NON-SPILT DUPLEX EQUITABLE DOMINATION NUMBER OF A GRAPH

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Abstract

Let G be a connected graph. A subset $D \subseteq V(G)$ is called duplex equitable dominating set if for each vertex $v \in V - D$, there exists two vertices $u_1, u_2 \in D$ such that u_1 dominates $v$and u_2 equitable dominates v. The smallest cardinality of duplex equitable domination set is known as duplex equitable domination number and it is represented by $\gamma_{de}(G)$. A duplex equitable dominating set D of a graph G is nonsplit duplex equitable domination set if the subgraphinduced by the vertices in V-D is connected. The smallest number of non-split duplex equitable dominating set is known as non split duplex equitable domination number and isrepresented by $\gamma_{nsde}(G)$. In this paper, we investigate the upper and lower bounds of $\gamma_{nsde}(G)$ and the exact values for some classes of graphs. Also we prove for any connected graph G, $\left\lceil \frac{2n}{\Delta+2} \right\rceil \leq \gamma_{nsde}(G)$and find the relationship between $\gamma_{nsde}(G)$ and other domination parameters like χ, κ and Δ.

Keywords: Equitable, domination number, duplex equitable, non-split

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1. Introduction

In Graph theory, the graphs that focuses on the link connecting edges and vertices in the field of Mathematics. Graph theory well-liked subject along with applications in languages, computer, biosciences, IT and mathematics. A graph G consists of a pair of sets (V, E), where V is the set of vertices and E is the set of edges, joining the pairs of vertices.In a connected graph, if two vertices are said to be adjacent, then there is an edge between that vertices.The number of vertices adjacent to a vertex is known as a Degree. A vertex having degree zero is called an isolated vertex. Also vertex having degree one is called a pendent vertex. In a complete graph every vertex has an edge with each other vertex. If the degree of a vertex in a graph is said to be two, then it is known as a cycle graph.A wheel graph is a graph obtained by adding new vertices in a cycle graph. A bipartite graph G is said to be a complete bipartite graph $K_{|V_1|,|V_2|}$with partition $V = (V_1, V_2)$ if every vertex in V_1 is connected to every vertex of V_2.A star graph is a complete bipartite graph $K_{1,n}$.A graph is connected if every pair of vertices has a path . The subgraph H of a connected graph G is said to be a spanning tree of G if H is a tree and H contains all vertices of G.Bistar is the graph obtained by joining the apex vertices of two copies of star $\boldsymbol{K_{1,n}}$.A book graph may be any of several kinds of graph formed by multiple cycles sharing an edge**.**A set (domination set) for a graph G is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number is the number of vertices in a smallest dominating set for G. A subset $D \subseteq V$ is said to be a dominating set of G if every vertex in V−D is adjacent to some vertex in D. The subset D of V is called equitable domination set if for every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in (G)$ and $|\deg(u) - \deg(v)| \leq 1$. The smallest cardinality of such a dominating set is represented by $\gamma_e(G)$ and is known as the equitable domination number of G [9]. A restrained dominating set is a set S where each vertex in $V-S$ is adjacent to a vertex in S and a vertex in $V-S$. The restrained domination number of G, $\gamma_r(G)$, is the minimum cardinality of a restrained dominating set of G. A set S in a graph G is said to be [1, 2] triple connected dominating set, $<S>$ is triple connected and if for every vertex $\upsilon \in V - S$, $1 \leq |N(\upsilon) \cap S| \leq 2$. The least number taken over all set is denoted by $\gamma_{[1,2]tc}(G)$. A subset D of V is called duplex equitable dominating set if for every vertex $v \in V - D$, there exists two vertices $u_1, u_2 \in D$ such that u_1 dominate v and u_2 equitable dominates v. The least cardinality of duplex equitable dominating set is called duplex equitable domination number and is represented $\gamma_{de}(G)$. This concept is presented by Harary and Haynes [5]. In the present article, new domination parameters γ_{de} are defined and we investigate the upper and lower bounds of $\gamma_{nsde}(G)$ and the exact values for some classes of graphs. Also we prove for any connected graph G, $\left\lceil \frac{2n}{\Delta+2} \right\rceil \leq \gamma_{nsde}(G)$ and find the relationship between $\gamma_{nsde}(G)$ and other domination parameters like χ, κ and Δ.

2. Main Results

Definition 1.1 a duplex equitable dominating set D of a graph G is non-split duplex equitable dominating set if the subgraph induced by the vertices in V-D is connected. The smallest number of non-split duplex equitable dominating set is called non-split duplex equitable domination number and is represented by $\gamma_{nsde}(G)$.

Example: 1.2 Consider the following graph

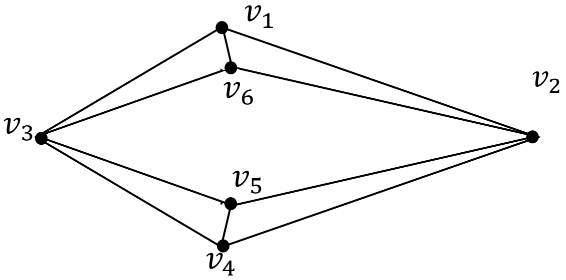

Fig: example of non-split duplex equitable domination number

Take D= $\{v_1, v_2, v_5\}$ and $V - D = \{v_3, v_4, v_6\}$
For $v_3 \in V - D$, there exists v_1, v_5 such that $v_3v_1, v_3v_5 \in E(G)$ and $|deg(v_3) - deg(v_1)| \leq 1$
For $v_4 \in V - D$, there exists v_2, v_6 such that $v_4v_2, v_4v_6 \in E(G)$ and $|deg(v_4) - deg(v_2)| \leq 1$
For $v_6 \in V - D$, there exists v_1, v_2 such that $v_6v_1, v_6v_2 \in E(G)$ and $|deg(v_6) - deg(v_2)| \leq 1$
Since $< V - D >$ is connected graph. Hence $\gamma_{nsde}(G) = 3$.

Remarks 1.3

1. For a complete graph K_m, $\gamma_{nsde}(K_m) = 2$
2. For a complete bipartite graph $K_{p,q}$, $\gamma_{nsde}(K_{pq}) = \begin{cases} 4 & if\ |p-q| \leq 1 \\ p+q-1 & if\ |p-q| \geq 1 \end{cases}$
3. For any star graph $K_{1,n-1}$, $\gamma_{nsde}(K_{1,n-1}) = n$
4. For any path P_n, $\gamma_{nsde}(P_n) = n - 1$
5. For any cycle C_n, $\gamma_{nsde}(C_m) = n - 1$
6. For any wheel $W_{1,n}$, $\gamma_{nsde}(W_{1,n}) = n - 1$
7. For any Helm graph H_n, $\gamma_{nsde}(H_n) = \begin{cases} n-1 \ if\ n \leq 4 \\ 2n\ if\ n \geq 4 \end{cases}$
8. For any bistar graph $B_{m,n}$, $\gamma_{nsde}(B_{m,n}) = \begin{cases} m+n+1\ \ if\ |m-n| \leq 1 \\ not\ \ exists\ \ if |m-n| \geq 1 \end{cases}$
9. For any crown graph $C_n{}^\circ P_2$, $\gamma_{nsde}(C_n{}^\circ P_2) = 2n - 2$
10. For Ladder graph, $\gamma_{nsde}(L_3) = 2n - 2$
11. For any comb graph $P_n{}^\circ P_2$, $\gamma_{nsde}(P_n{}^\circ P_2) = 2n - 2$
12. For any book graph, $B_n, \gamma_{nsde}(B_n) = n - 1$
13. For any Fan graph $F_{1,n}, \gamma_{nsde}(F_{1,n}) = n - 1$

Bounds of non-split duplex equitable domination number

Here some upper bounds are obtained for non-split duplex equitable domination number

Theorem: 2.1 Let G be a graph. Then $2 \leq \gamma_{nsd}(G) \leq n$ and the bound is exact

Proof: We know that duplex equitable dominating set has at least two vertices and at most n vertices, $2 \leq \gamma_{nsde}(G) \leq n$

Remark:

1. For any complete graph K_n, $\gamma_{nsde}(K_n) = 2$
2. For any star $K(K_{1,n-1})$, $\gamma_{nsde}(K_{1,n-1}) = n$

Theorem: 2.2 Let G be a graph. Then $(G)\ \gamma_c(G) \leq \gamma_t(G) \leq \gamma_e(G) \leq \gamma_{de}(G) \leq \gamma_{nsde}(G) \leq \gamma_{[1,2]tc}(G)$

Proof: Since every non-split duplex equitable domination set is a duplex domination set and each duplex equitable domination set is a domination set, $\gamma(G) \leq \gamma_c(G) \leq \gamma_t(G) \leq \gamma_e(G) \leq \gamma_{de}(G) \leq \gamma_{nsde}(G) \leq \gamma_{[1,2]tc}(G)$

Note 2.3 $\boldsymbol{Since}$ $\left\lceil\frac{n}{\Delta+1}\right\rceil \leq \boldsymbol{\gamma(G)}$ and by Theorem 2.2, it is clear that $\left\lceil\frac{n}{\Delta+1}\right\rceil \leq \boldsymbol{\gamma_{nsde}(G)}$.

Observation:2.4 Every duplex dominating set does not have to be a nonsplit duplex equitable domination set.

Theorem: 2.5 Every non-split duplex equitable dominating set consists of all pendent vertices in a graph.

Proof: Consider D is the non-split duplex equitable dominating set. Let v be a pendent vertex with support vertex u. Suppose $v \notin D$, then $v \in V - D$. Since D is non-split duplex equitable domination set, for each $v \in V - D$, ∃ two vertices u_1, $u_2 \in D$ such that $u_1 v$ and $u_2 v \in E(G)$ and $|\deg(u_1) - \deg(v)| \leq 1$ and $\langle V - D \rangle$ is connected. Therefore u_1 and u_2 are adjacent to v. So $\deg(v) \geq 2$, this contradicts the fact that the vertex v is pendent. So $v \in D$.

Observation:2.6 Every support vertex of a pendent vertex need not be in a non-split duplex equitable domination set.

In figure, $\gamma_{nsd}(G) = 4$. Here H= $\{v_1, v_2, v_4, v_5\}$ is a non-split duplex equitable dominating set such that $supp(v_3) \notin H$.

Remark2.7 If H is a spanning subgraph of a graph G such that $(H) \subseteq E(G)$, then $\gamma_{nsde}(G) \leq \gamma_{nsde}(H)$

Theorem :2.8 Let G= C_n ($n \geq 5$) and H be a connected spanning subgraph H of G. Then $\gamma_{nsd}(G) = \gamma_{nsde}(H)$.

Proof: We have $\gamma_{nsd}(C_n) = n - 1$, $n \geq 5$. We know that connected spanning sub graph H of C_n is the path and $\gamma_{nsde}(P_n) = n - 1$. Therefore $\langle H \rangle = P_n$ and $\gamma_{nsd}(H) = n - 1$. Hence $\gamma_{nsde}(G) = \gamma_{nsde}(H)$.

Theorem :2.9 For any graph G, then $\left\lceil\frac{2n}{\Delta+2}\right\rceil \leq \boldsymbol{\gamma_{nsde}(G)}$. The bound is sharp.

Proof: Since each vertex in $V - D$ takepart two in degree sum of D, $2|V - D| \leq \sum_{u \in D} \deg(u)$ where D is non split duplex equitable dominating set. Hence $2|V - D| \leq \gamma_{nsde}.\Delta$ which implies $2(|V| - |D|) \leq \gamma_{nsde}.\Delta$. Thus $(2n - 2)\,\gamma_{nsde} \leq \gamma_{nsde}.\Delta$. which implies $2n \leq \gamma_{nsde}(\Delta + 2)$. Hence $\left\lceil\frac{2n}{\Delta+2}\right\rceil \leq \gamma_{nsde}(G)$.

For K_4, $\gamma_{nsde}(K_4) = 2 \gamma_{nsde}(K_4) = \left\lceil\frac{2n}{\Delta+2}\right\rceil = \left\lceil\frac{2(4)}{3+2}\right\rceil$

Remark:2. 10 Let G be a graph and there exists a non-split duplex domination set which is not independent. Then $\gamma(G) + 1 \leq \gamma_{nsde}(G)$ and the bound is sharp.

It has a sharp bound for all K_n, $\gamma(K_n) = 1$ and $\gamma_{nsde}(K_n) = 2$.

Observation :2. 11 Let G be a graph. Suppose there is a non-split duplex dominating set which is independent. Then $\gamma(G) \leq \gamma_{nsde}(G)$ and its bound is acute

For helm graph H_4, $\gamma(H_4) = 5$ and $\gamma_{nsde}(H_4) = 5$.

Relation between non-split duplex equitable dominating set $\boldsymbol{\gamma_{nsde}}$ and Connectivity

Theorem 3.1 Let G be a graph. Then $\boldsymbol{\gamma_{nsde}(G) + \kappa(G) \leq 2n - 1}$ and equality holds if and only if G $\cong \boldsymbol{K_2}$

Proof: We know that $\gamma_{nsde}(G) \leq n$ and $\kappa(G) \leq n - 1$ then $\gamma_{nsde}(G) + \kappa(G) \leq n + n - 1 = 2n - 1$. Suppose $\gamma_{nsde}(G) + \kappa(G) = 2n - 1$ then $\gamma_{nsde}(G) = n$ and $\kappa(G) = n - 1$. Since $\gamma_{nsde}(G) = n$, G is a star and $\kappa(G) = n - 1$. G is complete graph and G $\cong K_2$.

Conversely $\gamma_{nsde}(K_2) = 2$ and $\kappa(K_2) = 1$ then $\gamma_{nsde}(G) + \kappa(G) = 3 = 2n - 1$.

Theorem :3.2 Let G be a graph. Then $\gamma_{nsde}(G) + \kappa(G) = 2n - 2$ if and only if G $\cong K_3$.

Proof: Suppose $\gamma_{nsde}(G) + \kappa(G) = 2n - 2$.

Then there exists $\gamma_{nsde}(G) = n - 1$ and $\kappa(G) = n - 1$

Since $\kappa(G) = n - 1$, G $\cong K_n$. For K_n, $\gamma_{nsde}(K_n) = 2$ which gives $n = 3$ then $G \cong K_3$

Conversely, $\gamma_{nsde}(K_3) = 2$ and $\kappa(K_3) = 2$ then $\gamma_{nsde}(G) + \kappa(G) = 4 = 2n - 2$.

Theorem :3.3 If G is a graph, then $\gamma_{nsde}(G) + \kappa(G) = 2n - 3$ iff G $\cong K_4$ or $K_{1,3}$ *or* C_4 or $K_4 - e$.

Proof: Suppose $\gamma_{nsde}(G) + \kappa(G) = 2n - 3$ then there exists three cases

(i) $\gamma_{nsde}(G) = n$ and $\kappa(G) = n - 3$

(ii) $\gamma_{nsde}(G) = n - 2$ and $\kappa(G) = n - 1$

(iii) $\gamma_{nsde}(G) = n - 1$ and $\kappa(G) = n - 2$

Case(i) $\gamma_{nsde}(G) = n$ and $\kappa(G) = n - 3$

Since $\gamma_{nsde}(G) = n$, G is a star ($K_{1,n}$) and $\kappa(K_{1,n}) = 1$ which gives $n = 4$ then $G \cong K_{1,3}$

Case(ii) $\gamma_{nsde}(G) = n - 2$ and $\kappa(G) = n - 1$

Since $\kappa(G) = n - 1$, G is a complete graph K_n and $\gamma_{nsde}(K_n) = 2$ which gives $n = 4$ then $G \cong K_4$.

Case(iii) $\gamma_{nsde}(G) = n - 1$ and $\kappa(G) = n - 2$
Since $\kappa(G) = n - 2, n - 2 \leq \delta(G)$. If $\delta(G) = n - 1$ then $G \cong K_n$ which is contrary to the assumption.If $\delta(G) = n - 2$, then $G \cong K_n - Q$ where Q is a matching in K_n. Then $\gamma_{nsde}(G) \leq 3$. Suppose $\gamma_{nsde}(G) = 3$. Then $n = 4$ and G is isomorphic to either C_4 or $K_4 - e$ .Suppose $\gamma_{nsde}(G) = 2$. Then $n = 3$ and hence $G \cong K_{1,3}$ which is a contradiction.
Conversely
If $G \cong K_{1,3}$ then $\gamma_{nsde}(K_{1,3}) = 4$ and $\kappa(K_{1,3}) = 1$ and hence $\gamma_{nsde}(G) + \kappa(G) = 5 = 2n - 3$
If $G \cong K_4$ then $\gamma_{nsde}(K_4) = 2$ and $\kappa(K_4) = 3$ and hence $\gamma_{nsde}(G) + \kappa(G) = 5 = 2n - 3$
If $G \cong C_4$ then $\gamma_{nsde}(C_4) = 3$ and $\kappa(C_4) = 2$ and hence $\gamma_{nsde}(G) + \kappa(G) = 5 = 2n - 3$

Theorem :3.4 If G is a graph, then $\gamma_{nsde}(G) + \kappa(G) = 2n - 4$ iff $G \cong K_5$ or $K_{1,4}$ or C_5
Proof: Suppose $\gamma_{nsde}(G) + \kappa(G) = 2n - 4$ then there exists four cases
(i) $\gamma_{nsde}(G) = n$ and $\kappa(G) = n - 4$
(ii) $\gamma_{nsde}(G) = n - 1$ and $\kappa(G) = n - 3$
(iii) $\gamma_{nsde}(G) = n - 2$ and $\kappa(G) = n - 2$
(iv) $\gamma_{nsde}(G) = n - 3$ and $\kappa(G)$ $n - 1$

Case (i) $\gamma_{nsde}(G) = n$ and $\kappa(G) = n - 4$
Since $\gamma_{nsde}(G) = n$, G is a star $K_{1,n}$ and $\kappa(K_{1,n}) = 1$ which gives $n = 5$then $G \cong K_{1,4}$

Case(ii)$\gamma_{nsde}(G) = n - 1$ and $\kappa(G) = n - 3$
Since $\kappa(G) = n - 3, n - 3 \leq \delta(G)$. If $\delta(G) = n - 1$ then $G \cong K_n$, which is opposition. If $\delta(G) = n - 2$ then $G \cong K_n - Q$ where Q is a matching in K_n. Then $\gamma_{nsde}(G) \leq 4$. Suppose $\gamma_{nsde}(G) = 4$. Then $n = 5$ and hence G is isomorphic to either C_5 .

Case(iii) $\gamma_{nsde}(G) = n - 2$ and $\kappa(G) = n - 2$
Since $\kappa(G) = n - 2, n - 2 \leq \delta(G)$. If $\delta(G) = n - 1$ then $G \cong K_n$,which is contrary to the statement. If $\delta(G) = n - 2$ then $G \cong K_n - Q$ where Q is a matching in K_n . Then $\gamma_{nsde}(G) \leq 3$. Suppose $\gamma_{nsde}(G) = 3$. Then $n = 5$ and hence $G \cong K_5 - Q$. Suppose $|Q|=1$ then $G \cong K_5 - Q$ where Q is a matching in K_5.

Case(iv)$\gamma_{nsde}(G) = n - 3$ and $\kappa(G) = n - 1$
Since $\kappa(G) = n - 1, G \cong K_n$. But $\gamma_{nsde}(G) = 2$ then $n = 5$ and hence $G \cong K_5$
Conversely
If $G \cong K_{1,4}$ then $\gamma_{nsde}(K_{1,4}) = 5$ and $\kappa(K_{1,4}) = 1$ and hence $\gamma_{nsde}(G) + \kappa(G) = 6 = 2n - 4$
If $G \cong K_5$ then $\gamma_{nsde}(K_5) = 2$ and $\kappa(K_5) = 4$ and hence $\gamma_{nsde}(G) + \kappa(G) = 6 = 2n - 4$
If $G \cong C_5$ then $\gamma_{nsde}(C_5) = 4$ and $\kappa(C_5) = 2$ and hence $\gamma_{nsde}(G) + \kappa(G) = 6 = 2n - 4$.

Relation between γ_{nsde} and χ.

Theorem 4.1 Let G be a graph. Then $\gamma_{nsde}(G) + \chi(G) \leq 2n$ and equality holds iff $G \cong K_2$
Proof : We know that $\gamma_{nsde}(G) \leq n$ and $\chi(G) \leq \Delta - 1$ then $\gamma_{nsde}(G) + \chi(G) \leq n + \Delta + 1 \leq n + n - 1 + 1 = 2n$. Now we assume that $\gamma_{nsde}(G) + \chi(G) = 2n$. This is possible only if $\gamma_{nsde}(G) = n$ and $\chi(G) = n$. Since $\chi(G) = n$, then G is a complete graph. But for K_n, $\gamma_{nsde}(G) = 2$. Thus $G \cong K_2$. The reverse statement is trivial.

Theorem 4.2 Let G be a graph. Then $\gamma_{nsde}(G) + \chi(G) = 2n - 1$ iff $G \cong K_3$.
Proof If G is K_3 , then $\gamma_{nsde}(G) + \chi(G) = 2n - 1$.
Conversely, assume that $\gamma_{nsde}(G) + \chi(G) = 2n - 1$. This is possible only if
$\gamma_{nsde}(G) = n - 1$ and $\chi(G) = n$. Thus G is complete. But for K_n , Since $\gamma_{nsde}(G) = 2$, and $\gamma_{nsde}(G) = n - 1$. This implies $n = 3$ and so $G \cong K_3$.

Theorem 4.3 If G is a graph, then $\gamma_{nsde}(G) + \chi(G) = 2n - 2$ iff $G \cong K_{1,3}$, $K_3(P_2)$, K_4.
Proof Suppose G is isomorphic to one of the following graphs: $K_{1,3}$, $K_3(P_2)$, K_4, then clearly $\gamma_{nsde}(G) + \chi(G) = 2n - 2$.
Conversely, assume that $\gamma_{nsde}(G) + \chi(G) = 2n - 2$. Then we have the following cases:
(i) $\gamma_{nsde}(G) = n$; $\chi(G) = n - 2$
(ii) $\gamma_{nsde}(G) = n - 1$; $\chi(G) = n - 1$
(iii) $\gamma_{nsde}(G) = n - 2$; $\chi(G) = n$

Case (i) $\gamma_{nsde}(G) = n$ and $\chi(G) = n - 2$.
Since $\gamma_{nsde}(G) = n$, G is a star graph. So $n = 4$. The only possibilities of G is $K_{1,3}$. We encounter an contradiction as the degree increases.

Case (ii)$\gamma_{nsde}(G) = n - 1$&$\chi(G) = n - 1$.
Then G has a clique K on $n - 1$ vertices. Take $D = \{v\}$ is the vertex apart from the vertices of the clique K_{n-1}.Then v is adjacent to u_i for some i in K_{n-1}. Now $\{v_1, u_i, u_j\}$ is a non-split duplex equitable dominating set. Hence $n = 3$. Therefore $K = K_3$.
If $\deg(v_1) = 1$ then $G \cong K_3(P_2)$. There is no graph by increasing the degree of v_1.

Case (iii) $\gamma_{nsde}(G) = n - 2$; $\chi(G) = n$.
Since $\chi(G) = n$, $G \cong K_n$. But for K_n, $\gamma_{nsde}(G) = 2$. Thus $n = 4$ and $G \cong K_4$.

Theorem 4.4 If G is a graph, then$\gamma_{nsde}(G) + \chi(G) = 2n - 3$ iff $G \cong K_{1,4}$, $K_3(P_3)$,$K_3(P_2, P_2, 0)$,K_5,$K_4(P_2)$.
Proof: Suppose that$\gamma_{nsde}(G) + \chi(G) = 2n - 3$. This is possible only if
(i) $\gamma_{nsde}(G) = n$ and $\chi(G) = n - 3$
(ii) $\gamma_{nsde}(G) = n - 1$and $\chi(G) = n - 2$
(iii) $\gamma_{nsde}(G) = n - 2$ and $\chi(G) = n - 1$

(iv) $\gamma_{nsde}(G)= n-3$and χ(G) = n

Case (i) If$\gamma_{nsde}(G) = n$and χ(G) = $n-3$.
Since $\gamma_{nsde}(G) = n$, G is a star. Therefore $n = 5$. Hence G ≅ $K_{1,4}$. We encounter an contradiction as the degree increases.

Case (ii)$\gamma_{nsde}(G) = n-1$ and χ (G) = $n-2$.
Then G has a complete graph K on $n-2$ vertices. Take $D = \{v_1, v_2\}$ is the vertex apart from the vertices of the K. Then the follow-up cases are $< D >= K_2$or $\overline{K_2}$.

Suppose $< D >= K_2$. Since G is a connected graph, either v_1 or v_2 is adjacent to u_i in $K_{n-2}$for some i. Then $\{v_1, v_2, u_i, u_j\}$ is a non-split duplex equitable dominating set so that n=5. Hence G=K_3. If deg$(v_1) = 2$ and deg$(v_2) = 1$, then G ≅ K_3 (P_3). No such graphs exists by increasing the degree.
Suppose $< D >= \overline{K_2}$. Since G is connected, either v_1 or v_2 is adjacent to $u_i$then G ≅ K_3 $(2P_2)$.
If v_1 is adjacent to u_i and v_2 is adjacent to u_j then G ≅ $K_3(P_2, P_2, 0)$. No such graphs exists by increasing the degree.

Case (iii) $\gamma_{nsde}(G) = n-2$ &χ(G) = $n-1$.
Since χ(G) = $n-1$, G has a complete graph K on n-1 vertices. Take D={v} is the vertex apart from the vertices of the K. If v is adjacent to u_i for some in K_{n-1}, then $\gamma_{nsde}(G) = 3$. So $n = 4$. Thus K=K_4. If deg(v)=1, then G ≅ K_4 (P_2). No such graphs exists by increasing the degree.

Case(iv)$\gamma_{nsde}(G) = n-3$&χ(G) = n
Then G is a complete graph and $\gamma_{nsde}(G) = n-3$ so that $n = 5$. Hence G ≅ K_5 Conversely, If G is isomorphic to anyone of the graph G : $K_{1,4}$, K_3 (P_3), $K_4(P_2)$, $K_3(P_2, P_2, 0)$, K_5 , then clearly$\gamma_{nsde}(G)$ + χ (G) = $2n-3$

Theorem 4.5 If G is a graph, then $\gamma_{nsde}(G)$ + χ (G) = $2n-4$iff G≅ $K_{1,5}$, $K_4(P_3)$, $C_4(P_2)$, $K_3(P_2, P_2, P_2)$, $K_3(2P_2, P_2, 0)$, K_4 $(2P_2)$,$K_4(P_2, 2P_2, 0,0)$, K_5 (P_2),K_6

Proof: Assume that $\gamma_{nsde}(G)$ + χ (G) = $2n-4$. This is possible for only in the following cases
$(i)\gamma_{nsde}(G) = n$ and $\chi(G) = n-4$
(ii) $\gamma_{nsde}(G) = n-1$ and $\chi(G) = n-3$
(iii) $\gamma_{nsde}(G) = n-2$ and $\chi(G) = n-2$
(iv)$\gamma_{nsde}(G) = n-3$ and $\chi(G) = n-1$
(v) $\gamma_{nsde}(G) = n-4$and$\chi(G) = n$

Case (i)$\gamma_{nsde}(G) = n$ and $\chi(G) = n-4$
Since $\gamma_{nsde}(G) = n$, G is a star and $\chi(G) = n-4$. Thus n=6 and G ≅ $K_{1,5}$.
We encounter an contradiction as the degree increases.

Case (ii)$\gamma_{nsde}(G) = n-1$ and $\chi(G) = n-3$
Then G has a complete graph K on $n-3$ vertices. Take $D = \{v_1, v_2, v_3\}$is the **set of** vertices apart from the vertices of the clique K_{n-3}. Then $< D >= P_3, K_3, \overline{K_3}, K_2 \cup K_1$

Subcase (i)$< D >= P_3$
Since G is a connected graph, we have the only possible cases are
(i) there exists a vertex u_i in K_{n-3} which is adjacent to any one of the end vertices
(ii) there exists a vertex u_i in K_{n-3} which is adjacent other than end vertices. If exists a vertex u_i in K_{n-3} which is adjacent any one of the end vertices then $\gamma_{nsde}(G) = 5$. Hence $n = 6$. Thus G ≅ K_3. If deg(v_1) 2, deg$(v_2) = 1$ & deg$(v_3) = 1$, then G ≅ K_3 (P_4)

Sub case(ii)$< D >= K_3$
Since G is a connected graph, there exists a vertex u_i in K_{n-3} adjacent any one of $\{v_1, v_2, v_3\}$. Clearly v_1 is adjacent to u_i, then $\gamma_{nsde}(G) = 5$. Therefore G ≅ K_3. We arrive a contradiction when increasing degree.

Subcase(iii)$< D >= \overline{K_3}$
Since G is a connected graph, let $u_i$be adjacent to all of the vertices of $\overline{K_3}$. Then $\gamma_{nsde}(G) = 5$. Hence $n = 6$ and G contains K_3. Let $V(K_3) = \{u_1, u_2, u_3\}$ .Suppose all of the vertices of $\overline{K_3}$ is adjacent to u_i. Then G ≅ K_3 $(3P_2)$. On increasing degree, we arriveda contrary. If any two vertices of $\overline{K_3}$ is adjacent to u_i and third vertex is adjacent to u_j , $i \neq j$. Then $\gamma_{nsde}(G) = 5$. Hence $n = 6$. Therefore K=K_3. Let $V(K_3) = \{u_1, u_2, u_3\}$. Then G ≅ K_3 $(2P_2, P_2, 0)$. On increasing degree, we arrived a contrary. If each of the vertices of $\overline{K_3}$ are adjacent to three distinct vertices of K_{n-3}.Let it be $u_i, u_j, u_k$for ,$i \neq j \neq k$. Then $\gamma_{nsde}(G) = 5$. Hence $n = 6$ and K=K_3. Take $V(K_3) = \{u_1, u_2, u_3\}$. Then G ≅ K_3 (P_2, P_2, P_2). On increasing degree, we arrived a opposition.

Case (iii) $\gamma_{nsde}(G) = n-2$ and $\chi(G) = n-2$
Then G has a clique K on $n-2$ vertices. Let $D = \{v_1, v_2, v_3, v_4\}$ be the set of vertices other than the clique K_{n-2}. Then $< D >= K_2, \overline{K_2}$

Subcase (i)$< D >= K_2$
As G is a connected graph, $v_1, v_2$are adjacent some u_i for some i in K_{n-2}. Then $\gamma_{nsde}(G) = 4$ so that $n = 6$ andK=K_4. Take $V(K_3) = \{u_1, u_2, u_3\}$. Then G ≅ K_4 (P_3). We arrive at a contradiction with increasing degree.

Subcase (ii)$< D >= \overline{K_2}$
As G is connected, both v_1 and v_2 is adjacent some u_i for some i in K_{n-2}. Then $\gamma_{nsde}(G) = 4$ so that $n = 6$ and K= K_4. Take $V(K_4) = \{u_1, u_2, u_3, u_4\}$. Then G ≅ K_4 (2). We arrive at a contradiction with

increasing degree. If each two vertices are adjacent to two distinct vertices of K_{n-2}, then $\gamma_{nsde}(G) = 4$ so that $n = 6$. Therefore K= K_4 . Hence G ≅ $K_4(P, P_2, 0,0)$. We arrive at a contradiction with increasing degree.

Case (iv) $\gamma_{nsde}(G) = n - 3$ and $\chi(G) = n - 1$
Then G has a clique k on $n - 1$ vertices. Let the vertex v_i be adjacent to u_i for some i in K_{n-1} . Therefore $\gamma_{nsde}(G) = n - 3$ and n=6. Thuse K=K_5&G ≅ K_5 (P_2).We arrive at a contradiction with increasing degree.

Case(v) $\gamma_{nsde}(G) = n - 4$ and $\chi(G) = n$
Since $\chi(G) = n$, then G ≅ K_n but for K_n, $\gamma_{nsde}(K_n) = 2$ so that n=6 . Therefore G ≅ K_6
Conversely, if G ≅ $K_{1,5}$, K_4 (P_3), $C_4(P_2)$, $K_3(P_2, P_2, P_2$), $K_3(2P_2, P_2, 0$), K_4 $(2P_2)$,$K_4(P_2, 2P_2, 0,0)$, K_5 (P_2),K_6
then clearly $\gamma_{nsde}(G)$ + χ (G) = $2n - 4$.

Observation: 4.6 In a graph if the non split duplex equitable domination number γ_{nsde} and the chromatic number χ are same then G ≅ $K_n(nP_n)$.

3. Conclusion

In this paper, the non split duplex equitable number is defined and determined for some specific graphs some upper bounds are also investigated, Further we like to process this research work for some other graphs and we investigate the bounds and applications of the non split duplex equitable domination numbers.

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