Section A-Research paper

Modeling of Student's Performance Evaluation using Fuzzy Clustering Techniques

Gopal Singh[1], Dr. Praveen Kumar[2]

[1]Research Scholar, Department of Mathematics, JV Jain College, Chaudhary Charan Singh University, Meerut, India
[2]Associate Professor, Department of Mathematics , JV Jain College, Chaudhary Charan Singh University, Meerut, India

Abstract: Fuzzy clustering techniques are used to solve data clustering problems by using partitioning approaches. K-Means and C-Means Clustering are two novel techniques, which are to be used for determining student's performance evaluation. The Bayesian method is also introduced for the same. For obtaining, a function of distance between marks and clusters, developed fuzzy rules from fuzzy membership values. These Clusters will regulate the system of fuzzy inference input and output space of the division of fuzzy inference system and also regulate the relative member of functions. This paper applies a new kind of approach using fuzzy clustering and the research paper tests the performance of these techniques and finds the better result for the student's better performance.

Keywords: Fuzzy logic, Clustering, Fuzzy- C means, Fuzzy K- Means

1. Introduction

Student's performance evaluations induce for better performance in institutions and help to make a better educational society. In India there are many subjects relate to educational detriment and needs to improvement in the concern matter. The diversity in the student community is comprehensive and many of them are related to evaluation process. In the identification of clustering system, Takagi- Sugani Model (T-K Model) developed By Takagi and Sugeno in 1985 [1]. Fuzzy clustering can be freely grasp algorithm which does not required any particular experience or prior knowledge but distinguished and classify the certain rules [2]. Fuzzy clustering most commonly based on the objective function which has advantage of simple algorithm and having fast convergence rate. Fuzzy clustering can handle large data set and theory of fuzzy c-means is better implemented and applied extensively in the given data sets. In the clustering analysis, data divided into groups where similar data objects to be held by the same cluster and non similar data objects to be held by different cluster [3].

2. Uses of Fuzzy Expert system

Modeling of students' performance evaluation using fuzzy logic techniques as in our previous publication [4] comprised the results in the following steps and methods [5].

Three steps are developed for evaluating student's performance evaluation.

a. The fuzzified value for input and output (Student's Performance level) result.
b. The inference techniques and applications.
c. The fuzzified outcome level.

3. Methodology:

3.1 Architecture of K- means cluster:

Iterative algorithm k-means clustering method comprise movable cluster starting with one then onto the next until the ideal set is obtain by classified information from a crisp perspective.

In a family set $\{B_i, i = 1, 2, 3, \ldots, c\}$ with a division of Y.

For the partition of Y, the set-theoretic forms is given as:

$\bigcup_{i=1}^{c} B_i$

$B_i \cap B_j = \emptyset, \forall\, i \neq j$

$\emptyset \subset B_i \subset Y, \forall\, i$

Where $Y=\{y_1, y_2, y_3, \ldots, y_n\}$ is a definite set which is universe of samples

It is defined as $2 \leq c \leq n$ here c is the number of cluster

Where $C = n$ classes, which only assign each data sample to a specific class.

The Classification criteria $K(U, v)$ is defined as:

$$K(U, v) = \sum_{j-1}^{n} \sum_{i=1}^{C} Y_{ij} (d_{ij})^2$$

Where U represent division matrix, v is vector of clusters centre, d_{ij} is the Eclidean distance between i^{th} cluster centre v_i and j^{th} data sample y_j .

$$d_{ij} = d(y_j - v_i) = \|y_j - v_i\| = \left[\sum_{k=1}^{m} (y_{jk} - v_{ik})\right]^{\frac{1}{2}}$$

The algorithm is explained below:

Step 1: start with basic prototype configuration $v_i, i = 1, 2, 3, \ldots, C$ (for instant randomly picked)

Step 2: To determine the value for d_{jj} or the distance that exist for y_j (a data set) to the centre C_i of the i^{th} class.

Step 3: create a factor matrix by giving numeric values to *U* according to the given formula:

$$Y_{ij} = \begin{cases} 1, & if\, d(y_j, v_i) = min_{k \neq i} d(y_j, v_i) \\ 0, & otherewise \end{cases}$$

Step 4: The prototype is now being updated by computing the weighted average, which contains the partition matrix's entries:

$$v_i = \frac{\sum_{j=1}^{n} Y_{ij} y_j}{\sum_{j=1}^{n} Y_{ij}}$$

Continue this process until convergence criteria is met.

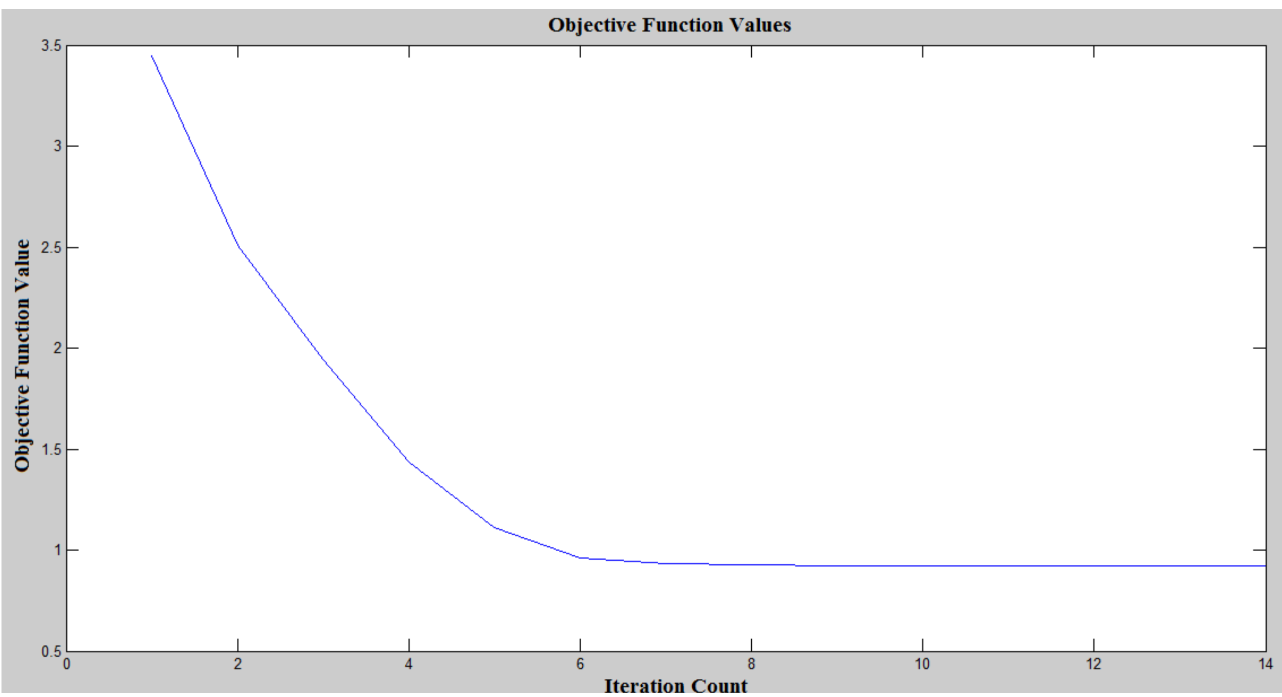

Fig. 1: Objective Function Values of the K-Means Method

Whenever the cluster centers have been identified, we must distribute the evaluation data vectors to the applicable clusters based on the distance between each vector and each cluster center. For the purpose of to calculate an error measure, the root mean square error (RMSE) is also used.

3.1.1 RESULT OF FUZZY K - MEANS

The results of this method and the values of the objective function are shown in Fig. 1. Here we have observed that three students are in the "Excellent" category clusters, no one student is in the "Good" category clusters, three students are in the "average" category clusters and four students are in the "low" category clusters shown in Table 1. The disadvantage of K-Means clustering methods is that it is unable to determine a student's overall marks and fuzzy membership value. The FCM might be able to resolve this issue. Using Excel, the calculation is completed.

$S.No.$	f_1	f_2	f_3	f_4	m_1 (0 0 0 30) *D*	m_2 (0 0 30 55 *C*	m_3 (0 55 75 80) *B*	m_4 (75 80 90 100) *A*	Grade based on K- Means
1	21.8	42.3	17.6	38	9.47	9.67	11.57	15.01	*D*
2	26.1	49.7	28.7	49.7	11.14	9.08	10.39	13.81	*C*
3	40.2	47.2	25	40.4	11.08	10.34	11.73	13.86	*C*
4	15	19.3	13.9	34.3	7.25	8.43	12.55	16.20	*D*
5	43.3	47.2	10.2	46.6	10.83	10.89	12.22	14.06	*D*
6	19.9	13.9	28.1	18.8	8.55	8.48	13.00	16.26	*C*
7	13.2	22.5	22.5	34.9	7.94	7.96	11.97	15.87	*D*
8	88	87.3	81.8	88.6	17.77	16.15	11.65	6.32	*A*
9	69	79.9	90.4	88.6	17.26	15.59	10.86	4.23	*A*
10	78.2	81.2	84.3	84.9	17.28	15.61	10.89	5.02	*A*

Table 1: Evaluation of Student's Academic Performance Results Using K-Means

3.2 Takagi- Sugeno (TS) fuzzy Model:

Takagi- Sugeno (TS) fuzzy model is one of the most fuzzy rule base type models which are used in problem solving domain for local linearity and identifying a non linear system through fuzzy reasoning. [7]

In general, a Takagi- Sugeno fuzzy model (TS) is according to rule base as of the form:

$R_i:\ if\ x_1$ is A_{i1}, x_2 is $A_{i2}, \dots, x_m$ is A_{im} (3.2.1)

$Then\ y_i : p_{i0} + p_{i1} + p_{i2} + \cdots + p_{im}$

$: p_{i0} + \sum_{j=1}^{m} p_{ij}\ \ ; j = 1,2,\dots,m$ (3.2.2)

Where R_i is the i^{th} fuzzy rule, $i = 1,2,\dots,c; c$ is the number of fuzzy rules; A_{ij} is $i^{th}$the fuzzy subset of the j^{th} input variable x_j; x is the input variable, $x = [1, x_1, x_2, \dots, x_m]^T$; y_i is the output variable of i^{th} fuzzy rule; p_{il} are the consequent parameters, $l = 0,1,2,\dots,m$.

The weighted average of the individual rules in the Takagi- Sugeno (TS) fuzzy model is as listed below:

$y = \sum_{i=1}^{c} w_i y_i = \sum_{i=1}^{c} w_i x^T \vartheta_i$ (3.2.3)

Here, $w_i = \frac{\sigma_i}{\sum_{j=1}^{c} \sigma_i}$ is the validity approach of i^{th} fuzzy rule and y_i, the output variable of i^{th} fuzzy rule sub model; $\sigma_i = [p_{i0}, p_{i1}, p_{i2}, \dots, p_{im}]^T; i = 1,2,\dots,c.$

Here, $\sigma_i = \mu_{i1}x_1 \times \mu_{i2}x_2 \times \dots \times \mu_{im}x_m$

$= \prod_{j=1}^{m} \mu_{ij} x_j$ (3.2.4)

This derived equation determines the contribution of i^{th} fuzzy rule to Takagi- Sugeno (TS) fuzzy model.

Triangular, trapezoidal and bell shaped membership functions can be implemented to the fuzzy set. In this research paper fuzzy set A_{ik} is used in the bell shaped.

$\mu_{ij}(x_j) = exp\left\{-\left(x_j - c_{ij}\right)^2 / b_{ij}^2\right\}$ (3.2.5)

Here the b_{ij} and c_{ij} are used as parameters for Gaussian membership function.

3.3 Determination of centre of Gaussian membership function:

An objective function for The Fuzzy C- Means is determined as:

$J_r(U, z) = \sum_{j=1}^{m} \sum_{i=1}^{c} (\mu_{ij})^r \|x_j - z_i\|^2$ (3.3.1)

Where, partition matrices U for the set of data X containing the membership of each vector for each cluster. The centre of clusters Z as $z = \{z_1, z_2, \dots, z_c\}, z_i \epsilon R^m$ according to the values of fuzzy rules; $\mu_{ij} \epsilon [0,1]$ is the membership degree of the j^{th} data pair pertaining to the i^{th} fuzzy subset. Let the equations consider μ_{ij} as follows:

$\sum_{i=1}^{c} \mu_{ij} = 1, 1 \le j \le n, \mu_{ij} \ge 0, 1 \le i \le c$ (3.3.2)

Here, the cluster centre number C and the input variables of dimensions n establish C- Means vectors. Now let we choose, the membership function for weighted index ($r > 1$) if the values of r is very least then we choose the membership values for input variable around to 1.Generally, we choose $r = 2$.

The clustering centre can be calculated using the following formula:

$$z_i = \frac{\sum_{j=1}^{m} (\mu_{ij})^r x_j}{\sum_{j=1}^{m} (\mu_{ij})^r}, \forall i \qquad (3.3.3)$$

Now, we can find the U as fuzzy membership function matrix as per the given formula::

$$\mu_{ij} = \left[\sum_{k=1}^{c} \left(\frac{d_{ij}}{d_{kj}} \right)^{\frac{2}{r-1}} \right]^{-1} \qquad (3.3.4)$$

$$d_{ij} = \| x_j - z_i \| > 0, \forall i, j$$

$if\ d_{ij} = 0$, then μ_{ij}=1, $\mu_{kj} = 0$, for all $k \neq i$ (3.3.5)

The main steps of Fuzzy C- means algorithm are given as follows:

Step 1: Set c such that $2 \leq c \leq m$, choose r and initialize the matrix U.

Step 2: Calculate the centre $\{ z_i^{(s)} \}$ for each c sets, $s = 0,1,2 \ldots$ iterations.

The centre of each cluster is calculated using the expression: $z_i^{(s)} = \frac{\sum_{j=1}^{m} (\mu_{ij})^{r^{(s)}} x_j}{\sum_{j=1}^{m} (\mu_{ij})^{r^{(s)}}}, \forall i$

Step 3: Update partition matrix $U^{(s)}$ for the iteration s by the equation (a) and (b)

$$\mu_{ij}^{(s+1)} = \left[\sum_{k=1}^{c} \left(\frac{(d_{ij})^{(s)}}{(d_{kj})^{(s)}} \right)^{\frac{2}{r^{(s)}-1}} \right]^{-1}, \text{ for } I_j = \emptyset$$

$$\mu_{ij}^{(s+1)} = 0\ \forall i$$

$$I_j = \{ i : 2 \leq c \leq m;\ (d_{ij})^{(s)} = 0$$

$$I_j' = \{1,2, \ldots, c\} - I_j \sum_{i \in I_j} \mu_{ij}^{(s+1)} = 1$$

Step 4: If $\| U^{(s+1)} - U^{(s)} \| \leq \epsilon$ stops, if not $s = s + 1$ (next iteration) and return to step 2.

By previous steps we can find the centre of Gaussian function or the clustering centre.

Step 5: Evaluate the resulting parameter according to orthogonal least squares (OLS) approach. Invert equation (3.14) to a vector form:

$$y = \varphi \theta$$

Here, $\varphi = [w_1, w_2, w_3, \ldots, w_m, w_1 x_1, \ldots, w_1 x_r]$

$$\theta = [p_{10}, p_{12}, \ldots, p_{1r}, p_{20}, p_{21}, \ldots, p_{2r}, \ldots, p_{m0}, p_{m2}, \ldots, p_{mr}]^T,$$

In regards the least square solutions:

$$\theta = (\varphi^T \varphi)^{-1} \varphi^T y$$

Now, by executing the iteration and conversion techniques, the equations Transform $[\varphi^T \varphi]$ into an orthogonal matrix$[K^T K]$ the the $[r+1] * m$ equations become mutually independent to calculate the resulting parameter θ.

3.4 Fuzzy C- means (FCM) cluster design

Fuzzy C- Means clustering (FCM) algorithm of data [6] , for which each data converted to a degree which is defined by membership function. In this process we will consider data is stored X and a form of matrix of $n \times m$ order, where n is the number of samples and m is the number of attributes.

Data clustering using fuzzy C- means algorithm follow the following steps:

1. find out the number of clusters (c), rank (τ), smallest error excepted ε, maximum iteration $(MaxIter)$, the primary objective function$(P_0 = 0)$ and iteration $(t = 1)$.
2. let an element of the matrix primary partition V; $i = 1,2, \dots, n$ and a random variable μ_{ir} ; $i = 1,2, \dots, n$, $r = 1,2, \dots, c$

   where,

$$R_i = \sum_{r=1}^{n} (\mu_{ir})^{\tau}$$

$$\mu_{ir} = \frac{\mu_{ir}}{R_i}$$

3. The cluster centers, U_{rj} ; $r = 1,2, \dots, c$ and $j = 1,2, \dots, m$ is calculated as

$$U_{rj} = \frac{\sum_{i=1}^{n} (\mu_{ir})^{\tau} \times X_{ij}}{\sum_{i=1}^{n} (\mu_{ir})^{\tau}}$$ is the cluster centre of U_{rj}

4. Calculate the objective function at iteration t

$$P_t = \sum_{i=1}^{n} \sum_{r=1}^{c} \sum_{j=1}^{m} (X_{ij} - U_{rj})^2 (\mu_{ir})^{\tau}$$

5. Calculate the partition matrix transition

$$\mu_{ir} = \frac{\left[\sum_{j=1}^{m} (X_{ij} - U_{rj})^2\right]^{-\frac{1}{\tau-1}}}{\sum_{r=1}^{c} \left[\sum_{j=1}^{m} (X_{ij} - U_{rj})^2\right]^{-\frac{1}{\tau-1}}}$$

6. If $|P_t - P_{t-1}| < \epsilon$ or $t > MaxIter$ then stop the process, otherwise replace t by $t+1$ and repeat the step 3.
7. Steps in Fuzzy C- Means algorithm can be illustrated by the following diagram (Figure 2)

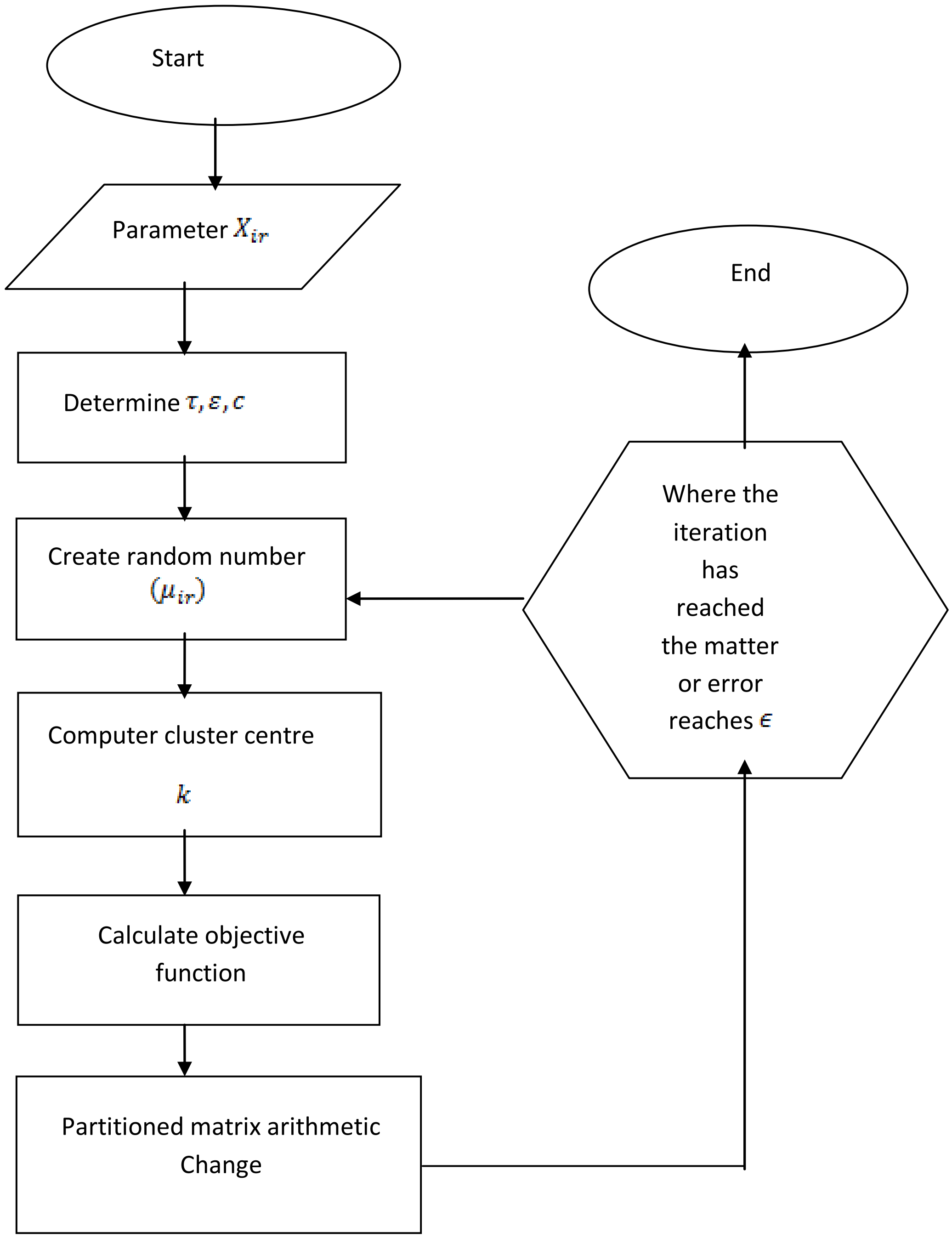

Figure 2: Diagram of Fuzzy $C-$ Means Algorithm

3.4.1 RESULT OF STUDENT'S PERFORMANCE USING FUZZY C-MEANS CLUSTERING

In this Method, the result is on rule –based fuzzy expert system and creates the fuzzy classes for the input parameters. The Fuzzy C- Means Clustering (FCM) algorithm uses to evaluate students performance evaluation.

The algorithms of proposed techniques are as follows:

3.4.1.1 (Fuzzification) : By using EXCEL for Fuzzy C- Means Clustering (FCM) algorithm, the data of the students performance was categorized into four classes (clusters) as namely Excellent(Ex), Good (G), Average (A) and Low (L). These are used for evaluating student's performance evaluation. The generalized membership values are in table 2 as follows.

In the present research, the use of fuzzy C-Means Algorithm (FCM) explained the dataset of students score marks described in Table 3.1. Values of elements of vector are shown in table 3.1 and the marks in the 8^{th} row of Table 3.1 are as follows:

Excellent = 0.844, Good = 0.155, Average = 0.016 Low = 0.004.

Max = (0.844, 0.155, 0.016, 0.004) = 0.844.

S.No.	X_1	X_2	X_3	X_4	Excellent	Good	Average	Low
1	21.8	42.3	17.6	38	0.0093	0.3584	0.5230	0.0975
2	26.1	49.7	28.7	49.7	0.0019	0.1717	0.4238	0.4425
3	40.2	47.2	25	40.4	0.0057	0.2025	0.5813	0.2450
4	15	19.3	13.9	34.3	0.0000	0.0352	0.4825	0.4876
5	43.3	47.2	10.2	46.6	0.0006	0.2265	0.4134	0.4470
6	19.9	13.9	28.1	18.8	0.0086	0.1116	0.0965	0.7500
7	13.2	22.5	22.5	34.9	0.0040	0.0000	0.5110	0.4890
8	88	87.3	81.8	88.6	0.8440	0.1550	0.0162	0.0038
9	69	79.9	90.4	88.6	0.5234	0.4906	0.0057	0.0010
10	78.2	81.2	84.3	84.9	0.3468	0.6475	0.0120	0.0027

Table 2: The Membership Functions Values

From the above four values, 8^{th} student is the most appropriate in class namely as (cluster) Excellent. Since, the student has maximum degree of membership, 10th student is the most appropriate in class namely as (cluster) Good, since the student has maximum degree of membership. Hence, this shows that the 8th student has progress constantly and the 10th student has depreciated consistently.

In the same way, all the remaining class (clusters) will resulted for the student's performance evaluation as follows:

Here,

a. 2 students will be consist by the first class namely as cluster (Excellent).
b. 1 students will be consist by the second class namely as cluster (Good).
c. 3 students will be consist by the third class namely as cluster (Average).
d. 4 students will be consist by the fourth class namely as cluster (Low).

3.4.1.2 Evaluation of Output:

Regression method derived the output values from input values. The output values defined the classes and the input values which are used from data base

For the present work, this method is implemented for categorization. Here input values are the values from database and the output data will defined the classes (clusters). Now, the Regression analyzes a set of data and formalizes the given data. The linear regression formula in two dimensional spaces is shown below:

$$y = ax + b$$

Here *a*, *b* are constant values, which are derived for best fit in linear relationship of input and output data by the normal equations. This method is used to approximate the definite relationship between input and output variable. This defined linear regression model has been utilized to estimate an output value on given input value. The regression analysis method, used to estimate the output values for rule based Fuzzy Expert System in student's Performance evaluation is derived. The EXECL software is used to estimate the output data. Here, the paper shows the output data as clusters (Excellent, Good, Average and Low) is given below:

$$Y(Excellent) = 0.0103 * X - 0.366$$

$$Y(Good) = -0.00652 * X + 0.649$$

$$Y(Average) = 0.00522 * X - 0.0343$$

$$Y(Low) = -0.00869 * X + 0.753$$

3.4.1.3 Rules for generalization: The FCM approach offered more precise and faster convergence and accuracy in the evaluation of performance. The evaluation is based on five rules which are as follows:

Sr. No.	IF	THEN
1	cluster -Very High	Performance-Very High
2	cluster -High	Performance—High
3	cluster -Average	Performance—Average
4	cluster -Low	Performance—Low
5	cluster -Very Low	Performance--Very Low

3.4.1.4 Estimation for Student's performance evaluation – Defuzzification:

The final estimated value for student's performance is evaluated by the formula:

$$Y = \frac{\mu_{Excellent}(x) \times y_1 + \mu_{Good}(x) \times y_2 + \mu_{Average}(x) \times y_3 + \mu_{Low}(x) \times y_4}{\mu_{Excellent}(x) + \mu_{Excellent}(x) + \mu_{Average}(x) + \mu_{Low}(x)}$$

$$Y = \frac{(0.3659 \times 0.0093) + (0.6407 \times 0.3845) + (-0.0316 \times 0.5230) + (0.7522 \times 0.0975)}{0.0093 + 0.3584 + 0.5230 + 0.0975}$$

$$Y = 0.2955$$

Hence, in the similar pattern, the evaluated student's performance is shown in table 3 and the fig. 4 shows objective function for the performance evaluation of the students. This approach shows that the FCM method is more reliable than fuzzy K-means method.

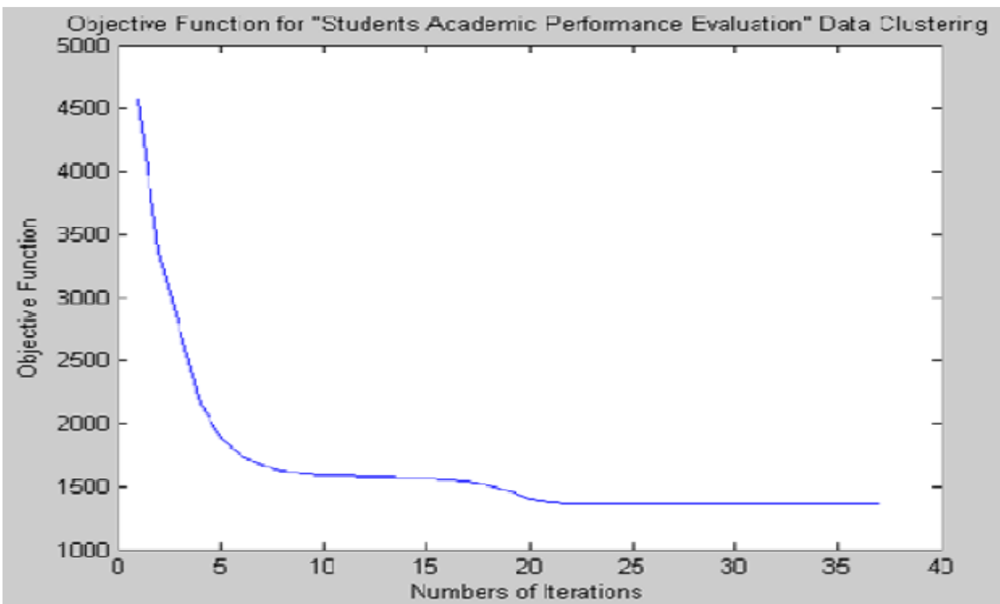

Fig. 4: Performance of Objective Function

S. No.	X_1	X_2	X_3	X_4	**Fuzzy-1**		**Fuzzy-2**		**Grade based on K-means**	**FCM**	
					Output	**Grade**	**Output**	**Grade**	**Grade**	**Output**	**Grade**
1	21.8	42.3	17.6	38	31.9	*C*	31.9	*C*	*D*	29.55	*D*
2	26.1	49.7	28.7	49.7	32	*C*	42.5	*C*	*C*	41.33	*D*
3	40.2	47.2	25	40.4	42.5	*C*	42.5	*C*	*C*	28.91	*D*
4	15	19.3	13.9	34.3	17	*D*	17	*D*	*D*	37.06	*D*
5	43.3	47.2	10.2	46.6	42.5	*C*	42.5	*C*	*D*	43.08	*D*
6	19.9	13.9	28.1	18.8	15.1	*D*	16	*D*	*C*	65.39	*B*
7	13.2	22.5	22.5	34.9	16.4	*D*	16.4	*D*	*D*	35.00	*D*
8	88	87.3	81.8	88.6	75	*B*	75	*B*	*A*	39.68	*D*
9	69	79.9	90.4	88.6	72	*B*	75	*B*	*A*	49.59	*D*
10	78.2	81.2	84.3	84.9	74.6	*B*	74.6	*C*	*A*	53.99	*C*

Table 3: Comprised results of Fuzzy-1, Fuzzy-2, K-Means and FCM

4. CONCLUSION

In this research paper, the main focus is on the development of fuzzy logic system and fuzzy C- Means clustering (FCM). These approaches are used to evaluate student's performance. There are more differences are in the results at the time of assessment in the classical and proposed fuzzy logic expert systems. To deal with this result, this research work will help to evaluate the student's performance.

In this proposed work, the crisp data is formed into fuzzy set by fuzzy expert system and the result is compared according to qualitative manner to contrast the predictive power of clustering algorithm. The Euclidean distance is as obvious in the end result. Some models are improved from basic limited exist results. The FCM approaches are the most excellent approaches in the modeling of the student's performance as compare to classical fuzzy logic methods as shown in the research work.

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