin{cases} \frac{x-l}{m-l}, & l \le x \le m \\ \frac{u-x}{u-m}, & m \le x \le u \\ 0, & otherwise \end{cases} \qquad (1)$$

(ii)A fuzzy number is called a trapezoidal fuzzy number $\tilde{c} = (l, m_1, m_2, u)$if

$$\tilde{c}(x) = \begin{cases} \frac{x-l}{m_1-l}, & l \le x \le m_1 \\ 1, & m_1 \le x \le m_2 \\ \frac{m_2-x}{u-m_2}, & m_2 \le x \le u \\ 0, & otherwise \end{cases} \qquad (2)$$

2.3 Fuzzification of an interval number using Arithmetic Progression

Consider an interval number [l, u]. Then by the definition of arithmetic progression, the mandatory triangular fuzzy number can be taken as

$$(l, m, u) \qquad (3)$$

Where ‘ m’ is given by$\frac{l+u}{2}$.

Consider an interval number [l, u]. Then by the definition of arithmetic progression, the required trapezoidal fuzzy number can be taken as

$(l, l+d, l+2d, u)$ (3a)

Where ' d' gives the $\frac{1}{3}^{rd}$ length of the interval.

2.4 Defuzzification using Beta distribution

Consider the triangular fuzzy number $\tilde{c}$ = (l, m, u). The crisp real number $\mu\tilde{c}$ conforming to the triangular fuzzy number $\tilde{c}$ = (l, m, u) using Beta distribution is attained from the following relation:

$$\mu\tilde{c} = \frac{l+m+u}{3}. \quad (4)$$

Consider the trapezoidal fuzzy number $\tilde{c}$ = (l, m_1, m_2, u). The Beta distribution and uniform distribution are combined to find the ranking of the generalized trapezoidal fuzzy number which maps the set of all fuzzy number to a real number and is defined by

$$\mu\tilde{c} = \frac{2l+7m_1+7m_2+2u}{18} \quad (5)$$

which gives the defuzzified number of the corresponding trapezoidal fuzzy number[10].

2.5 The Fuzzy Number Ranking Algorithm.

In this section the ranking of generalized fuzzy numbers is done as follows:

Consider the fuzzy numbers ã = ($l_{ã}, m_{1ã}, m_{2ã}, u_{ã}$) and $\tilde{b}$= ($l_{\tilde{b}}, m_{1\tilde{b}}, m_{2\tilde{b}}, u_{\tilde{b}}$).

Following factors are used to determine the ranking of the numbers::

Corresponding to $\tilde{a}$ and $\tilde{b}$ find the crisp real numbers $\mu\tilde{a}$ and $\tilde{\mu}b$.

(i) If $\mu\tilde{a}$ is greater than $\mu\tilde{b}$, then $\tilde{a}$ is greater than $\tilde{b}$.

(ii) If $\mu\tilde{a}$ is equal to $\mu\tilde{b}$ and $u\tilde{a}$ is greater than $u$then $\tilde{a}$greater than $\tilde{b}$.

(iii) If $\mu\tilde{a}$ is equal to$\mu\tilde{b}$, $u\tilde{a}$ equal to$u\tilde{b}$and $m\tilde{a}$ is greater than $m\tilde{b}$ then $\tilde{a}$is greater than $\tilde{b}$.

(iv) If $\mu\tilde{a}$ is equal to$\mu\tilde{b}$, $u\tilde{a}$ is equal to$u\tilde{b}$ and $m\tilde{a}$ is equal to$m\tilde{b}$ then $\tilde{a}$is equal to$\tilde{b}$.

Again, for an arbitrary finite subset S of E

1) $\tilde{a}$∈S ,$\tilde{a}$is greater than $\tilde{a}$.

2) $(\tilde{a}, \tilde{b}) \in S^2$, $\tilde{a}$is greater than $\tilde{b}$, and $\tilde{b}$is greater than $\tilde{a}$ then $\tilde{a}$is equal to $\tilde{a}$
3) $(\tilde{a}, \tilde{b}, \tilde{c}) \in S^3$,$\tilde{a}$is greater than $\tilde{b}$, and $\tilde{b}$ is greater than $\tilde{c}$ then $\tilde{a}$is greater than$\tilde{c}$.
4) $(\tilde{a}, \tilde{b}) \in S^2$if inf sup($\tilde{a}$) is greater than sup sup($\tilde{b}$) then $\tilde{a}$ *is greater than are equal to* $\tilde{b}$.
5) Let S and S' be two arbitrary finite subsets of E in which $\tilde{a}$ and $\tilde{b}$ are in $S \cap S$'. The ranking order $\tilde{a}$is greater than$\tilde{b}$is obtained by Beta distribution method on S' if and only if $\tilde{a}$*is greater than are equal to* $\tilde{b}$by Beta distribution method on S.
6) Let $\tilde{a}, \tilde{b}$,$\tilde{a} + \tilde{c}$ and $\tilde{b} + \tilde{c}$ be elements of E. If $\tilde{a}$*is greater than are equal to* $\tilde{b}$ then $\tilde{a} + \tilde{c}$*is greater than are equal to* $\tilde{b} + \tilde{c}$.

3. Fuzzy Assignment Problem

Let's say there are n activities that need to be executed and n people are available to carry out them. Suppose if every person is eligible to do all the activites in a specific time with unreliable grade of efficiency. Let Cij signify the cost interval if the ith individual is tasked with the jth activity, then the problem is to find a minimum fuzzy cost with fuzzy assignment. The Generalized form of FPA is given as follows:

Job / Person	J_1	J_2	J_3	….	J_j	….	J_n
P_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\tilde{c}_{13}$		$\tilde{c}_{1j}$		$\tilde{c}_{1n}$
P_2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\tilde{c}_{23}$		$\tilde{c}_{2j}$		$\tilde{c}_{2n}$
P_3	$\tilde{c}_{31}$	$\tilde{c}_{32}$	$\tilde{c}_{33}$		$\tilde{c}_{3j}$		$\tilde{c}_{3n}$
…							
P_i	$\tilde{c}_{i1}$	$\tilde{c}_{i2}$	$\tilde{c}_{i3}$		$\tilde{c}_{ij}$		$\tilde{c}_{in}$
…							
P_n	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	$\tilde{c}_{n3}$		$\tilde{c}_{nj}$		$\tilde{c}_{nn}$

3.1 Algorithm to solve Fuzzy Assignment Problem using classical approach

The algorithm for finding the optimal solution using Hungarian algorithm has been proposed by Kuhn in 1955. The same approach is being employed after converting the fuzzy cost to the corresponding crisp cost.

The steps to find the solution for a FPA:

Step 1: Ensure the FPA is balanced

Step2: If it is an unbalanced FPA make it as a balanced one by adding dummy row and column

Step 3: Express the interval cost as a trapezoidal fuzzy number using equation (3)

Step 4: Defuzzify the fuzzy assignment cost using equation (4/5)

Step5: In each row subtract every elements of the row by the smallest cost present in it.

Step 6: From the reduced matrix subtract each element in a column by its minimal cost .

Step 7: Assign a zero in each row and cancel all other zero's found in the corresponding row or column, in such a way that every row has a zero and every column has a zero. This gives the optimal solution.

Step 8: If the count of assigned zero is not equal to the number of rows or columns the current solution is not optimal, then apply the classical Hungarian method to find the solution

(i) Tick the row which has no assignment

(ii) Tick the column which has zero in the ticked row

(iii) Tick the row which has zero assignment in the corresponding ticked column

(iv) Draw lines through ticked row and unticked column until all the zeroes are covered by minimum number of lines

(v) Subtract all the left over elements with the lowest value among them. Add the value at the joint of the lines

Step 9: Go to step 7. Repeat the process until an optimal solution is obtained.

4. Numerical example

Using Triangular fuzzy number

Let us solve the following assignment problem with upper and lower limit cost matrix, using beta distribution approach to identify the best option and to allocate the four jobs to four different person:

$$\begin{bmatrix} (1,4) & (1,7) & (8,14) & (5,8) \\ (0,3) & (-1,2) & (5,8) & (2,5) \\ (3,9) & (5,11) & (12,18) & (6,12) \\ (5,11) & (1,10) & (1,7) & (1,4) \end{bmatrix}$$

Solution:

The given problem is a balanced assignment problem. Now reduce the matrix to a triangular FPA by converting each cost to a triangular fuzzy number using (3) (i.e) $(l, \frac{l+u}{2}, \underline{u})$

$$\begin{bmatrix} (1,2.5,4) & (1,4,7) & (8,11,14) & (5,6.5,8) \\ (0,1.5,3) & (-1,0.5,2) & (5,6.5,8) & (2,3.5,1.5) \\ (3,6,9) & (5,8,11) & (12,15,18) & (6,9,12) \\ (5,8,11) & (1,5.5,10) & (1,4,7) & (1,2.5,4) \end{bmatrix}$$

Defuzzify the FPA using (6),

$$\begin{bmatrix} 2.5 & 4 & 11 & 6.5 \\ 1.5 & 0.5 & 6.5 & 2.33 \\ 6 & 8 & 15 & 9 \\ 8 & 5.5 & 4 & 2.5 \end{bmatrix}$$

The Reduced matrix after row and Column reduction is

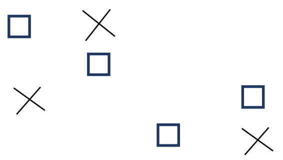
$$\begin{bmatrix} 0 & 1.5 & 7 & 4 \\ 1 & 0 & 4.5 & 1.83 \\ 0 & 2.0 & 7.5 & 3.0 \\ 5.5 & 3 & 0 & 0 \end{bmatrix}$$

Proceeding by Hungarian algorithm for optimality we have

$$\begin{bmatrix} 0 & 1.5 & 7 & 4 \\ 1 & 0 & 4.5 & 1.83 \\ 0 & 2.0 & 7.5 & 3.0 \\ 5.5 & 3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 5.5 & 2.5 \\ 2.5 & 0 & 4.5 & 1.83 \\ 0 & 0.5 & 6 & 1.5 \\ 7 & 3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 4 & 1 \\ 2.5 & 0 & 3 & 0.33 \\ 0 & 0.5 & 4.5 & 0 \\ 8.5 & 4.5 & 0 & 0 \end{bmatrix}$$



Thus the optimal allocation is

A ---- I; B ----- II; C----- IV; D-----III

The fuzzy optimal cost is

Opt z = (1,2.5,4)+ (−1,0.5,2) +(6,9,12)+(1,4,7) = (7,16,25)

Using the Beta distribution technique we have optimum z = $\frac{7+16+25}{3}$ =16

Using Trapezoidal fuzzy number

Let us solve the following assignment problem with upper and lower limit cost matrix, using beta distribution approach to identify the best option and to allocate the four jobs to four different person:

$$\begin{bmatrix} (1,4) & (1,7) & (8,14) & (5,8) \\ (0,3) & (-1,2) & (5,8) & (2,5) \\ (3,9) & (5,11) & (12,18) & (6,12) \\ (5,11) & (1,10) & (1,7) & (1,4) \end{bmatrix}$$

Solution:

The given problem is a balanced assignment problem. Now reduce the matrix to a trapezoidal FPA by converting each cost to a trapezoidal fuzzy number using (2) (i.e) (*l*, *l*,+d, *l*,+ 2d,U)

$$\begin{bmatrix} (1,2,3,4) & (1,3,5,6) & (8,10,12,14) & (5,6,7,8) \\ (0,1,2,3) & (-1,0,1,2) & (5,6,7,8) & (2,3,4,5) \\ (3,5,7,9) & (5,7,9,11) & (12,14,16,18) & (6,8,10,12) \\ (5,7,9,11) & (1,4,7,10) & (1,3,5,7) & (1,2,3,4) \end{bmatrix}$$

Defuzzify the FPA using (6),

$$\begin{bmatrix} 2.5 & 3.9 & 11 & 6.5 \\ 1.5 & 0.50 & 6.5 & 3.5 \\ 6 & 8 & 15 & 9 \\ 8 & 5.5 & 4 & 2.5 \end{bmatrix}$$

The Reduced matrix after row and Column reduction is

$$\begin{bmatrix} 0 & 1.4 & 7 & 4 \\ 1 & 0 & 4.5 & 3 \\ 0 & 2 & 7.5 & 3 \\ 5.5 & 3 & 0 & 0 \end{bmatrix}$$

Proceeding by Hungarian algorithm for optimality we have

$$\begin{bmatrix} 0 & 1.4 & 7 & 4 \\ 1 & 0 & 4.5 & 3 \\ 0 & 2 & 7.5 & 3 \\ 5.5 & 3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 5.6 & 2.6 \\ 2.4 & 0 & 4.5 & 3 \\ 0 & 0.6 & 6.1 & 1.6 \\ 6.9 & 3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 4 & 1 \\ 2.4 & 0 & 2.9 & 1.4 \\ 0 & 0.6 & 4.5 & 0 \\ 8.5 & 4.6 & 0 & 0 \end{bmatrix}$$

Thus the optimal allocation are

A ---- I; B ----- II; C----- IV; D-----III

The fuzzy optimal cost is

Opt z = (1,2,3,4)+ (-1,0,1,2) +(6,8,10,12)+(1,3,5,7) = (7,13,19,25)

Using the Beta distribution technique we have optimum z $=\frac{2(7)+7(13)+7(19)+2(25)}{18}$ =16

Optimum Allocation Entries using existing Row Minimum Allocation Method [20]

$$\begin{bmatrix} (\mathbf{1,2,3,4}) & (1,3,5,6) & (8,10,12,14) & (5,6,7,8) \\ (0,1,2,3) & (\mathbf{-1,0,1,2}) & (5,6,7,8) & (2,3,4,5) \\ (3,5,7,9) & (5,7,9,11) & (12,14,16,18) & (\mathbf{6,8,10,12}) \\ (5,7,9,11) & (1,4,7,10) & (\mathbf{1,3,5,7}) & (1,2,3,4) \end{bmatrix}$$

Thus the optimal allocation are

A ---- I; B ----- II; C----- IV; D-----III

The fuzzy optimal cost is

Opt z = (1,2,3,4)+ (-1,0,1,2) +(6,8,10,12)+(1,3,5,7)

=(7,13,19,25)

Using the Beta distribution technique, we have Opt z = $\frac{8+7(13)+7(19)+25}{18}$ =14.22

5. Conclusion

On comparing with the existing Row Minimum Allocation Method studied in [20] this method is found to give a better result for FPAs

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