INFLATIONARY LRS BIANCHI TYPE I UNIVERSE WITH FLAT POTENTIAL

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Abstract.
Our study focused on the LRS Bianchi Type-I inflationary universe, where a flat potential was taken into consideration. To get an inflationary solution .We have looked of a region that is flat has constant potential V. to obtain a deterministic model of the universe taking a condition $A = lB^n$. This model's geometrical and physical behaviour is also examined.

Keywords: Inflationary universe, Bianchi Type-I, general relativity

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DOI: 10.48047/ecb/2023.12.si10.00461

1. Introduction

Einstein's theory [1] has attracted interest in recent years due to its success in describing the universe's accelerating expansion. The FRW model describes a universe that is perfectly homogenous and isotropic, is the basis for present cosmology. This is consistent with observable findings regarding the universe's large-scale structure.

The inflationary universe is a theory in modern cosmology that proposes a period of rapid and exponential expansion of the universe in its very early stages. This theory was first proposed by Guth [2] in the 1980s to address within the framework of the standard Big Bang model, issues such as the horizon and flatness problems arise.

According to the inflationary theory, the universe underwent a brief period of expansion driven by a repulsive force generated by a scalar field, known as the inflation. This rapid expansion caused the universe to increase in size by an incredible factor, making it almost homogeneous and isotropic. This expansion also smoothed out any irregularities or fluctuations that existed before, laying the foundation for the large-scale structure of the universe we observe today.

The inflationary universe theory is backed by multiple observations, including those of the cosmic microwave background radiation and the distribution of galaxies on a large scale. It has become a cornerstone of modern cosmology and continues to be an active area of research, with many questions still unanswered [3-5].

The Bianchi type 1 model is often used as a simplification of more complex models, such as (FLRW) model, which assumes isotropy and homogeneity. The Bianchi type 1 model assumes that the universe has three orthogonal axes of symmetry, and that the expansion of the universe is different in each direction. In the Bianchi type 1 model, the universe can expand or contract in each direction independently of the other directions. This leads to the possibility of a universe that is expanding in one direction while contracting in another direction. This is in contrast to the FLRW model, which assumes that the universe expands or contracts uniformly in all directions. The Bianchi type 1 model has found utility in studying the universe's growth, analysing how matter and radiation affect its expansion, and evaluating cosmological hypotheses. Moreover, it has been used to investigate the universe's structure on a large scale. Bali and Jain [6] have explored the Bianchi type I inflationary universe as described by general relativity. Bali and Anjali [7] have explored the Bianchi Type-I cosmological model with bulk viscous fluid, string dust, and magnetization. Bali [8] has explored the inflationary scenario within the framework of the Bianchi Type I model. Bali and Singh [9] have explored the inflationary universe in the LRS Bianchi Type I model with stiff fluid**. Poonia** and Sharma [10] have explored the LRS Bianchi type I space-time model, taking into account the effect of bulk viscosity. Bulk Viscous Bianchi Type-I is study by Parashar [11].

In the present study examines LRS Bianchi type I Universe with the presence of flat potential. The objective is to develop a model of the universe that is deterministic taking a condition $A = lB^n$. This model's geometrical and physical behaviour is also examined.

2. The equations for the metric and fields

The metric of LRS Bianchi Type I take the form

$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2)$ (1)

A and B, which serve as metric coefficients, are functions of the parameter t

$L = \int \sqrt{-g}\ [R - \frac{1}{2}g^{ij}\ \partial_i\phi\ \partial_j\phi - V(\phi)]d^4x$ (2)

Einstein Field Equation is given by

$R_{ij} = \frac{1}{2}Rg_{ij} = -T_{ij}$ (3)

With

$T_{ij} = \partial_i\varphi\partial_j\varphi - \left[\frac{1}{2}\partial_p\varphi\partial^p\varphi + V(\varphi)\right]g_{ij}$ (4)

$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\ \ \partial^\mu\varphi) = -\frac{dV}{d\varphi}$ (5)

For the metric (1), the Einstein field equation (3) is

$\frac{B_4^2}{B^2} + 2\frac{B_{44}}{B} = -\frac{1}{2}\varphi_4^2 + V(\varphi)6)$

$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} = -\frac{1}{2}\varphi_4^2 + V(\varphi)$ (7)

$\frac{B_4^2}{B^2} + 2\frac{A_4B_4}{AB} = -\frac{1}{2}\varphi_4^2 + V(\varphi)$ (8)

And is the scalar field equation

$\varphi_{44} + \left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)\varphi_4 + \frac{dV}{d\varphi} = 0$ (9)

The average scale factor (a) for the LRS Bianchi type I model is given by

$a = (AB^2)^{\frac{1}{3}}$ (10)

$V = a^3 = AB^2$ (11)

Hubble parameter is

$H = \frac{1}{3}\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)$ (12)

As well as being specified as, the θ (expansion scalar), σ (shear scalar), and Am (mean anisotropy parameter) are

$\theta = 3H = \frac{A_4}{A} + 2\frac{B_4}{B}$ (13)

$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H - \frac{1}{3}\theta^2\right)$ (14)

$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$ (15)

3 The field equations yield solutions that

The flat region is taken into consideration because we're interested in inflationary solutions i.e.

$V(\varphi) = constant$ (16)

From equation (9)

$\varphi = \frac{k}{AB^2}$ (17)

Where k is constant

We suppose that the θ (expansion) and σ (shear) in the model are proportional. This condition provides to

$A = \lambda B^n$ (18)

Where λ and n are constant

From equation (6), (7) and (18)

$\frac{B_{44}}{B} + (n+1)\frac{B_4^2}{B^2} = 0$ (19)

Solution of equation (19) is

$B = (k_1 t + k_2)^{\frac{1}{n+2}}$ (20)

From equation (18) and (20)

$A = \lambda(k_1 t + k_2)^{\frac{n}{n+2}}$ (21)

By the using transformation

$k_1 t + k_2 = T$, x=X, y=Y, z=Z

$ds^2 = -\frac{dT^2}{k_1^2} + \lambda^2 T^{\frac{2n}{n+2}} dX^2 + T^{\frac{2}{n+2}}(dY^2 + dZ^2)$

(22)

4. Some physical and geometrical characteristics

The proper volume V Is

$V = \lambda T$ (23)

The expansion rates along the x, y, and z axes are given by

$H_x = \frac{A_4}{A} = \frac{nk_1}{n+2}\frac{1}{T}$ (24)

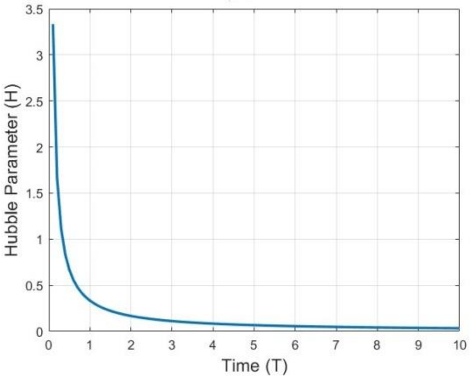
$H_y = H_z = \frac{k_1}{n+2}\frac{1}{T}$ (25)

The average value of the Hubble parameter (H) is

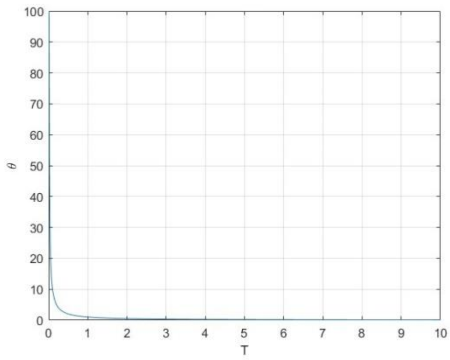
$H = \frac{k_1}{3T}$ (26)

The expansion scalar (θ) is

$\theta = \frac{k_1}{T}$ (27)



Fg 1. The plot of hubble parameter H vs Time



Fg 2. The plot of expansion scalar θ vs Time

Shear scalar (σ) is

$\sigma^2 = \frac{(n-1)^2 k_1^2}{3(n+2)^2}\frac{1}{(T)^2}$ (28)

And

$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(n+2)} = constant$ (29)

Mean anisotropy parameter (Am) is

$A_m = \frac{2(n-1)^2}{(n+2)^2}$ (30)

5. Discussion

LRS Bianchi type I inflationary Universe with the presence of flat potential has been examined in this study. The field equations have been solved exactly to yield a solution. The universe's model has been determined, and its physical behaviour is investigated. When T → ∞ then spatial volume V→ ∞.As time (T) increases, the expansion rate in the model decreases until it stops at T=∞. When T → ∞, then $\sigma \to 0$. In this study, a model is proposed that accelerated, shears, and begins expanding with a big bang singularity. This singularity is of the point type.

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