DETERMINATION OF THE CONCENTRATIONS OF THE REACTANTS OF FIRST ORDER CONSECUTIVE CHEMICAL REACTION USING ANUJ TRANSFORM

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ABSTRACT: The study of chemical reactions plays the vital role for understanding the problems of science and engineering such as photosynthesis; heat mass transfer; nuclear reactor; photon emission and radioactive decay. The main objective of this manuscript is to make use of Anuj transform in order to analytically determine the solution of the problem of concentrations of the reactants of first order consecutive chemical reaction. Results suggested that Anuj transform is efficient method for determining the solution of the problem of concentrations of the reactants of first order consecutive chemical reaction. Results of the manuscript demonstrate that Anuj transform has good accuracy and provide exact results without doing intricate calculation work.

KEYWORDS: Anuj Transform; Inverse Anuj Transform; Cramer Rule; Chemical Reaction; Concentration; Reactant.

MATHEMATICS SUBJECT CLASSIFICATION: 35A22; 44A05; 34A30; 65L05; 80A30

1.INTRODUCTION: In the recant years, integral transforms are the first preference of the researchers for tackling the problem of growth of the species [1-12]; decay problem of radioactive substance [1-10]; Abel's problem of mechanics [13-20]; heat problem [21]; circuit problems of electronics communication [22]; problem of infected cells during infection of HIV-1 [23]; concentration problem of drug during intravenous injection of drug [24-25] and vibration problem of string [21-22] because integral transforms provide the exact primitives of these problems. Researchers are also very interested for developing new integral transforms [26-27] nowadays due to their high-yielding characteristic of providing results of the problems with good accuracy.

Researchers [28-35] developed the duality relation among the different integral transforms and successfully utilized these relations for developing new properties and theories of integral transforms. Higazy and Aggarwal [36] applied Sawi transform on the mathematical model of the chemical reaction in series and estimated the concentration of chemical substances. Murphy [37] analyzed the consecutive chemical reactions of first and second orders. Lin [38] analyzed the consecutive reactions (homogeneous) performed in an annular reactor with non Newtonian flow.

Chrastil [39] obtained the value of rate constants of consecutive chemical reaction of first order by the aid of final product. The mathematical models of the consecutive reactions were suggested by Westman and DeLury [40]. Erdogdn and Sahmurat [41] obtained the kinetic constants of first-order consecutive chemical reactions. The main interest of this manuscript is to determine the solution of the problem of concentrations of the reactants of first order consecutive chemical reaction via Anuj transform. This study has a great practical importance due to achieve maximum production by eliminating waste or useless products in the transitional phase of chemical reaction.

2. NOMENCLATURE OF SYMBOLS

$\mathcal{F}$, family of piecewise continuous and exponential order function;

$\mathcal{A}$, Anuj transform operator;

$\mathcal{A}^{-1}$, inverse Anuj transform operator;
$\in$, belongs to;
!, the usual factorial notation;
Γ, the classical Gamma function;
$\mathcal{L}$, Laplace transform operator;
N, the set of natural numbers;
R, the set of reals;
$\theta_1(t)$, concentration of a chemical reactant P at any time t;
$\theta_2(t)$, concentration of a chemical reactant Q at any time t;
$\theta_3(t)$, concentration of a chemical reactant R at any time t;
$\theta_1(0) = \omega$, initial concentration of a chemical reactant P;
$\theta_2(0)$, initial concentration of a chemical reactant Q;
$\theta_3(0)$, initial concentration of a chemical reactant R;
$\beta_1, \beta_2 > 0$, rate constants

3. DEFINITION OF ANUJ TRANSFORM

If $H(t) \in \mathcal{F}, t \geq 0$ then the Anuj transform of $H(t)$ is defined as [26]

$$\mathcal{A}\{H(t)\} = r^2 \int_0^\infty H(t)e^{-\left(\frac{1}{r}\right)t} dt = h(r),\ r > 0 \tag{1}$$

4. INVERSE ANUJ TRANSFORM

The inverse Anuj transform of $h(r)$, denoted by $\mathcal{A}^{-1}\{h(r)\}$, is another function $H(t)$ having the characteristic that $\mathcal{A}\{H(t)\} = h(r)$.

5. RELATION BETWEEN LAPLACE AND ANUJ TRANSFORMS

If $\mathcal{L}\{H(t)\} = \int_0^\infty H(t)e^{-rt} dt = \Psi(r)$, (2)

then $h(r) = r^2 \Psi\left(\frac{1}{r}\right)$ (3)

and $\Psi(r) = r^2 h\left(\frac{1}{r}\right)$ (4)

Proof: Equation (1) gives

$$h(r) = r^2 \int_0^\infty H(t)e^{-\left(\frac{1}{r}\right)t} dt = r^2 \left\{\int_0^\infty H(t)e^{-\left(\frac{1}{r}\right)t} dt\right\} = r^2 \Psi\left(\frac{1}{r}\right)$$

Now equation (2) gives

$$\Psi(r) = \int_0^\infty H(t)e^{-rt} dt = r^2 \left\{\frac{1}{r^2}\int_0^\infty H(t)e^{-rt} dt\right\} = r^2 h\left(\frac{1}{r}\right).$$

6. PROPERTIES OF ANUJ TRANSFORM: In this part, we will describe the properties of Anuj transform that will be used in later section of this manuscript.

6.1 Linearity [24]: If $H_j(t) \in \mathcal{F},\ t \geq 0, j = 1, 2, 3, \dots\dots, n$ with $\mathcal{A}\{H_j(t)\} = h_j(r), j = 1, 2, 3, \dots\dots n$ then $\mathcal{A}\{\sum_{j=1}^{n} \ell_j H_j(t)\} = \sum_{j=1}^{n} \ell_j\, \mathcal{A}\{H_j(t)\} = \sum_{j=1}^{n} \ell_j\, h_j(r)$, where ℓ_j are arbitrary constants.

6.2 Change of Scale [24]: If $H(t) \in \mathcal{F}, t \geq 0$ with $\mathcal{A}\{H(t)\} = h(r)$ then $\mathcal{A}\{H(\ell t)\} = \frac{1}{\ell^3} h(\ell r)$, where ℓ is arbitrary constant.

6.3 Translation [24]: If $H(t) \in \mathcal{F}, t \geq 0$ with $\mathcal{A}\{H(t)\} = h(r)$ then

$\mathcal{A}\{e^{\ell t} H(t)\} = (1 - \ell r)^2 h\left(\frac{r}{1-\ell r}\right)$, where ℓ is arbitrary constant.

Remark 1: Equations (3) and (4) can be use for establishing the further properties of Anuj transform.

7. ANUJ TRANSFORMS OF THE DERIVATIVES OF A FUNCTION [12]: If $H(t) \in \mathcal{F}, t \geq 0$ with $\mathcal{A}\{H(t)\} = h(r)$ then

a) $\mathcal{A}\{H'(t)\} = \frac{1}{r} h(r) - r^2 H(0)$.
b) $\mathcal{A}\{H''(t)\} = \frac{1}{r^2} h(r) - rH(0) - r^2 H'(0)$.
c) $\mathcal{A}\{H'''(t)\} = \frac{1}{r^3} h(r) - H(0) - rH'(0) - r^2 H''(0)$.

Remark 2: Tables 1-2 visualized the Anuj transforms and inverse Anuj transforms of fundamental functions respectively.

Table-1: Anuj transforms of fundamental functions [26]

S.N.	$H(t) \in \mathcal{F}, t > 0$	$\mathcal{A}\{H(t)\} = h(r)$
1	1	r^3
2	$e^{\ell t}$	$\left(\frac{r^3}{1-\ell r}\right)$
3	$t^\lambda, \lambda \in N$	$\lambda!\, r^{\lambda+3}$
4	$t^\lambda, \lambda > -1, \lambda \in R$	$r^{\lambda+3}\Gamma(\lambda+1)$
5	$sin(\ell t)$	$\left(\frac{\ell\, r^4}{1+r^2\ell^2}\right)$
6	$cos(\ell t)$	$\left(\frac{r^3}{1+r^2\ell^2}\right)$
7	$sinh(\ell t)$	$\left(\frac{\ell r^4}{1-r^2\ell^2}\right)$
8	$cosh(\ell t)$	$\left(\frac{r^3}{1-r^2\ell^2}\right)$

Table-2: Inverse Anuj transforms of fundamental functions

S.N.	$h(r)$	$H(t) = \mathcal{A}^{-1}\{h(r)\}$
1	r^3	1
2	$\left(\frac{r^3}{1-\ell r}\right)$	$e^{\ell t}$
3	$r^{\lambda+3}, \lambda \in N$	$\frac{t^\lambda}{\lambda!}$
4	$r^{\lambda+3}, \lambda > -1, \lambda \in R$	$\frac{t^\lambda}{\Gamma(\lambda+1)}$
5	$\left(\frac{r^4}{1+r^2\ell^2}\right)$	$\frac{sin(\ell t)}{\ell}$
6	$\left(\frac{r^3}{1+r^2\ell^2}\right)$	$cos(\ell t)$
7	$\left(\frac{r^4}{1-r^2\ell^2}\right)$	$\frac{sinh(\ell t)}{\ell}$
8	$\left(\frac{r^3}{1-r^2\ell^2}\right)$	$cosh(\ell t)$

8. DETERMINATION OF THE CONCENTRATIONS OF THE REACTANTS OF FIRST ORDER CONSECUTIVE CHEMICAL REACTION USING ANUJ TRANSFORM:

The concentrations $\theta_1(t), \theta_2(t)$ and $\theta_3(t)$ of three chemical reactants P, Q and R of first order consecutive chemical reaction

$$Chemical\ reactant\ \ P ---\rightarrow Chemical\ reactant\ \ Q ---\rightarrow Chemical\ reactant\ \ R \qquad (5)$$

at any time t is determined by the following system of linear ordinary differential equations as [36]

$$\left.\begin{aligned} \frac{d\theta_1}{dt} &= -\beta_1\theta_1 \\ \frac{d\theta_2}{dt} &= \beta_1\theta_1 - \beta_2\theta_2 \\ \frac{d\theta_3}{dt} &= \beta_2\theta_2 \end{aligned}\right\} \quad (6)$$

with $\theta_1(0) = \omega$, $\theta_2(0) = 0$ and $\theta_3(0) = 0$ (7)

Performing Anuj transform on equation (6), we have

$$\left.\begin{aligned} \mathcal{A}\left\{\frac{d\theta_1}{dt}\right\} &= -\mathcal{A}\{\beta_1\theta_1\} \\ \mathcal{A}\left\{\frac{d\theta_2}{dt}\right\} &= \mathcal{A}\{\beta_1\theta_1 - \beta_2\theta_2\} \\ \mathcal{A}\left\{\frac{d\theta_3}{dt}\right\} &= \mathcal{A}\{\beta_2\theta_2\} \end{aligned}\right\} \quad (8)$$

Use of 6.1 in equation (8) gives

$$\left.\begin{aligned} \mathcal{A}\left\{\frac{d\theta_1}{dt}\right\} &= -\beta_1\mathcal{A}\{\theta_1\} \\ \mathcal{A}\left\{\frac{d\theta_2}{dt}\right\} &= \beta_1\mathcal{A}\{\theta_1\} - \beta_2\mathcal{A}\{\theta_2\} \\ \mathcal{A}\left\{\frac{d\theta_3}{dt}\right\} &= \beta_2\mathcal{A}\{\theta_2\} \end{aligned}\right\} \quad (9)$$

Use of 7 (a) in equation (9) gives

$$\left.\begin{aligned} \frac{1}{r}\mathcal{A}\{\theta_1\} - r^2\theta_1(0) &= -\beta_1\mathcal{A}\{\theta_1\} \\ \frac{1}{r}\mathcal{A}\{\theta_2\} - r^2\theta_2(0) &= \beta_1\mathcal{A}\{\theta_1\} - \beta_2\mathcal{A}\{\theta_2\} \\ \frac{1}{r}\mathcal{A}\{\theta_3\} - r^2\theta_3(0) &= \beta_2\mathcal{A}\{\theta_2\} \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \left(\frac{1}{r}+\beta_1\right)\mathcal{A}\{\theta_1\} - r^2\theta_1(0) &= 0 \\ -\beta_1\mathcal{A}\{\theta_1\} + \left(\frac{1}{r}+\beta_2\right)\mathcal{A}\{\theta_2\} - r^2\theta_2(0) &= 0 \\ \beta_2\mathcal{A}\{\theta_2\} + \frac{1}{r}\mathcal{A}\{\theta_3\} - r^2\theta_3(0) &= 0 \end{aligned}\right\} \quad (10)$$

Use of equation (7) in equation (10) provides

$$\left.\begin{aligned} \left(\frac{1}{r}+\beta_1\right)\mathcal{A}\{\theta_1\} - r^2\omega &= 0 \\ -\beta_1\mathcal{A}\{\theta_1\} + \left(\frac{1}{r}+\beta_2\right)\mathcal{A}\{\theta_2\} &= 0 \\ \beta_2\mathcal{A}\{\theta_2\} + \frac{1}{r}\mathcal{A}\{\theta_3\} &= 0 \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \left(\frac{1}{r}+\beta_1\right)\mathcal{A}\{\theta_1\} &= r^2\omega \\ -\beta_1\mathcal{A}\{\theta_1\} + \left(\frac{1}{r}+\beta_2\right)\mathcal{A}\{\theta_2\} &= 0 \\ \beta_2\mathcal{A}\{\theta_2\} + \frac{1}{r}\mathcal{A}\{\theta_3\} &= 0 \end{aligned}\right\} \quad (11)$$

Equation (11) represents a system of three non-homogeneous linear equations in $\mathcal{A}\{\theta_1\}, \mathcal{A}\{\theta_2\}$ and $\mathcal{A}\{\theta_3\}$ unknowns. Now use of Cramer's rule in equation (11) gives the values of unknowns $\mathcal{A}\{\theta_1\}, \mathcal{A}\{\theta_2\}$ and $\mathcal{A}\{\theta_3\}$ as:

$$\mathcal{A}\{\theta_1\} = \frac{\begin{vmatrix} r^2\omega & 0 & 0 \\ 0 & \left(\frac{1}{r}+\beta_2\right) & 0 \\ 0 & \beta_2 & \frac{1}{r} \end{vmatrix}}{\begin{vmatrix} \left(\frac{1}{r}+\beta_1\right) & 0 & 0 \\ -\beta_1 & \left(\frac{1}{r}+\beta_2\right) & 0 \\ 0 & \beta_2 & \frac{1}{r} \end{vmatrix}}, \begin{vmatrix} \left(\frac{1}{r}+\beta_1\right) & 0 & 0 \\ -\beta_1 & \left(\frac{1}{r}+\beta_2\right) & 0 \\ 0 & \beta_2 & \frac{1}{r} \end{vmatrix} \neq 0$$

$$\Rightarrow \mathcal{A}\{\theta_1\} = \left(\frac{\omega r^3}{1+\beta_1 r}\right) \quad (12)$$

$$\mathcal{A}\{\theta_2\} = \frac{\begin{vmatrix} \left(\frac{1}{r}+\beta_1\right) & r^2\omega & 0 \\ -\beta_1 & 0 & 0 \\ 0 & 0 & \frac{1}{r} \end{vmatrix}}{\begin{vmatrix} \left(\frac{1}{r}+\beta_1\right) & 0 & 0 \\ -\beta_1 & \left(\frac{1}{r}+\beta_2\right) & 0 \\ 0 & \beta_2 & \frac{1}{r} \end{vmatrix}}, \begin{vmatrix} \left(\frac{1}{r}+\beta_1\right) & 0 & 0 \\ -\beta_1 & \left(\frac{1}{r}+\beta_2\right) & 0 \\ 0 & \beta_2 & \frac{1}{r} \end{vmatrix} \neq 0$$

$$\Rightarrow \mathcal{A}\{\theta_2\} = \left(\frac{\omega\beta_1}{\beta_2-\beta_1}\right)\left[\frac{r^3}{1+\beta_1 r} - \frac{r^3}{1+\beta_2 r}\right] \quad (13)$$

$$\mathcal{A}\{\theta_3\} = \frac{\begin{vmatrix} \left(\frac{1}{r}+\beta_1\right) & 0 & r^2\omega \\ -\beta_1 & \left(\frac{1}{r}+\beta_2\right) & 0 \\ 0 & \beta_2 & 0 \end{vmatrix}}{\begin{vmatrix} \left(\frac{1}{r}+\beta_1\right) & 0 & 0 \\ -\beta_1 & \left(\frac{1}{r}+\beta_2\right) & 0 \\ 0 & \beta_2 & \frac{1}{r} \end{vmatrix}}, \begin{vmatrix} \left(\frac{1}{r}+\beta_1\right) & 0 & 0 \\ -\beta_1 & \left(\frac{1}{r}+\beta_2\right) & 0 \\ 0 & \beta_2 & \frac{1}{r} \end{vmatrix} \neq 0$$

$$\Rightarrow \mathcal{A}\{\theta_3\} = \omega\left[r^3 - \left(\frac{\beta_2}{\beta_2-\beta_1}\right)\left(\frac{r^3}{1+\beta_1 r}\right) + \left(\frac{\beta_1}{\beta_2-\beta_1}\right)\left(\frac{r^3}{1+\beta_2 r}\right)\right] \quad (14)$$

Performing inverse Anuj transform on equations (12), (13) and (14) gives the required concentrations $\theta_1(t), \theta_2(t)$ and $\theta_3(t)$ as:

$$\theta_1 = \mathcal{A}^{-1}\left\{\frac{\omega r^3}{1+\beta_1 r}\right\}$$

$$\Rightarrow \theta_1 = \omega \mathcal{A}^{-1}\left\{\frac{r^3}{1+\beta_1 r}\right\} = \omega e^{-\beta_1 t} \quad (15)$$

$$\theta_2 = \mathcal{A}^{-1}\left\{\left(\frac{\omega\beta_1}{\beta_2-\beta_1}\right)\left[\frac{r^3}{1+\beta_1 r} - \frac{r^3}{1+\beta_2 r}\right]\right\}$$

$$\Rightarrow \theta_2 = \left(\frac{\omega\beta_1}{\beta_2-\beta_1}\right)\left[\mathcal{A}^{-1}\left\{\frac{r^3}{1+\beta_1 r}\right\} - \mathcal{A}^{-1}\left\{\frac{r^3}{1+\beta_2 r}\right\}\right]$$

$$\Rightarrow \theta_2 = \left(\frac{\omega\beta_1}{\beta_2-\beta_1}\right)\left[e^{-\beta_1 t} - e^{-\beta_2 t}\right] \quad (16)$$

$$\theta_3 = \mathcal{A}^{-1}\left\{\omega\left[r^3 - \left(\frac{\beta_2}{\beta_2-\beta_1}\right)\left(\frac{r^3}{1+\beta_1 r}\right) + \left(\frac{\beta_1}{\beta_2-\beta_1}\right)\left(\frac{r^3}{1+\beta_2 r}\right)\right]\right\}$$

$$\Rightarrow \theta_3 = \omega\left[\mathcal{A}^{-1}\{r^3\} - \left(\frac{\beta_2}{\beta_2-\beta_1}\right)\mathcal{A}^{-1}\left\{\frac{r^3}{1+\beta_1 r}\right\} + \left(\frac{\beta_1}{\beta_2-\beta_1}\right)\mathcal{A}^{-1}\left\{\frac{r^3}{1+\beta_2 r}\right\}\right]$$

$$\Rightarrow \theta_3 = \omega\left[1 - \left(\frac{\beta_2}{\beta_2-\beta_1}\right)e^{-\beta_1 t} + \left(\frac{\beta_1}{\beta_2-\beta_1}\right)e^{-\beta_2 t}\right] \quad (17)$$

Remark 3: Results given by equations (15), (16) and (17) have perfect agreement with [36].

9. CONCLUSIONS: In this manuscript, we have profitably attained the solution of the problem of concentrations of the reactants of first order consecutive chemical reaction by using Anuj transform. It is observed that Anuj transform provide the exact analytical primitive of this problem with good accuracy. We can use Anuj transform to study the problems of chemical engineering in future that involve different order reversible and parallel chemical reactions.

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REFERENCES

1. Aggarwal, S., Gupta, A.R., Singh, D.P., Asthana, N. and Kumar, N., Application of Laplace transform for solving population growth and decay problems, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(9), 141-145, 2018.
2. Aggarwal, S., Gupta, A.R., Asthana, N. and Singh, D.P., Application of Kamal transform for solving population growth and decay problems, Global Journal of Engineering Science and Researches, 5(9), 254-260, 2018.
3. Aggarwal, S., Pandey, M., Asthana, N., Singh, D.P. and Kumar, A., Application of Mahgoub transform for solving population growth and decay problems, Journal of Computer and Mathematical Sciences, 9(10), 1490-1496, 2018.
4. Aggarwal, S., Sharma, N. and Chauhan, R., Solution of population growth and decay problems by using Mohand transform, International Journal of Research in Advent Technology, 6(11), 3277-3282, 2018.
5. Aggarwal, S., Asthana, N. and Singh, D.P., Solution of population growth and decay problems by using Aboodh transform method, International Journal of Research in Advent Technology, 6(10), 2706-2710, 2018.
6. Aggarwal, S., Singh, D.P., Asthana, N. and Gupta, A.R., Application of Elzaki transform for solving population growth and decay problems, Journal of Emerging Technologies and Innovative Research, 5(9), 281-284, 2018.
7. Aggarwal, S., Sharma, S.D. and Gupta, A.R., Application of Shehu transform for handling growth and decay problems, Global Journal of Engineering Science and Researches, 6(4), 190-198, 2019.
8. Aggarwal, S. and Bhatnagar, K., Sadik transform for handling population growth and decay problems, Journal of Applied Science and Computations, 6(6), 1212-1221, 2019.
9. Singh, G.P. and Aggarwal, S., Sawi transform for population growth and decay problems, International Journal of Latest Technology in Engineering, Management & Applied Science, 8(8), 157-162, 2019.
10. Aggarwal, S., Sharma, S.D., Kumar, N. and Vyas, A., Solutions of population growth and decay problems using Sumudu transform, International Journal of Research and Innovation in Applied Science, 5(7), 21-26, 2020.
11. Priyanka and Aggarwal, S., Solution of the model of the bacteria growth via Rishi transform, Journal of Advanced Research in Applied Mathematics and Statistics, 7(1 & 2), 5-11, 2022.
12. Bansal, S., Kumar, A. and Aggarwal, S., Application of Anuj transform for the solution of bacteria growth model, GIS Science Journal, 9(6), 1465-1472, 2022.
13. Sharma, N. and Aggarwal, S., Laplace transform for the solution of Abel's integral equation, Journal of Advanced Research in Applied Mathematics and Statistics, 4(3&4), 8-15, 2019.
14. Aggarwal, S. and Sharma, S.D., Application of Kamal transform for solving Abel's integral equation, Global Journal of Engineering Science and Researches, 6(3), 82-90, 2019.
15. Aggarwal, S., Sharma, S.D. and Gupta, A.R., A new application of Mohand transform for handling Abel's integral equation, Journal of Emerging Technologies and Innovative Research, 6(3), 600-608, 2019.
16. Aggarwal, S. and Sharma, S.D., Solution of Abel's integral equation by Aboodh transform method, Journal of Emerging Technologies and Innovative Research, 6(4), 317-325, 2019.
17. Aggarwal, S. and Gupta, A.R., Sumudu transform for the solution of Abel's integral equation, Journal of Emerging Technologies and Innovative Research, 6(4), 423-431, 2019.
18. Aggarwal, S. and Gupta, A.R., Shehu transform for solving Abel's integral equation, Journal of Emerging Technologies and Innovative Research, 6(5), 101-110, 2019.
19. Aggarwal, S. and Bhatnagar, K., Solution of Abel's integral equation using Sadik transform, Asian Resonance, Vol. 8 No. 2, (Part-1), 57-63, 2019.
20. Aggarwal, S. and Upadhyaya, L.M., An application of the Laplace-Carson transform method for the solution of the generalized Abel's integral equation, Bulletin of Pure and Applied Sciences-Math & Stat. 40E(2), 155-163, 2021.

21. Debnath, L. and Bhatta, D., Integral transforms and their applications, 2nd ed., Chapman & Hall/CRC, Boca Raton.
22. Spiegel, M.R., Schaum's outline of theory and problems of Laplace transforms, McGraw-Hill, USA, 1965.
23. Higazy, M., Aggarwal, S. and Hamed, Y.S., Determination of number of infected cells and concentration of viral particles in plasma during HIV-1 infections using Shehu transform, Journal of Mathematics, 2020, 1-13, 2020. https://doi.org/10.1155/2020/6624794
24. Kumar, A., Bansal, S., Aggarwal, S., Determination of the blood glucose concentration of a patient during continuous intravenous injection using Anuj transform, Neuroquantology, 19(12), 303-306, 2021.
25. Higazy, M., Aggarwal, S. and Taher, A.N., Sawi decomposition method for Volterra integral equation with application, Journal of Mathematics, 2020, 1-13, 2020. https://doi.org/10.1155/2020/6687134
26. Kumar, A., Bansal, S. and Aggarwal, S., A new novel integral transform "Anuj transform" with application, Design Engineering, 2021(9), 12741-12751, 2021.
27. Kumar, R., Chandel, J. and Aggarwal, S., A new integral transform "Rishi Transform" with application, Journal of Scientific Research, 14(2), 521-532, 2022.
28. Aggarwal, S., Sharma, N. and Chauhan, R., Duality relations of Kamal transform with Laplace, Laplace-Carson, Aboodh, Sumudu, Elzaki, Mohand and Sawi transforms, SN Applied Sciences, 2(1), 135, 2020. https://doi.org/10.1007/s42452-019-1896-z
29. Aggarwal, S. and Gupta, A.R., Dualities between Mohand transform and some useful integral transforms, International Journal of Recent Technology and Engineering, 8(3), 843-847, 2019.
30. Aggarwal, S. and Gupta, A.R., Dualities between some useful integral transforms and Sawi transform, International Journal of Recent Technology and Engineering, 8(3), 5978-5982, 2019.
31. Aggarwal, S., Bhatnagar, K. and Dua, A., Dualities between Elzaki transform and some useful integral transforms, International Journal of Innovative Technology and Exploring Engineering, 8(12), 4312-4318, 2019.
32. Chauhan, R., Kumar, N. and Aggarwal, S., Dualities between Laplace-Carson transform and some useful integral transforms, International Journal of Innovative Technology and Exploring Engineering, 8(12), 1654-1659, 2019.
33. Aggarwal, S. and Bhatnagar, K., Dualities between Laplace transform and some useful integral transforms, International Journal of Engineering and Advanced Technology, 9(1), 936-941, 2019.
34. Chaudhary, R., Sharma, S.D., Kumar, N. and Aggarwal, S., Connections between Aboodh transform and some useful integral transforms, International Journal of Innovative Technology and Exploring Engineering, 9(1), 1465-1470, 2019.
35. Mishra, R., Aggarwal, S., Chaudhary, L. and Kumar, A., Relationship between Sumudu and some efficient integral transforms, International Journal of Innovative Technology and Exploring Engineering, 9(3), 153-159, 2020.
36. Higazy, M. and Aggarwal, S., Sawi transformation for system of ordinary differential equations with application, Ain Shams Engineering Journal, 12(3), 3173-3182, 2021. https://doi.org/10.1016/j.asej.2021.01.027
37. Murphy, R.D., Exact solution of rate equations for consecutive first and second order reactions, Indian Journal of Chemistry, 32A, 381-382, 1993.
38. Lin, S.H., Consecutive chemical reactions in an annular reactor with non Newtonian flow, Applied Scientific Research, 30, 113-126, 1974.
39. Chrastil, J., Determination of the first order consecutive reaction rate constants from final product, Computers & Chemistry, 12(4), 289-292, 1988.
40. Westman, A.E.R. and DeLury, D.B., The differential equations of consecutive reactions, Canadian Journal of Chemistry, 34(8), 1134-1138, 1956.
41. Erdogdn, F. and Sahmurat, F., Mathematical fundamentals to determine the kinetic constants of first-order consecutive reactions, Journal of Food Process Engineering, 30(4), 407-420, 2007.