A STUDY ON IFM/IFM/1 INTUITIONISTIC VACATION QUEUEING SYSTEM WITH SERVER START-UP AND TIME-OUT

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Abstract

In this research paper we construct the membership and non-membership functions with intuitionistic fuzzy model with single server queueing system. Also we construct the triangular and trapezoidal fuzzy numbers for N-policy IFM/IFM/1queueing system with server vacation start-up and time-out with the help of IFs α-cut method. Where the arrival rate, service rate and vacation time are intuitionistic fuzzy numbers. IFS queues are reduced to a family of intuitionistic Crisp queues based on IFS α-cut approach. The main objective of the paper is to analyze the expected system length of the queue in L. Finally to illustrated numerical example of this model.

Keywords: Vacation queue, membership function, non-membership function, intuitionistic fuzzy set, performance measures.

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1. Introduction

The queuing theory is a tool used to make queues or lines. How check-out people behave when they have to stand in a queue to make a purchase or receive a service within a given time period and what kind of organization can effectively operate on queue lines. It also studies how many people can do it and how many people must line up, most of the people can do a certain queueing arrangement process in given time period.

Queuing theory can also be applied to moving information through a waiting line. For example, ants follow queue and travel one after the other to bring food. People lose patience to stand in line but ants go anywhere in queue. A transport company, can use queuing theory to determine the most operationally efficient method of transferring packages from one transport vehicle to another.

Vacation queueing theory is application of classical queueing theory. Vacation queue means if the system is idle the server takes a vacation, when the vacation ends, if there are customers in the queue, the server becomes active, otherwise it takes another vacation. The idea of queueing system with server vacation was first introduced by Yonatan Levy and Uri Yechiali [1] they developed solution for the problem of system idle time in an M/G/1 queueing system for utilization of the server idle time. G.Ayyappan, Gopal Sekar and A.Muthu Ganapathi [2] introduced an interrupt for M/M/1 retrial queueing system approach with Erlang K-types service.

Time out concept was first introduced by Oliver C. Ibe [3] derived on expression for mean waiting time of M/G/1 vacation queueing system with server time-out. K. Satish kumar A. Ankama Rao and K.Chandan[5] derived expression for expected system length of N-policy M/M/1 queue with server start-up and time-out and recently Dr.V N Rama Devi, K. Satish Kumar G.Sridhar and K.Chandan [4] anlyze N-policy finite capacity and calculated various parameters for cost function of M/M/1 vacation queueing system in transient state with server start-up and time-out.

In practical situation fuzzy queueing models are more useful than deterministic queueing model, the inter arival times and service times are fuzzy nature and described by linguistic quantifiers such as very low,low, moderate,quick and very quick followed by particular probability distribution. Thus fuzzy queues are more reasonable than family of crisp values. For example arrivals as very low,moderate, quick and very quick. Like a crisp queueing system, the main objective of the fuzzy queue is to find fuzzy exponential arrival rate and service rate for performance measures. That is in fuzzyness, the membership values of a fuzzy set lies between zero and one. Queue with fuzzy model have been described by several researchers like Li and Lee [6], Buckley [7] and S.P. Chen [8]. All are analyzed fuzzy queueing models and it is based on Zedeh's [9] extension principle. K. Julia Rose Mary, P. Monica and S. Mythili [10] developed the membership function of system characteristic for FM/FM/1 queueing system with N-policy and find out the expected number of customers in system. V. Ashok Kumar[11] derived the membership function of the system performance for N-policy FM/ FM/1queueing model with infinite capacity and G. Kannadasan & N. Sathiyamoorth [12] analyze the performance measures for FM/FM/1 queue with working vacation. Recently K. Venu Gopal, P. Sudhakara Babu, K. Satish Kumar and K.Chandan [13] studied membership functions of the triangular and trapezoidal fuzzy numbers for N-policy FM/FM/1 vacation queueing system with server start-up and time-out.

The intuitionistic fuzzy sets are an extension of the concept of fuzzy sets that define degree of membership non-membership of an element to a set. Both membership and non-membership function can be assigned on the interval [0,1] so complement 1 is degree of hesitation. For example Corona vaccination drive is done people are coming for vaccine taking vaccine most of those who took it are successful. But very few have side effects. But some are reluctant to take the vaccine. in the above example we call those who are successful in bringing vaccine as degree of membership and we call those who have side effects after taking the vaccine as degree of nonmember ship. we call those who are hesitant to take the vaccine the degree of hesitation.

Intuitionistic fuzzy set was developed by Krassimir T. Atanassov [14-17] is a greatful work to deal with vague set. Dr. P Rajarajeswari and M. Sangeetha [18] construct the system characteristics of the membership and non-membership functions for queueing system with fuzzy intuitionistic, Chandan Sing Ujarari and Arun Kumar[19] derived both membership and non-membership functions of the performance calculations of priority queueing model for intuitionistic fuzzy, Dr.A. Tamilarasi[20] construction for the membership and non-membership functions of the trapezoidal fuzzy number for different queueing models

M/IF/1,IF/M/1and IF/IF/1. Recently Y. Saritha and K. Chandan [21] derived triangular, trapezoidal and pentagonal membership and non-membership functions for IFM/IFG/1 vacation queue with break downs, repair and server time out.

Our aim this paper is we construct the membership and non-membership function of the triangular and trapezoidal fuzzy numbers and system characteristics of expected system length L, optimal threshold N* and minimum expected cost T(N) for N-policy IFM/IFM/1 vacation queueing system with server start-up and time-out. To illustrate the numerical example applying IFS α-cuts properties for IF.

2. Fuzzy set Theory

2.1 *Fuzzy set*

Let A be a traditional set, $\mu_A(x)$ be a function from A to [0,1]. A fuzzy set A with the membership function $\mu_A(x)$ is defined as A={x,$\mu_A(x)$;x ∈ X} where X is a non-empty set and $\mu_A(x) \in [0,1]$.

2.2 *Membership Functions*

Triangular Fuzzy Number: triangular fuzzy number $\tilde{A}$ is defined by(n1, n2, n3),where ni∈R and n1≤ n2≤ n3.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-n_1}{n_2-n_1} & for\, n_1 \le x \le n_2 \\ \frac{n_3-x}{n_3-n_2} & for n_2 \le x \le n_3 \\ 0 & Otherwise. \end{cases}$$

Trapezoidal Fuzzy Number: A trapezoidal fuzzy number Ã is defined by (n1, n2, n3, n4) where ni∈R and n1≤n2≤ n3≤n4.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-n_1}{n_2-n_1} & for\, n_1 \le x \le n_2 \\ 1 & for\ \ n_2 \le x \le n_3 \\ \frac{n_4-x}{n_4-n_3} & for\ \ n_3 \le x \le n_4 \\ 0 & Otherwise. \end{cases}$$

2.3 *Intutionistic Fuzzy Set*

(IFS) Intuitionistic fuzzy set was introduced by Krassimir T. Atanassov and it was extension of the fuzzy set by Lofti Zadeh's fuzzy set. It defined as degree of membership non-membership of an element to a set [0,1].

Definition: let X be a given set. An intuitionistic fuzzy set B in X is given by B = {(x,$\mu_B(x)$, $\eta_B(x)$) / x ∈ X}where $\mu_B(x)$, $\eta_B(x)$: X → [0,1], where $\mu_B(x)$is membership of the element x in B , $\eta_B(x)$ is the non-membership of the element x in B and 0 ≤ $\mu_B(x)$+ $\eta_B(x)$ ≤ 1. for each x ∈ X, $\pi_B(x)$ = 1- $\mu_B(x)$- $\eta_B(x)$ is the degree of hesitation.

2.5 *Non-Membership Functions*

Non-membership function of triangular fuzzy number is given by

$$\eta_{\bar{B}}(x) = \begin{cases} \frac{n_2-x}{n_2-n_1} & for\, n_1 \le x \le n_2 \\ \frac{x-n_3}{n_3-n_2} & for n_2 \le x \le n_3 \\ 0 & Otherwise. \end{cases}$$

Non-membership function of trapezoidal fuzzy number is given by

$$\eta_{\bar{B}}(x) = \begin{cases} \frac{n_2-x}{n_2-n_1} & for\, n_1 \le x \le n_2 \\ 1 & for\ \ n_2 \le x \le n_3 \\ \frac{x-n_3}{n_4-n_3} & for\ \ n_3 \le x \le n_4 \\ 0 & Otherwise. \end{cases}$$

3. Model Description

Let us consider N-policy M/M/1 vacation queueing model with server start-up and time-out. Here customers arrives at a Poisson arrival rate λ.The server spends a random initial time t pre-service when the queue size N(≥1) is reached which assumed to exponential service rate μ followed by probability distribution and served in FIFS discipline. Immediately the start period is over,the server served at all the waiting customer in the queue. When the system become empty, the server waits for fixed amount of time c, where c is called server time out.

Let

C_h = holding cost per unit time for each customer in the system,

C_b =cost per unit time to keep the server on and in operation

C_m= initial cost per unit time per cycle,

C_t= expiration cost per unit time per cycle,

C_s=set up cost per cycle,

C_v= benefit per unit time for the server being on vacation and doing secondary work.

The following performance measures are taken from ref [5].

1. Optimal N-policy (N*)

$$N^* = \sqrt{\left(\frac{2\lambda\mu\gamma+2\lambda\mu c+c\lambda\gamma}{2c\mu\gamma}\right)^2 - \frac{2\lambda^2\mu-2\lambda\gamma\mu+2\lambda^2\gamma-\lambda\mu c+c^2\lambda}{c\mu\gamma} + \frac{2(\mu-\lambda)\left\{c_b\frac{\lambda}{\mu}+c_v\left(\frac{\lambda}{c}+\frac{\lambda}{\gamma}\right)+c_m\frac{\lambda}{\gamma}+c_t\frac{\lambda}{c}+c_s\lambda\right\}}{\mu c_h}} - \frac{2\lambda\mu\gamma+2\lambda\mu c+c\lambda\gamma}{2c\mu\gamma}$$

2.The expected system length

$$L = \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

3.The optimum expected cost for the model is

$$T(N) = C_h\left\{\frac{N(N-1)}{2}\frac{\left(1-\frac{\lambda}{\mu}\right)}{\left[N+\frac{\lambda}{\gamma}+\frac{\lambda}{C}\right]} + \frac{\lambda}{\gamma}\left[N+\frac{\lambda}{\gamma}\right]\frac{\left(1-\frac{\lambda}{\mu}\right)}{\left[N+\frac{\lambda}{\gamma}+\frac{\lambda}{C}\right]} + \frac{\lambda}{\mu-\lambda}\left[\frac{\lambda}{\mu-\lambda}\left(N+\frac{\lambda}{\gamma}+\frac{\lambda}{C}\right) + \frac{\lambda}{C} + N + \frac{\lambda}{\gamma} + \frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(N+\frac{\lambda}{\gamma}\right)\right] + C_m\frac{\lambda}{\gamma} + C_t\frac{\lambda}{C} + C_s\lambda - C_v N\right\}\frac{\left(1-\frac{\lambda}{\mu}\right)}{\left[N+\frac{\lambda}{\gamma}+\frac{\lambda}{C}\right]} + \frac{\lambda}{\mu}C_b$$

4. N-policy IFM/IFM/1 Model

Here, we applying intuitionistic fuzzy model for the above performance measures to optimal threshold N* and expected system length L for triangular and trapezoidal fuzzy numbers by using α-cut intervals. Let$\tilde{\lambda}$,$\tilde{\mu}$ and $\tilde{\gamma}$be the Poisson arrival rate with fuzzy, fuzzy service rate and fuzzy with vacation rate respectively. Let $\mu_{\tilde{\lambda}}(x)$, $\mu_{\tilde{\mu}}(x)$ and $\mu_{\tilde{\gamma}}(x)$ are membership function between $\tilde{\lambda}$,$\tilde{\mu}$ and $\tilde{\gamma}$ respectively and Let $\eta_{\tilde{\lambda}}(x)$, $\eta_{\tilde{\mu}}(x)$ and $\eta_{\tilde{\gamma}}(x)$ denote the non- membership function between $\tilde{\lambda}$,$\tilde{\mu}$ and $\tilde{\gamma}$ respectively. Let us consider the N-policy IFM/IFM/1 vacation queueing system with server start-up and time-out. The inter-arrival rate A and service rate S and vacation rate V. The IF sets are given by

$\tilde{A}$ = { $\tilde{\lambda}$, μ($\tilde{\lambda}$),η($\tilde{\lambda}$) / $\tilde{\lambda}$ ∈ X}
$\tilde{S}$ = { $\tilde{\mu}$, μ($\tilde{\mu}$),η($\tilde{\mu}$) / $\tilde{\mu}$ ∈ Y}
$\tilde{V}$ = { $\tilde{\gamma}$, μ($\tilde{\gamma}$),η($\tilde{\gamma}$) / $\tilde{\gamma}$ ∈ Y_1}

Where X, Y and Y_1 are crisp universal sets of arrival rate, service rate and vacation rate respectively and $0 \le \mu(x) + \eta(x) \le 1$.
the α-cuts of arrival rate, service rate and vacation rate are represented as

A(α) = {$\tilde{\lambda}$ ∈ X/μ($\tilde{\lambda}$)≥α, $\tilde{\lambda}$ ∈ X/η($\tilde{\lambda}$)≥α}
S(α) = {$\tilde{\mu}$ ∈ Y/μ($\tilde{\mu}$)≥α, $\tilde{\mu}$ ∈ Y/η($\tilde{\mu}$)≥α}
V(α) = {$\tilde{\gamma}$ ∈ Y_1/μ($\tilde{\gamma}$)≥α, $\tilde{\gamma}$ ∈ Y_1/η($\tilde{\gamma}$)≥α}

4.1 *Membership functions*

A triangular membership function P (A,S,V) is defined by(n_1, n_2, n_3),where $n_i \in R$ and $n_1 \le n_2 \le n_3$. The membership function of Triangular fuzzy number is given by

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$$\mu_{P(A,S,V)}(x) = \begin{cases} \frac{x-n_1}{n_2-n_1} & for\ n_1 \le x \le n_2 \\ \frac{n_3-x}{n_3-n_2} & for n_2 \le x \le n_3 \\ 0 & Otherwise. \end{cases}$$

A trapezoidal membership function P (A, S, V) is defined by(n_1, n_2, n_3,n_4),where $n_i \in R$ and $n_1 \le n_2 \le n_3 \le n_4$. The membership function of trapezoidal fuzzy number is given by

$$\mu_{P(A,S,V)}(x) = \begin{cases} \frac{x-n_1}{n_2-n_1} & for\ n_1 \le x \le n_2 \\ 1 & for\ n_2 \le x \le n_3 \\ \frac{n_4-x}{n_4-n_3} & for\ n_3 \le x \le n_4 \\ 0 & Otherwise. \end{cases}$$

4.2 *Non-membership functions*

A triangular insuitionistic fuzzy number P (A,S,V) is defined by(n_1, n_2, n_3),where $n_i \in R$ and $n_1 \le n_2 \le n_3$. The non-membership function of Triangular fuzzy number is given by

$$\eta_{P(A,S,V)}(x) = \begin{cases} \frac{n_2-x}{n_2-n_1} & for\ n_1 \le x \le n_2 \\ \frac{x-n_3}{n_3-n_2} & for n_2 \le x \le n_3 \\ 0 & Otherwise. \end{cases}$$

A trapezoidal insuitionistic fuzzy number P(A,S,V) is defined by(n_1, n_2, n_3,n_4),where $n_i \in R$ and $n_1 \le n_2 \le n_3 \le n_4$. The non- membership function of trapezoidal fuzzy number is given by

$$\eta_{P(A,S,V)}(x) = \begin{cases} \dfrac{n_2 - x}{n_2 - n_1} & for\ n_1 \le x \le n_2 \\ 1 & for\ n_2 \le x \le n_3 \\ \dfrac{x - n_3}{n_4 - n_3} & for\ n_3 \le x \le n_4 \\ 0 & Otherwise. \end{cases}$$

Now let us consider in single server IFM/IFM/1 fuzzy N-policy queueing system of expected system length L and optimal threshold N* minimum expected cost T (N).

5. BASIC PROPERTIES ON INTUITIONISTIC FUZZY SETS

1. [inclusion] A⊆B ↔ $\mu_A(x) \le \mu_B(x)$and$\eta_A(x) \ge \eta_B(x) \forall\, x \in$ X
2. [equality] A=B ↔ $\mu_A(x) = \mu_B(x)$and$\eta_A(x) = \eta_B(x) \forall x \in$ X
3. [compliment] A^c = {$(x, \mu_A(x), \mu_B(x)$):$x \in$ X }
4. [union] A∪B= {$(x, max(\mu_A(x), \mu_B(x)$), $min(\eta_A(x), \eta_B(x)): x \in$ X }
5. [intersection] A∩B= {$(x, min(\mu_A(x), \mu_B(x)$), $max(\eta_A(x), \eta_B(x)): x \in$ X }
6. [addition] A⊕B = {$(x, (\mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \eta_A(x), \eta_B(x)): x \in$ X }
7. [multiplication] A⊗B = {$(x, (\mu_A(x)\mu_B(x), \eta_A(x) + \eta_B(x) - \eta_A(x), \eta_B(x)): x \in$ X }

etc.

B. Algorithm

The DSW algorithm consists of the following steps:

1) Select a α-cut value where 0≤ α≤1.
2) Find the intervals in the input membership function and also the input non-membership function that correspond to this α.
3) Using standard binary interval operations compute the interval for the output of both the membership and non-membership functions for the selected α-cut level.
4) Repeat steps 1 – 3 for different values of α to complete α-cut representation of both membership and non-membership functions of the solution.

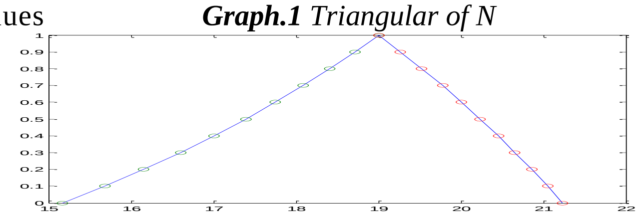
6. Numerical analysis

6.1 Triangular fuzzy number with graphical representation

Consider an IFM/IFM/1, Queueing System with Server-Up and Timeout where the arrival rate and service rate are triangular fuzzy numbers represented by λ=[1,2,3]; μ=[5,6,7]; =[2,3,4]. C=1 and non-monetary parameters as C_b=300, C_m=200, C_t=30, C_s=500, C_v =15, C_h=5. The interval of confidence at possibility level α as [1+ α, 3− α]; [5+ α, 7− α]; [2+ α, 4− α].

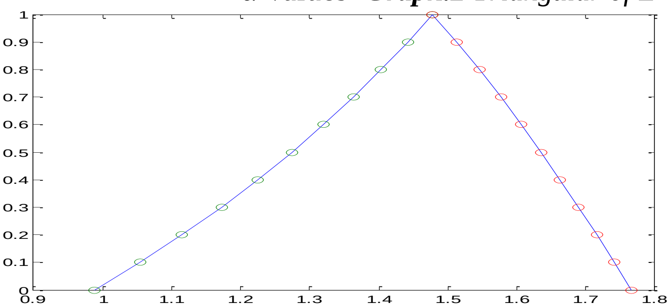
α	N Lower	N Upper	L Lower	L Upper	T(N) Lower	T(N) Upper
0	15.16443207	21.23330677	0.988472307	1.765506562	1.83428832	162.7140665
0.1	15.68224089	21.04918795	1.054423315	1.741018687	87.64262089	159.8999673
0.2	16.15928648	20.85803517	1.11542473	1.715834451	93.1810985	156.9947881
0.3	16.6010868	20.65933451	1.171998422	1.689875896	98.47113669	153.9940877
0.4	17.01210545	20.45251957	1.224614418	1.663056418	103.5313397	150.8931152
0.5	17.3960098	20.23696415	1.273693453	1.635279698	108.378056	147.6867802
0.6	17.75585343	20.01197361	1.319610824	1.606438487	113.0257892	144.3696185
0.7	18.09420831	19.77677451	1.362700682	1.576413217	117.4875083	140.9357526
0.8	18.41326277	19.53050224	1.403260323	1.545070426	121.7748883	137.3788466
0.9	18.71489524	19.27218608	1.441554249	1.512260951	125.8984994	133.6920545
1.0	19.00073098	19.00073098	1.477817888	1.477817888	129.8679588	129.8679588

α-values **Graph.1** *Triangular of N*



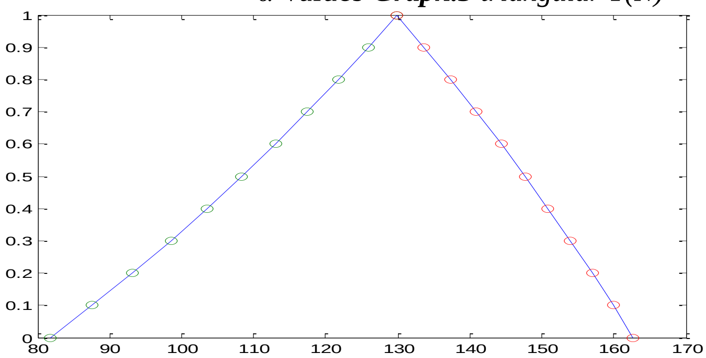
Membership function of triangular N

α-values ***Graph.2*** *Triangular of L*



Membership function of triangular L

α-values ***Graph.3*** *triangular T(N)*



Membership function of triangular T(N)

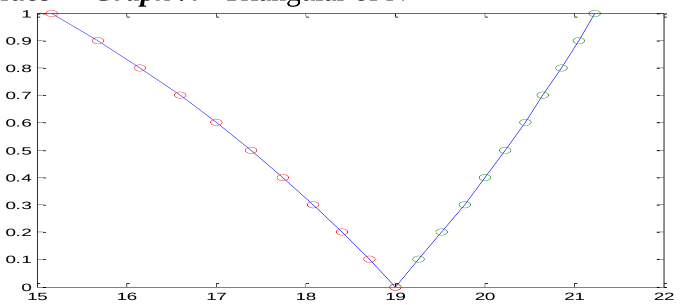
We have calculated expected system length L for the given values by using intuitionistic fuzzy numbers. The value of α is substituted in above performance measures and the results are in the following table.2 and a graphical representation shown graph.3,4.

The possibility interval of confidence at level α as [2- α, 2+ α]; [6- α, 6+ α];[3- α, 3+ α].

Table .2 *Non-membership function of triangular fuzzy*

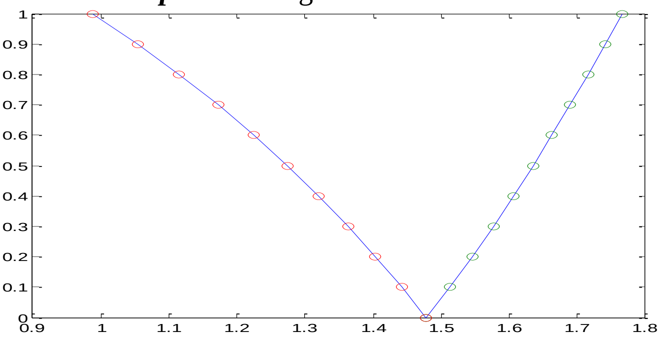
β	N Lower	N upper	L lower	L Upper	T(N) Lower	T(N) Upper
0	19.00073098	19.00073098	1.477817888	1.477817888	129.8679588	129.8679588
0.1	19.27218608	18.71489524	1.512260951	1.441554249	125.8984994	133.6920545
0.2	19.53050224	18.41326277	1.545070426	1.403260323	121.7748883	137.3788466
0.3	19.77677451	18.09420831	1.576413217	1.362700682	117.4875083	140.9357526
0.4	20.01197361	17.75585343	1.606438487	1.319610824	113.0257892	144.3696185
0.5	20.23696415	17.3960098	1.635279698	1.273693453	108.378056	147.6867802
0.6	20.45251957	17.01210545	1.663056418	1.224614418	103.5313397	150.8931152
0.7	20.65933451	16.6010868	1.689875896	1.171998422	98.47113669	153.9940877
0.8	20.85803517	16.15928648	1.715834451	1.11542473	93.1810985	156.9947881
0.9	21.04918795	15.68224089	1.741018687	1.054423315	87.64262089	159.8999673
1.0	21.23330677	15.16443207	1.765506562	0.988472307	81.83428832	162.7140665

α-values ***Graph .4*** Triangular of N



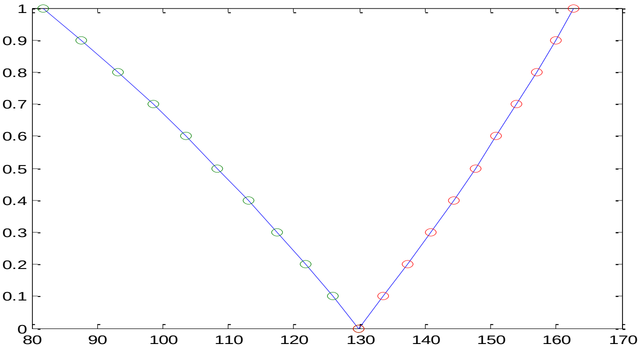
Non-Membership function of triangular N

α-values ***Graph . 5*** Triangular of L



Non-membership function of triangular L

α-values ***Graph . 6*** Triangular of T(N)



Non-membership function of triangular T(N)

From the above tables 1,2 noted the (α,β)-cuts of the performance measures at 11 distinct points 0,0.1,0.2,0.3,……..1.0. The membership value for expected length L is 1.4778, impossible to outside the range [0.9884, 1.7655]. The non-membership value for expected length L lies between [0.9884, 1.7655] never falls at the outside.

6.2 Trapezoidal fuzzy number with graphical representation

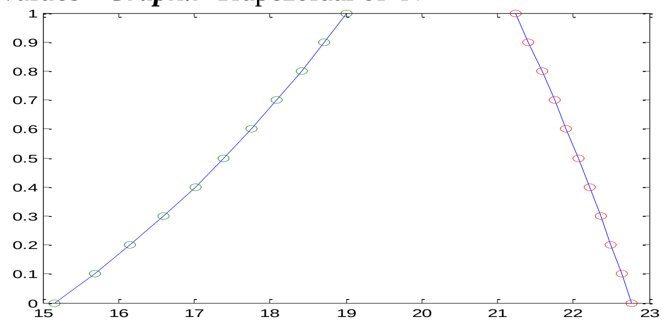
Take both the arrival rate and service rate are trapezoidal fuzzy numbers represented by by λ= [1,2,3,4]; μ = [5,6,7,8]; ϒ=[2,3,4,5]. C=1and nonmonetary parameters as C_b=300,C_m= 200,C_t= 30,C_s= 500, C_v= 15,C_h= 5.The interval of confidence at possibility level α as[1+ α,4−α];[5+α, 8−α];[2+α,5−α].

We calculated expected system length L for the mention values by using intuitionistic fuzzy numbers. The value of α is substituted in above performance measures and the results are in the following table.4 and a graphical representation shown graph.7,8.

Table.3 *membership function for trapezoidal fuzzy number*

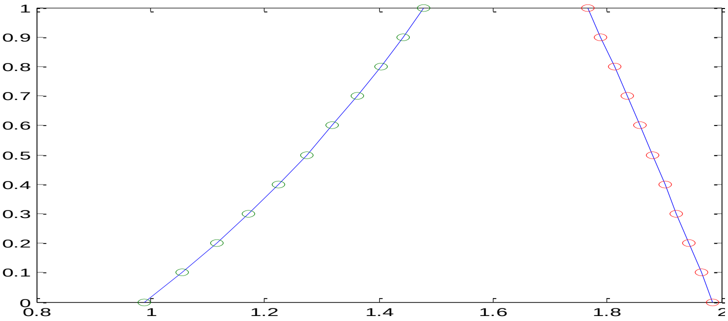
α	N Lower	N Lower	L Lowere	L Upper	T(N) Lower	T(N) Upper
0	15.16443207	22.77339456	0.988472307	1.984646438	81.83428832	186.6472965
0.1	15.68224089	22.63937788	1.054423315	1.964186158	87.64262089	184.5519208
0.2	16.15928648	22.5016035	1.11542473	1.94349494	93.1810985	182.3975368
0.3	16.6010868	22.35984898	1.171998422	1.922544059	98.47113669	180.1817092
0.4	17.01210545	22.21387457	1.224614418	1.901301888	103.5313397	177.9018647
0.5	17.3960098	22.06342149	1.273693453	1.879733583	108.378056	175.5552814
0.6	17.75585343	21.90820991	1.319610824	1.857800725	113.0257892	173.1390772
0.7	18.09420831	21.74793662	1.362700682	1.835460926	117.4875083	170.6501973
0.8	18.41326277	21.58227247	1.403260323	1.812667373	121.7748883	168.0854008
0.9	18.71489524	21.41085934	1.441554249	1.78936833	125.8984994	165.4412444
1.0	19.00073098	21.23330677	1.477817888	1.765506562	129.8679588	162.7140665

α-values ***Graph.7*** Trapezoidal of N



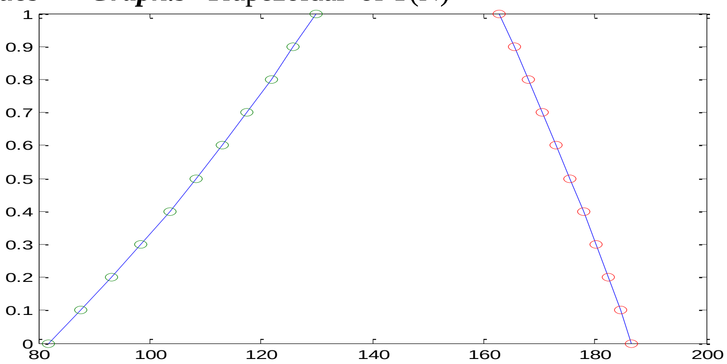
Membership function trapezoidal N

α-values ***Graph.8*** Trapezoidal of L



Membership function trapezoidal L

α-values ***Graph.9*** Trapezoidal of T(N)



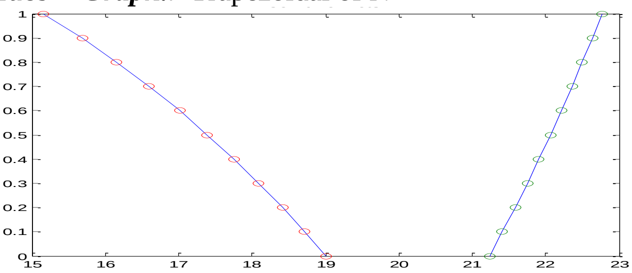
Membership function trapezoidal T(N)

The possibility interval of confidence at level α as [2- α, 3+ α]; [6- α, 7+ α]; [3- α, 4+ α].

Table.4 *Non- membership function for trapezoidal fuzzy number*

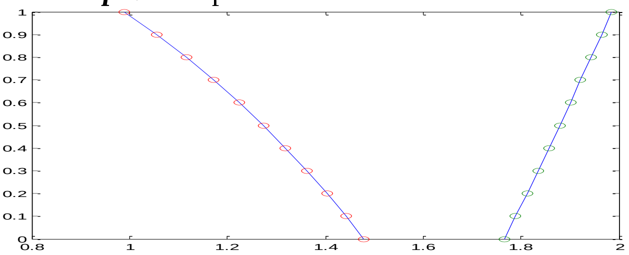
β	N Lower	N Upper	L low	L Upper	T(N) Lower	T(N) Upp
0	21.23330677	19.00073098	1.765506562	1.477817888	129.8679588	162.7140665
0.1	21.41085934	18.71489524	1.78936833	1.441554249	125.8984994	165.4412444
0.2	21.58227247	18.41326277	1.812667373	1.403260323	121.7748883	168.0854008
0.3	21.74793662	18.09420831	1.835460926	1.362700682	117.4875083	170.6501973
0.4	21.90820991	17.75585343	1.857800725	1.319610824	113.0257892	173.1390772
0.5	22.06342149	17.3960098	1.879733583	1.273693453	108.378056	175.5552814
0.6	22.21387457	17.01210545	1.901301888	1.224614418	103.5313397	177.9018647
0.7	22.35984898	16.6010868	1.922544059	1.171998422	98.47113669	180.1817092
0.8	22.5016035	16.15928648	1.94349494	1.11542473	93.1810985	182.3975368
0.9	22.63937788	15.68224089	1.964186158	1.054423315	87.64262089	184.5519208
1.0	22.77339456	15.16443207	1.984646438	0.988472307	81.83428832	186.6472965

α-values ***Graph.7*** Trapezoidal of N



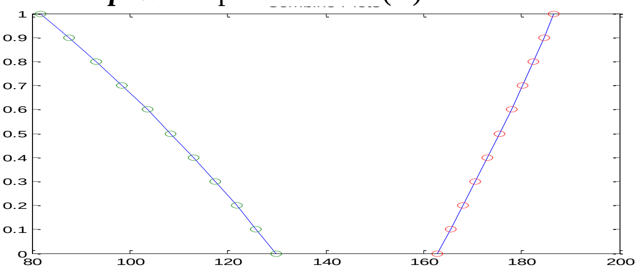
Non-membership function trapezoidal N

α-values ***Graph.8*** Trapezoidal of L



Non-membership function trapezoidal L

α-values ***Graph.9*** Trapezoidal of T(N)



Non-membership function trapezoidal T(N)

From the above tables 3, 4 noted the (α,β)-cuts of the performance measures at 11 distinct points 0,0.1,0.2,0.3,……..1.0. The membership value for expected length is 0.9884, impossible to outside the range [1.4778, 1.7655]. The non-membership value for expected length L lies between [0.9884, 1.9846] and impossible to falls at the outside the range [1.4778, 1.7655].

7. Conclusion

In this paper we studied and perform N-policy IFM/IFM/1 vacation queuing system with server start-up and time-out for (triangular and trapezoidal) intuitionistic fuzzy set by using IFS α-cut method. The arrival rate, service rate and vacation time are in IFS natured for different IFS models.

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