CHEMICAL REACTION AND HALL EFFECTS ON UNSTEADY FLOW PAST AN ISOThERMAL VERTICAL PLATE IN A ROTATING FLUID WITH VARIABLE MASS DIFFUSION WITH HEAT SOURCE

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Abstract:
The present paper investigated chemical reaction effect and hall current on electrically conducting rotating fluid with variable mass diffusion on an infinite vertical plate which is an isothermal has been analyzed. Using perturbation technique to find exact solution of nonlinear partial differential equation such as momentum, energy, and mass with boundary conditions. The result of velocity, temperature and concentration represent by graphically with different parameter. Combined analysis of Hall current, chemical reaction and rotating fluid plays very important role in science and technology like space research, fluid flow sensor, current sensor, geo physics etc.

Keywords: Chemical reaction, Isothermal vertical plate, Heat and Mass transfer, MHD

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Introduction

In industrial, scientific and engineering application, heat transfer and thermal radiation has play an important role in the boundary layer flow. The mutual heat and mass transfer are increases with the control of buoyancy forces. The characteristic condensing and boiling in separation processes such as drying, evaporation, distillation, condensation, rectification and absorption of chemical engineering has significant role in fluid mechanics. MHD heat transfer and radiation with chemical reaction has influence of liquid metal flows and ionized gas flow into the electrolytes. [1-18].

Considering MHD flow along with heat and mass transfer on rotating fluid with radiation(R) on an IVP – isothermal vertical channel plate is having variable mass diffusion has attracted the researchers to make more attention of its industrial, scientific, and engineering application viz. Involving high temperature, solar power technology, space technology, space vehicles etc. magneto hydrodynamic flow with Thermal radiation and Hall effect is very important in science and technology like Hall effect sensors can be used in pressure sensors, fluid flow sensors, rotating speed sensors, and electric sensors. Hall effect principal used in brushless DC motors, nuclear power reactors, control of hypersonic flows, efficient Hall thrusters in magnetic propulsion, measure magnetic field, development of plasma actuator, control hydraulic valves by Hall effect joysticks, Hall current accelerators etc. [19-38].

MHD flow problems associated with heat and mass transfer plays important roles in different areas of science and technology, like chemical engineering, mechanical engineering, biological science, petroleum engineering and biomechanics. Such problems frequently occur in petro-chemical industry, chemical vapor deposition on surfaces, cooling of nuclear reactors, heat exchanger design, forest fire dynamics and geophysics. The influence of magnetic field on viscous, incompressible and electrically conducting fluid is of great importance in many applications such as magnetic material processing, glass manufacturing control processes and purification of crude oil. [39-58]

The present paper investigated chemical reaction effect and hall current on electrically conducting rotating fluid with variable mass diffusion on an infinite vertical plate which is an isothermal has been analyzed. Using Laplace transform to find exact solution of nonlinear partial differential equation such as momentum, energy, and mass with boundary conditions. The result of velocity, temperature and concentration represent by graphically with different parameter. Combined analysis of Hall current, chemical reaction and rotating fluid plays very important role in science and technology like space research, fluid flow sensor, current sensor, geo physics etc.

1. Mathematical Formulation

An unsteady hydromagnetic flow of fluid past an infinite isothermal vertical plate with varying mass diffusion exists. The fluid and the plate rotate in unison with a uniform angular velocity \( \Omega' \) about the \( z' \) – axis normal to the plate. Initially the fluid is assumed to be at rest and surrounds an infinite vertical plate with temperature \( T'_\infty \) and concentration \( C'_\infty \). A magnetic field of uniform strength \( B_0 \) is transversely applied to the plate. The \( \chi' \) – axis is taken along the plate in the vertically upward direction and the \( \zeta' \) – axis is taken normal to the plate. The physical model of the problem shown in fig. (1). At time \( t' > 0 \), the plate and the fluid are at the same temperature \( T'_\infty \) in the stationary condition with concentration level \( C'_\infty \) at all the points. At time \( t' > 0 \), the plate is subjected to a uniform velocity \( u = u_0 \) in its own plane against the gravitational force.
The plate temperature and concentration level near the plate are raised uniformly and are maintained constantly thereafter. All the physical properties of the fluid are considered to be constant except the influence of the body force term. Then under the usual Boussinesq’s approximation the unsteady flow equations are momentum equation, energy equation, and mass equation respectively.

Equation of Momentum:
\[
\frac{\partial u'}{\partial t'} - 2\Omega' v = \frac{\partial^2 u}{\partial z^2} - \frac{1}{\rho} \frac{\partial \rho}{\partial x} + g + \frac{B_0}{\rho} j_y, \quad (1)
\]
\[
\frac{\partial v}{\partial t} - 2\Omega' u = \frac{\partial^2 v}{\partial z^2} - \frac{B_0}{\rho^2} j_x, \quad (2)
\]

Equation of Energy
\[
\rho \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z^2} + \frac{q}{\rho} - Q_0 (T' - T_w), \quad (3)
\]

Equation of diffusion
\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z^2} - Kr' (C - C_0') + \frac{D_M K_T}{T_M} \frac{\partial^2 T'}{\partial z^2}, \quad (4)
\]
As, no large velocity gradient here, the viscous term in equation (1) vanishes for small and hence for the outer flow, beside there is no magnetic field along x-direction gradient, so this results in,
\[
0 = D \frac{\partial \rho}{\partial x} - p_x g, \quad (5)
\]
By eliminating the pressure term from equation (1) and (5), we obtain
\[
\frac{\partial u'}{\partial t'} - 2\Omega' v = \frac{\partial^2 u}{\partial z^2} - \frac{1}{\rho} \frac{\partial \rho}{\partial x} + (\rho' - \rho) g + \frac{B_0}{\rho} j_y, \quad (6)
\]
The Boussinesq approximation gives
\[
\rho' - \rho = \rho_{\infty} \beta (T' - T_{\infty}) + \rho_{\infty} \beta (C' - C_{\infty}) \quad (7)
\]
On using (7) in the equation (6) and noting that \( \rho_{\infty} \) is approximately equal to 1, the momentum equation reduces to
\[
\frac{\partial u'}{\partial t'} - 2\Omega' v = \frac{\partial^2 u}{\partial z^2} + \frac{B_0}{\rho} j_y + g \beta (T' - T_w) + g \beta' (C' - C_{\infty}) \quad (8)
\]
The generalized Ohm’s law with Hall currents is taken into account and ion – slip and thermo-electric
\[
j + \frac{\omega T}{B_0} (j \times B) = \sigma [E + q \times B] \quad (9)
\]
The equation (9) gives
\[
j_x - m_j = \sigma \nu B_0 \quad (10)
j_y - m_j = \sigma \nu B_0 \quad (11)
\]
where \( m = \omega \nu T \) is Hall parameter;
Solving (10) and (11) for \( j_x \) and \( j_y \), we have
\[
j_x = \frac{\sigma B_0}{1 + m^2} (v - mu) \quad (12)
j_y = \frac{\sigma B_0}{1 + m^2} (u - mv) \quad (13)
\]
where \( B_0 \) – Imposed magnetic field, \( m \) – Hall parameter, \( \nu \) – Kinematic viscosity, \( \Omega \) – Component of angular viscosity, \( \Omega \) – Non-dimensional angular velocity, \( J \) – component of current density \( j \), \( \rho \) – Fluid density, \( \sigma \) – Electrical conductivity, \( \omega \) – Time, \( \mu \) – Coefficient of viscosity, \( T \) – Temperature of the fluid near the plate, \( T_w \) – Temperature of the plate, \( \theta \) – Dimensionless temperature,
\[ T_\infty - \text{Temperature of the fluid far away from the plate, } \]
\[ C - \text{Dimensionless concentration, } \]
\[ \kappa - \text{Thermal conductivity, } \]
\[ \beta - \text{Volumetric coefficient of thermal expansion, } \]
\[ \beta' - \text{Volumetric coefficient of expansion with concentration, } \]
\[ C' - \text{Species concentration in the fluid, } \]
\[ C_\infty - \text{Concentration for away from the plate, } t - \]

Non-dimensional time \((u,v,w) - \text{Components of velocity field } F, (U,V,W) - \text{Non dimensional velocity components, } (x,y,z) - \text{Cartesian coordinates.} \)

On the use of (12) and (13), the momentum equations (8) and (2) become

\[ \frac{\partial u'}{\partial t} = v \frac{\partial^2 u}{\partial z^2} + 2\Omega' v - \frac{\sigma \mu_e H_0^2}{\rho \left(1 + m^2\right)} \left( u + mv \right) + g \beta \left( T' - T_\infty \right) + g \beta' \left( C' - C_\infty \right) \]  

(14)

\[ \frac{\partial v'}{\partial t} = v \frac{\partial^2 u}{\partial z^2} + 2\Omega' v - \frac{\sigma \mu_e H_0^2}{\rho \left(1 + m^2\right)} \left( v - mu \right) \]  

(15)

Due to small Coriolis force, the second term on the right side of the equation (14) and (15) comes into existence.

The boundary conditions are given by:

\[ u = 0, \quad T = T_\infty, \quad C = C_\infty, \quad \forall \quad y, t' \leq 0 \]  

(18)

\[ u' > 0: \quad u = u_0, T \rightarrow T_\infty, C' = C_\infty + \left( C'_\infty - C_\infty \right) \quad \text{at} \quad y = 0 \]  

(19)

The dimensionless quantities are introduced as follows:

\[ U = \frac{u}{u_0}, V = \frac{v}{v_0}, t = \frac{t u_0^3}{\rho}, Z = \frac{z u_0^3}{\rho}, \Omega = \Omega v, Gr = \frac{g \beta v \left( T_\infty - T_\infty \right)}{u_0^3} \]  

(20)

\[ Gc = \frac{g \beta' v \left( C'_\infty - C_\infty \right)}{u_0^3}, M^2 = \frac{\sigma \mu_e H_0^2 v}{2 \rho u_0^2}, R = \frac{16 \mu_c}{\kappa k u_0^2}, Pr = \frac{\mu_c}{\kappa} \]  

\[ Kr = \frac{K r' v u_0^3}{\rho}, Q = \frac{Q u_0 v}{\rho C_p u_0^2}, Sr = \frac{D_M K_T \left( T'_\infty - T_\infty \right)}{V T_M \left( C'_\infty - C_\infty \right)} \]  

Together with the equation (1), (2), (3) and (4), boundary conditions (18), (19), using (20), we have

\[ \frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Z^2} + 2V \left( \Omega - \frac{2 m^2}{1 + m^2} \right) + \frac{2 m^2}{1 + m^2} U + Gr \theta + Gc C \]  

(21)

\[ \frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial Z^2} - 2U \left( \Omega + \frac{2 m^2}{1 + m^2} \right) + \frac{2 m^2}{1 + m^2} V \]  

(22)

with the boundary conditions

\[ U = 0, \quad \theta = 0, \quad C = 0, \quad v = 0 \quad \forall \quad Z, t \leq 0 \]  

(23)

\[ U \rightarrow 1, \quad \theta \rightarrow 1, \quad C \rightarrow t, \quad v \rightarrow 0 \quad \forall \quad t > 0 \]  

(24)
Now equations (21), (22) and the boundary conditions (23), (24) can be combined to give:

\[
\frac{\partial F}{\partial t} + \frac{\partial^2 F}{\partial Z^2} = F \alpha + Gr \theta + Gc C
\]

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Z^2} - \frac{1}{Pr} (R + Q) \theta
\]

\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Z^2} - Kr C + Sr \frac{\partial^2 \theta}{\partial Z^2}
\]

where \( F = U + i \nu \) and \( a = 2 \left[ \frac{M^2}{1 + m^2} + i \Omega \frac{M^2 m}{(1 + m^2)} \right] \)

In this study the value of (rotation parameter) is taken to be \( \Omega = -\frac{M^2 m}{1 + m^2} \), as a result of this the transverse velocity vanishes

with the boundary conditions

\[
F = 0, \quad \theta = 0, \quad C = 0 \quad \forall \ Z, t \leq 0
\]

\[
F \rightarrow 1, \quad \theta \rightarrow 1, \quad C \rightarrow t, \quad at \ Z = 0 \quad \forall \ t > 0
\]

\[
F \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0, \quad at \ Z \rightarrow \infty \quad \forall \ t > 0
\]

2. Method of Solution

Equation (25) – (27) are coupled, non – linear partial differential equations and these cannot be solved in closed form using the initial and boundary conditions (28). However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighborhood of the fluid in the neighborhood of the plate as

\[
F(z,t) = F_0(z) e^{i\omega t}
\]

\[
\theta(z,t) = \theta_0(z) e^{i\omega t}
\]

\[
C(z,t) = C_0(z) e^{i\omega t}
\]

Substituting (29) in Equation (25) – (27) and equating the harmonic and non – harmonic terms, we obtain

\[
F_{0}^{*} - \beta_{0}^{2} F_{0} = -Gr \theta_{0} - Gm C_{0}
\]

\[
\theta_{0}^{*} - \beta_{0}^{2} \theta_{0} = 0
\]

\[
C_{0}^{*} - \beta_{0}^{2} C_{0} = -Sr \beta c \theta_{0}
\]

The corresponding boundary conditions can be written as

\[
F_0 = 1, \quad \theta_0 = 1, \quad C_0 = 0 \quad \text{at} \ Z = 0
\]

\[
F_0 = 0, \quad \theta_0 = 0, \quad C_0 = 0 \quad \text{as} \ Z \rightarrow \infty
\]

Solving the equations (30) – (32) under the boundary condition (33), we get the solution for fluid velocity; temperature; concentration is expressed below using perturbation method:

\[
F_0 = B_1 e^{-\beta_1 \nu} + B_2 e^{-\beta_2 \nu} + B_3 e^{-\beta_3 \nu} + B_4 e^{-\beta_4 \nu}
\]

\[
\theta_0 = e^{-\beta_0 \nu}
\]

\[
C_0 = A_1 e^{-\beta_1 \nu} + A_2 e^{-\beta_2 \nu}
\]

In view of the above equation (29) becomes

\[
F(z,t) = \left\{ B_1 e^{-\beta_1 \nu} + B_2 e^{-\beta_2 \nu} + B_3 e^{-\beta_3 \nu} + B_4 e^{-\beta_4 \nu} \right\} e^{i\omega t}
\]

\[
\theta(z,t) = \left\{ e^{-\beta_0 \nu} \right\} e^{i\omega t}
\]

\[
C(z,t) = \left\{ A_1 e^{-\beta_1 \nu} + A_2 e^{-\beta_2 \nu} \right\} e^{i\omega t}
\]

Coefficient of Skin-Friction

The coefficient of skin-friction at the vertical porous surface is given by

\[
C_f = \left( \frac{\partial F}{\partial Z} \right)_{z=0} = -(B_1 \beta_1 + B_2 \beta_2 + B_3 \beta_3)
\]

Coefficient of Heat Transfer

The rate of heat transfer in terms of Nusselt number at the vertical porous surface is given by

\[
Nu = \left( \frac{\partial T}{\partial Z} \right)_{z=0} = -\beta_1
\]

Sherwood number

\[
Sh = \left( \frac{\partial C}{\partial Z} \right)_{z=0} = -(A_1 \beta_1 + A_2 \beta_2)
\]
3. Results and Discussions
Final results are shown graphically for various parameters like thermal Grashof number (Gr), rotation parameter (Ω), modified Grashof number (Gc), Prandtl number (Pr), Schmidt number (Sc), Chemical reaction parameter (Kr), Hartman number (M), Radiation parameter (R), Soret number (Sr), on the velocity, temperature and concentration profiles can be analyzed from Fig. (2) – (17). The influence of thermal buoyancy force parameter (Gr) on the axial velocity shows in Fig. (2). As can be seen from this figure, the axial velocity profile increases with increases in the values of the thermal buoyancy. We actually observe that the axial velocity overshoot in the boundary layer region. Buoyancy force acts like a favourable pressure gradient which accelerates the fluid within the boundary layer therefore the solutal buoyancy force parameter has the same effect on the velocity as thermal Grashof number. From this figure we observe that the effect of magnetic field is to decrease the value of velocity profile throughout the boundary layer which results in the thinning of the boundary layer thickness. The influences of the Schmidt number on the axial velocity profiles are plotted in Fig. (3) respectively. It is noticed from this figure that, the axial velocity decrease on increasing Sc. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) boundary layer. Fig. (4) display the effect of magnetic field parameter or Hartmann number on axial velocity. It is seen from these figures that the axial velocity increases when Hartmann number increases. That is the axial velocity fluid motion is retarded due to application of transverse magnetic field. This phenomenon clearly agrees with the fact that Lorentz force that appears due to interaction of the magnetic field and fluid axial velocity resists the fluid motion. The influence of the hall parameter on axial velocity profiles is as shown in Figs. (5) respectively. It is observed from these figures that the axial velocity profiles increase with an increase in the hall parameter m. This is because, in general, the Hall currents reduce the resistance offered by the Lorentz force. This means that Hall currents have a tendency to increase the fluid velocity components. Fig. (6) illustrates the influence of rotation parameter on the velocity. Physically, the presence of rotation parameter effect has the tendency in resulting in a net reduction in the flow velocity. This behaviour is seen from this figure in which the velocity decreases as rotation parameter increases. Fig. (7) illustrates the behaviour of axial velocity profiles for different values of the chemical reaction parameter. It is pertinent to mention that (Kr > 0) corresponds to a destructive chemical reaction. It can be seen from the profiles that the axial velocity increases in the degenerating chemical reaction in the boundary layer. This is due to the fact that the increase in the rate of chemical reaction rate leads to thinning of a momentum in a boundary layer in degenerating chemical reaction. It can be seen from the profiles that the cross flow axial velocity reduces in the degenerating chemical reaction. It is evident from Fig. (8) shows the effect of Soret number. It is observed that, the increasing in Soret number the axial velocity profiles also increases. Fig. (9) that the effects of rotation on the axial respectively. It is evident from this figure that, axial velocity increases on increasing in reaction parameter. This implies that rotation retards fluid flow in the axial velocity flow direction and accelerates fluid flow in the axial velocity flow direction in the boundary layer region. It is evident from Fig. (10) that, the heat source parameter leads to increases in the axial velocity with increasing values of thermal radiation parameter. It is evident from Fig. (11) that, the thermal radiation parameter leads to increases in the axial velocity with increasing values of thermal radiation parameter. Thus, the Fig. (11) are in excellent agreement with the laws of Physics. Thus as thermal radiation parameter increases, the axial velocity increases. Now, from this figure, it may be inferred that radiation has a more significant effect on temperature than on velocity. Thus, the heat source parameter does not have a significant effect on the velocities but produces a comparatively more pronounced effect on the temperature of the mixture. Fig. (12) indicates that effect of heat source parameter on the temperature profiles. It is deduced that temperature profiles decrease of the fluid near the plate decrease when radiation parameter are increased. Physically, thermal radiation causes a fall in temperature of the fluid medium and thereby causes a fall in kinetic energy of the fluid particles. This results in a corresponding decrease in fluid velocities. Fig. (13) indicates that effect of radiation parameter on the temperature profiles. It is deduced that temperature profiles
decrease of the fluid near the plate decrease when radiation parameter are increased. Physically, thermal radiation causes a fall in temperature of the fluid medium and thereby causes a fall in kinetic energy of the fluid particles. This results in a corresponding decrease in fluid velocities. Fig. (14) Shows the temperature profile for different values of Prandtl number. It is observed that temperature increases with decrease in values of Prandtl number and also heat transfer is predominant in air when compared to water. Figs. (15) - (17) depict the influence of the non-dimensional chemical reaction parameter, Schmidt number and Soret number concentration profiles, respectively. From fig. (15), the effect of chemical reaction parameter is very important in the concentration field. Chemical reaction increases the rate of interfacial mass transfer. Reaction reduces the local concentration, thus increases its concentration gradient and its flux. In fig. (16) we see that the concentration profiles decrease with increasing values of the Schmidt number. Fig. (17) shows the effect of Soret number. It is observed that, the increasing in Soret number the concentration profiles also increases.

4. Appendix

\[
\begin{align*}
\beta_1^2 &= (i\omega + R + Q) Pr, \\
\beta_2^2 &= (i\omega + Kr) Sc, \\
\beta_3^2 &= (i\omega + a), \\
A_1 &= -\frac{S_0 \beta_1^2}{\beta_1^2 - \beta_2^2}, \\
A_2 &= (t - A_1), \\
B_1 &= -\frac{Gr}{\beta_1^2 - \beta_3^2}, \\
B_2 &= -\frac{GcA_i}{\beta_1^2 - \beta_3^2}, \\
B_3 &= -\frac{GcA_i}{\beta_1^2 - \beta_3^2}, \\
B_4 &= (1 - B_1 - B_2 - B_3).
\end{align*}
\]

5. References

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Fig. (2): Axial velocity profiles for different values of Gr, Gc
Fig. (3): Axial velocity profiles for different values of $Sc$

Fig. (4): Axial velocity profiles for different values of $M$

Fig. (5): Axial velocity profiles for different values of $m$
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Section A - Research paper

Fig. (6): Axial velocity profiles for different values of $\Omega$

Fig. (7): Axial velocity profiles for different values of $Kr$

Fig. (8): Axial velocity profiles for different values of $Sr$
Fig. (9): Axial velocity profiles for different values of $K$

Fig. (10): Axial velocity profiles for different values of $Q$

Fig. (11): Axial velocity profiles for different values of $R$
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**Fig. (12):** Temperature profiles for different values of Q

**Fig. (13):** Temperature profiles for different values of R

**Fig. (14):** Temperature profiles for different values of Pr
Fig. (15): Concentration profiles for different values of Kr

Fig. (16): Concentration profiles for different values of Sc

Fig. (17): Concentration profiles for different values of Sr