Section: Research Paper

# Bitopological Properties of δ(δg)* - Closed Sets

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**ABSTRACT**

*The concept of $\delta(\delta g)^*$ - closed set in bitopological spaces (briefly BTS) was introduced in this paper. It specifically looked into and characterized concepts related like $\delta(\delta g)^*$- interior and $\delta(\delta g)^*$- closure of a set, and $\tau_i\,\tau_j$– closed sets. This paper proves that every $\tau_j$ - $\delta$-closed set and $\tau_i\,\tau_j\delta g^*$ -closed sets are $\tau_i\,\tau_j\delta(\delta g)^*$-closed. It is also shows that the family of all $\tau_i\,\tau_j\delta(\delta g)^*$-closed (resp. $\tau_i\,\tau_j\delta(\delta g)^*$-open) sets is not equal to $\tau_j\,\tau_i\delta(\delta g)^*$-closed.*

**Keywords:** *$\delta(\delta g)^*$-closed set, $\delta(\delta g)^*$- interior, $\delta(\delta g)^*$-closure, $\tau_i\,\tau_j\delta g^*$-closed sets, and $\tau_j\,\tau_i$ $\delta(\delta g)^*$-closed set*

## I. Introduction

Various kinds of closed and open sets have been introduced in an arbitrary topological space over time. Sets that are stronger than open sets are known as -open sets, according to Velicko (1968) [28]. Generalized closed sets, often known as g-closed sets, were first introduced and studied by Norman Levine [14] in 1970. Julian Dontchev (1996) [4] proposed a class of generalized closed sets known as "g-closed sets" by fusing the concepts of "$\delta$-closedness" and "g-closedness." In 2010, Thivagar et al [25] established a class termed "$\delta\hat{g}$-closed sets" that falls in between the categories of "$\delta$-closed" and "$\delta$g-closed" sets.

By the end of 2012, Sudha R. likewise Sivakamasundari, K. [19] introduced and studied the $\delta g^*$-closed set, another universal closed set. In 2014, K. K and Meena. (g)* - closed set in topological spaces is a new class of generalized closed sets that Sivakamasundari presented. The concept of bitopological spaces, or the triple (X, $P_1$, $P_2$) with $P_1$ and $P_2$ being two topologies on X, was instead presented by Kelly [12].

These ideas serve as the inspiration for the author's introduction of the idea of *$\delta(\delta g)^*$* - closed sets in Bitopological spaces (also known as BTS) and his investigation of them. He then uses them to introduce new classes of mappings and BTS. This paper presents a number of characterizations, properties, and examples linked to the new notions.

## II. *Bitopological Properties of δ(δg)* - Closed Sets*

The principles and characterization of the $\delta(\delta$g)*-closed set in the BTS are introduced in this part, and the connections between the $\delta(\delta$g)*-closed set and other closed sets existing in the BTS are also established.

### 1.1 *$\delta(\delta g)^*$ - Closed Sets*

**Defintion 2.1** A subset A of a BTS (X, $\tau_{1.}$ $\tau_2$) is said to be $\tau_{1.}$ $\tau_2$ *$\delta(\delta g)^*$ - closed set* if $\delta cl_2(A) \subseteq$ U whever A$\subseteq$ U, and U is $\delta g_1$-open in *X*. The complement of $\tau_{1.}$ $\tau_2$ - *$\delta(\delta g)^*$ - closed set* is said to be $\tau_{1.}$ $\tau_2$ - *$\delta(\delta g)^*$ - open set*.

**Example 2.2** Suppose X = {a, b, c} then let us consider the topologies $\tau_1 = \{\emptyset$, X, {a}, {b}, {a, b}} and $\tau_2 = \{\emptyset$, X, {b}, {c}, {b, c}} .

So, $\tau_1$-closed sets in X are ∅, X, {b, c}, {a, c} and {c}, $\tau_2$-closed sets in X are ∅, X, {a}, {a, b} and {a, c}. The $\tau_1$ - *regular open* sets and *$\delta_1$-open sets* in X are ∅, X, {a}. Hence, $\tau_2$ - *regular open* sets in X are ∅, X, {b}, and {c}, the *$\delta_2$-open sets* are ∅, X, {b}, {c} and {b, c}. So, the *$\delta_1$-closed sets* in X are ∅, X, {b, c}, while the *$\delta_2$-open sets* in X are ∅, X, {a}, {a, b} and {a, c} and the *$\delta g_1$-closed sets* in X are ∅, X, {c}, {a, c} and {b, c}.

Therefore, ∅, X, {a}, {c}, {a, b}, {a, c} and {b, c} are the $\tau_1$ $\tau_2$ *$\delta$ $(\delta g)^*$ - closed set*.

### 1.2 *Closure and Interior in BTS*

**Defintion 2.3** Let X be an empty set and $\tau_1$ and $\tau_2$ be two topologies on X. The triple (X$_{,1,}$$\tau_2$) is said to be a *bitopological space (briefly, BTS)*. Let (X$_{,1,}$$\tau_2$) be a BTS and A$\subseteq$ X. Respectively, $\tau_i$ - *cl(A)* and $\tau_i$ - *int (A) denotes* the cl(A) and int(A) with respect to $\tau_i$ .

**Theorem 2.4** Suppose A and B are nonempty subsets of X. If A ≠ ∅, so $a \in \delta(\delta g)^*$-*cl(A) if and only if for every $\delta(\delta g)^*$-open set* U with $a \in$ U, U ∩ A ≠ ∅.

**Theorem 2.5** Let (X, $\tau$) be a topological space and A, B, and F be subsets of X.

- (i) If A is $\tau$ - *$\delta(\delta g)^*$-closed, then $A = \delta(\delta g)^*\text{-}cl(A) = \delta(\delta g)^*cl(\delta(\delta g)^*\text{-}cl(A))$.*
- (ii) If A ⊆ B, then ($\delta(\delta g)^*\text{-}cl(A) \subseteq \delta(\delta g)^*\text{-}cl(B)$).
- (iii) $\delta(\delta g)^*\text{-}cl(A) \subseteq \delta(\delta g)^*\text{-}cl(\delta(\delta g)^*\text{-}cl(A))$
- (iv) $\delta(\delta g)^*\text{-}cl(A) \cup \delta(\delta g)^*\text{-}cl(B) \subseteq \delta(\delta g)^*\text{-}cl(A \cup B)$.

**Theorem 2.6** Let (X, $\tau$) be a topological space and A, B, and F be subsets of X.

- (i) If A is *$\delta(\delta g)^*$-open, then $A = \delta(\delta g)^*\text{-}cl(A) = \delta(\delta g)^*cl(\delta(\delta g)^*\text{-}cl(A))$.*
- *(ii)* *$x \in \delta(\delta g)^*$- int (A) if and only if there exists a $\delta(\delta g)^*$-open set U with $x \in$* U ⊆ A.
- (iii) If A ⊆ B, then ($\delta(\delta g)^*\text{-}int(A)$ $\delta(\delta g)^*\text{-}cl(B)$).

**Theorem 2.7** Let A ⊆ X. Then $\delta(\delta g)^*\text{-}int(A) = x\backslash[\ \delta(\delta g)^*\text{-}cl(x\backslash A)]$.
**Corollary 2.8** Let A ⊆ X. Then $\delta\ (\delta g)^*\text{-}cl(A) = X\backslash[\ \delta(\delta g)^*\text{-}int(X\backslash A)]$.

## III. *$\tau_i$, $\tau_j$- $\delta(\delta g)^*$- Open Set*

**Theorem 3.1** Suppose (X, $\tau_i$ $\tau_{j,}$) is a BTS and A$\subseteq$ X. It follows that A is $\tau_i$ $\tau_j$ - *$\delta(\delta g)^*$-open set iff $U \subseteq \tau_j$ - $\delta int(A)$ when $U \subseteq$* A and U is $\tau_i$ - *$\delta g$ closed set in X.*

Results

**Theorem 3.2** There is $\tau_i\ \tau_j$ *- δ(δg)*- closed* in every $\tau_j$ *– δ-closed set.*
**Theorem 3.3** There is $\tau_i\tau_{,j}$ *- δ(δg)*- closed* in every $\tau_i\ \tau_j$ *– δg* closed set.*
**Theorem 3.5** There is $\tau_{i,}\tau_j$ *- gδ- closed* in every $\tau_i\ \tau_j$ *-δ(δg)*- closed set.*
**Theorem 3.6** There is $\tau_{i,}\tau_j$ *– rg- closed in* every $\tau_i\ \tau_j$ *– δ(δg)*- closed set.*
**Theorem 3.7** There is $\tau_{i,}\tau_j$ *– gpr- closed in* every $\tau_i\ \tau_j$ *– δ(δg)*- closed set.*
**Theorem 3.8** There is $\tau_{i,}\tau_j$ *– δg# -closed in* every $\tau_i\ \tau_j$ *– δ(δg)*- closed set.*
**Theorem 3.9** There is $\tau_{i,}\tau_j$ *– rwg- closed* in every $\tau_i\ \tau_j$ *– δ(δg)*- closed set.*
**Theorem 3.10** There is $\tau_{i,}\tau_j$ *– gspr- closed* in very $\tau_i\ \tau_j$ *– δ(δg)*- closed set.*
**Theorem 3.11** There is $\tau_{i,}\tau_j$ *– πg- closed* in every $\tau_i\ \tau_j$ *– δ(δg)*- closed set.*
**Theorem 3.12** There is $\tau_{i,}\tau_j$ *– πgp- closed* in every $\tau_i\ \tau_j$ *– δ(δg)*- closed set.*
**Theorem 3.13** There is $\tau_{i,}\tau_j$ *– πgsp- closed* in every $\tau_i\ \tau_j$ *– δ(δg)*- closed set.*

## Recommendations

Further investigate *δ(δg)*- closed set in bigeneralized topological spaces.*

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