



SOLVING ASSIGNMENT PROBLEM USING COMPLETELY RANDOMISED DESIGN, RANDOMISED BLOCK DESIGN, LATIN SQUARE AND GAME THEORY: A COMPARATIVE APPROACH WITH R STUDIO CODING

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Abstract

Assignment problem can also be applied in statistical techniques. Completely Randomized design is a very useful model in statistics. Many complex experimental situations can often be considered as completely Randomized design. A Randomized block design tests the effect of two independent variables on a dependent variable. A Randomized block design test analyzes the effect of the independent variables on the expected outcome along with their relationship to the outcome itself. Latin square is the square matrix are also solved in this paper. Agricultural experiment for certain area of land to solve a problem dealing with the different varieties of a particular paddy the method of block and treatment is used. Comparing with these types of problem which are solved in completely Randomized design, Randomized block design, Latin square, Assignment problem and Game theory, then we get the optimal solution by a new approach. Also the methods are solved using R language.

Keywords: Assignment problem, Completely Randomized, Randomized blocks design, Latin square, Game Theory, new-proposed using R.

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I INTRODUCTION

Assignment problem is selected having Lower cost and highest profit in [1,2,3,4,5] (time of distance) assigned to the j-th job. The problem is to find an assignment (which job should be assigned to which person one on-one basis) So that the total cost of performing [6,7,8,9]all jobs is minimum, problem of this kind are known as assignment problem. There is a technique that will enable us to test for the significance of the difference among than two sample means. There is a difference in average yield by fertilizer type and planting density. Whether the effects of some fertilizers on the yields are significantly different. which of various training methods produces the fastest learning record, that is swayam , NPTEL. Each of the samples is drawn from a normal population. The variation of each value around its own grand mean should be independent for each value. Agricultural research is the one which gave rise to the[6,7,10] development of theory and practice of experimental design. We compare the Assignment problem with different approaches and the results are tested using in R- Language. [11] if it is possible then we solve in Matlab. Agricultural research is the one which gave rise to

the development of theory and practice of experimental design. Credit goes to R.A Fisher for starting and solving these problems and creating a new branch of science from which experimentation in many fields of research has since been benefited. Important examples of experimental design are to Completely randomised design, Randomised block design , Latin square design. Now comparing the problem based on complete randomised, Randomised block design and Latin square design with assignment problem to testing the hypothesis. If it is possible to solve in statistics based on Completely randomised design, Randomized block design, Latin square design and Game Theory. Then it is good comparison in assignment problem. In future we can solve problem in Assignment Problem in statistics based on Completely Randomized design, Randomized blocks design Latin square and Game Theory.

II Mathematical Formulation of Assignment Problem

Definition 1. Assignment Problem [4,12] can be expressed as an $n \times n$ cost matrix with C real elements as shown in the below table:

Table 1.1 Approach of Assignment Problem

		<i>chemical substance</i>						<i>Available</i>	
		1	2	3	·	<i>j</i>	·	<i>n</i>	
<i>duplication</i>	1	p_{11}	p_{12}	p_{13}	·	p_{1j}	·	p_{1n}	1
	2	p_{21}	p_{22}	p_{23}	·	p_{2j}	·	p_{2n}	1
	3	p_{31}	p_{32}	p_{33}	·	p_{3j}	·	p_{3n}	1
	·								
	<i>i</i>	p_{i1}	p_{i2}	p_{i3}	·	p_{ij}	·	p_{in}	1
	<i>n</i>	p_{n1}	p_{n2}	p_{n3}	·	p_{nj}	·	p_{nn}	1
<i>required</i>	1	1	1		1		1		

III Different approaches utilized to work out Assignment problems:

Apply different approaches utilized to work out Assignment problems[15] are connected with completely Randomised design, Randomised block design, Latin square design and Game Theory [16,17,18,19,20] but differ from both. This novel-strategy offer a continuous approach in Assignment problem. This different approaches utilized in the distribution of the outcomes are correlated in Completely Randomized design, Randomized blocks design , Latin square and Game Theory. Here, we Frame the assignment matrix[2,13] where p_{ij} is the charge of passing on i^{th} job to j^{th} appliance.

3.1 Proposed Method: Assignment problem is solved by the method of Hungarian[14].

3.2 Design of experiments

Suppose there is an agricultural experiment for a certain area of land. Let there be four different chemical treatments of soil. These different chemical treatments may produce different yields. IN order to obtain as much information as possible the details of an experiment must be carefully planned in advance. This planning is referred to as the design of experiment. To design such as experiment , we may divide the land into, say 25 plots (5X5) indicated by squares although physically any shape can be used. Assign each treatment, denoted by L,M,N,O,P to five blocks eliminate various sources of error such as fertility of the soil. This is indicated by the following

Table1.2 Complete randomization design

K	M	O	N	K
N	L	M	L	O
O	K	N	O	M
L	N	K	M	N
M	O	L	M	K

3.3 Randomised blocks design

Consider an agricultural experiment consisting of examining the yields per hectare of 5 different varieties of a particular crop, say paddy, where each variety is grown on 5 different plots of land. Then a total of $5 \times 5 = 25$ plots are needed. It is in fact, convenient in such a case to combine plots into blocks say 5 plots to a block. This is represented in the following

Table1.3 Randomized blocks design

Blocks					
I	K	L	N	M	O
II	O	K	L	N	M
III	M	O	K	L	N
IV	N	M	O	K	L
V	L	N	M	O	K

3.4 Plan of an experiment:

To plan an experiment the following three are essential. A statement of the objective. Statement should clearly mention the hypothesis to be tested. A description of the experiment. Description should include the type of experimental material, size of the experiment and the number of replications. The outline of the method of analysis. The outline of method consists of analysis of variance.

3.5 Condition of the design of experiment:

Experimental design methods can play a major role in engineering design activities, where new products are developed and the existing ones improved. Some applications of statistical experimental design in engineering include: Evaluation and comparison of basic design configuration. Evaluation of material alternatives. Determination of key product design parameters that affect performance.

Use of experimental design in these areas can result in improved manufacturability of the product, enhanced field performance and reliability, lower product cost and shorter product development time.

3.6 Basic principles of design of experiments:

Replication: It is the repetition of the experimental situation by replicating the experimental. Randomised : It is a technique by which we can randomly assign the treatments to the experimental units. The purpose of randomness is

to ensure that no particular treatment is unnecessarily favorable or handicapped.

3.7 Analysis of variance (ANOVA):

Analysis of Variance is a technique that will enable us to test for the significance of the difference among more than two samples means. Analysis of variance is useful, for example, (i) which of various training methods produces the fastest learning record. (ii) whether the effects of some fertilizers on the yields are significantly different. (iii) whether the mean qualities of outputs of various machines differs significantly etc., In fact this technique finds application in nearly every type of experimental design in natural sciences as well as in social sciences. In general, analysis of variance studies mainly homogeneity of populations by separating the total variance into its various components. According to R.A. Fisher. The separation of variance ascribable to one group of causes from the various ascribable to other group.”

3.8 Assumptions in ANOVA:

In order to use ANOVA we make the following assumptions. The samples are drawn from normal populations. The samples are independently drawn from these populations. All the populations have the same variance.

- (i) Completely Randomized design (One-way classification): In one way classification the observations or experimental units are classified according to one factor of interest.
- (ii) Randomized block design (Two-way classification): In two-way classification of analysis of variance we consider one classification along column-wise and the other along row-wise.
- (iii) Latin Square design: Latin square design have an equal number of rows & columns one blocking factor is represented in the column of the other in rows.

IV. NUMERICAL ILLUSTRATIONS:

In this section, we provide numerical examples to illustrate the proposed algorithm:

Problem based on Assignment problem, Completely randomized design, Randomized block design, Latin square and game theory these are the types of algorithm are solved in this paper. We can take a problem solved on varieties type of condition solved in the new technique. First we can testing the problem with assignment problem that is different between paddy and yields. In the technique we can consider the paddy as the row operation, rearranging the order as K, L, M, N, O

then we can solve the problem based on Hungarian method in assignment problem. we can find the result of the optimal cost. Testing the cost is less than or greater than then we comparing with other varieties types. Second we can find result for completely randomized design type based on duplication (paddy) that is row (L,M,N,O,P) operation. Five varieties L,M,N,O,P of a chemical substance are tested in with a five duplication the ground gives in pound. Analyse the experimental yield. To find the result for 1%,5% and 10% comparing the statistics table is agree or eliminated, Fourth type that is Latin square is square matrix we can solved problem. Fifth type we rearranged in K,L,M,N,O strategy of problem using game theory. Finally we checking the optimal cost is within the range.

4.1 Problems based on different approaches:

Five varieties K, L, M, N, O of a fertilizer are tested Assignment problem with 5 replication cost. Find the yields in paddy of the cost are as follows. Paddy depends on rows and yields depends on columns.

Table: 1.4 different approaches

paddy	Yields				
	L90	O80	M134	K112	N92
	O85	N84	L70	M141	K82
	M110	K90	N87	L84	O69
	K81	M125	O85	N76	L72
	N82	L60	K94	O85	M88

First we can changing the paddy order that is first row K, second row L, third row M, fourth row N, Fifth row O. we can rearrange order into K, L,M,N & O solved problem based on Assignment problem.

Solution:

Table: 1.5 Optimal cost

K	81	90	94	112	82
L	90	60	70	84	72
M	110	125	134	141	88
N	82	84	87	76	92
O	85	80	85	85	69

Step:1 Optimal Assignment

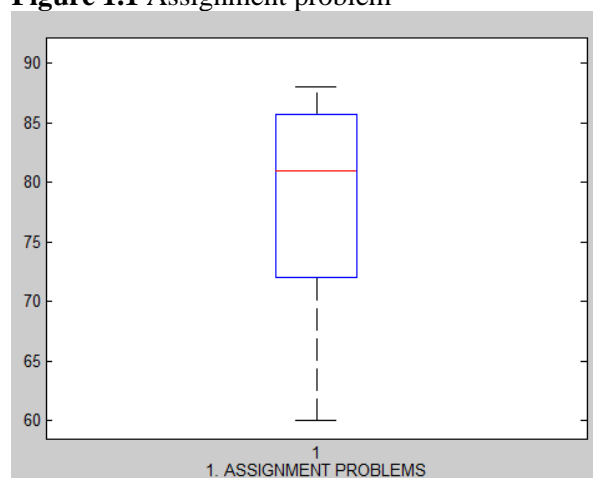
K	[0]	9	3	31	7
L	30	[0]	0	24	18
M	16	31	30	47	[0]
N	6	8	1	[0]	22
O	10	5	[0]	10	0

Optimal solution problem for paddy K→81,L→60,M→88,N→76,O→85. Total =390costs.

Result:

Based on the Assignment problem we plot the graph using R studio in the Figure 1.1.

Figure 1.1 Assignment problem



4.2 Problems based on different approaches. Five varieties L,M,N,O,P of a chemical substance (yields) are tested in with a five duplication the ground gives in pound. Observe the duplication (paddy).

Table : 1.6 chemical substance CRD

Duplication (paddy)	Chemical substance(yields)				
	M90	P80	N134	L112	O92
	P85	O84	M70	N141	L82
	N110	L90	O87	M84	P69
	L81	N125	P85	O76	M72
	O82	M60	L94	P85	N88

Solution: **H₀**: Testing the condition is satisfied
H₁: Testing the condition is not satisfied

Table: 1.7 Rearrange CRD

Duplication (Paddy)	Chemical substance(yields)					(Y1) ²	(Y2) ²	(Y3) ²	(Y4) ²	(Y5) ²
	Y1	Y2	Y3	Y4	Y5					
L	81	90	94	112	82	6561	8100	8836	12544	6724
M	90	60	70	84	72	8100	3600	4900	7056	5184
N	110	125	134	141	88	12100	15625	17956	19881	7744
O	82	84	87	76	92	6724	7056	7569	5776	8464
P	85	80	85	85	69	7225	6400	7225	7225	4761
total	$\sum y_1 = 448$	$\sum y_2 = 439$	$\sum y_3 = 470$	$\sum y_4 = 498$	$\sum y_5 = 403$	$\sum (Y1)^2 = 40710$	$\sum (Y2)^2 = 40781$	$\sum (Y3)^2 = 46486$	$\sum (Y4)^2 = 52482$	$\sum (Y5)^2 = 32877$

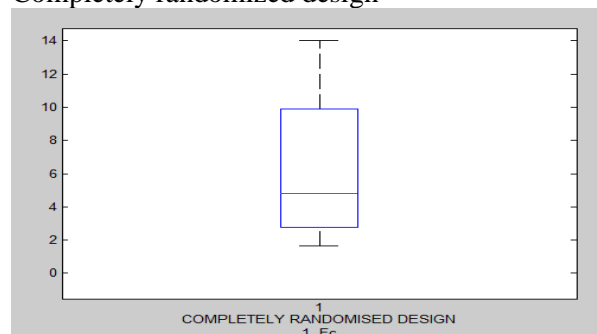
Table: 1.8 ANOVA table

Source of Variation	Sums of Squares SS	Degrees of freedom DF	Mean Squares MS	F	p-value		
					1%	5%	10%
Duplication (paddy)	SSC = 1005.04	C-1 = 4	MSC = 251.26	1.669	14.020	5.8025	3.844
Error	SSE = 8388.4	N-C = 20	MSE = 419.42				
Total	SST = 9393.44	N-1 = 24					

Result:

$F_c(20,4)$ at 1% level of significance is 14.020. Calculated value is $F=1.669$. So, H_0 is agree. Hence testing the condition is satisfied. $F_c(20,4)$ at 5% level of significance is 5.8025. Calculated value is $F=1.669$. So, H_0 is eliminated. Hence testing the condition is satisfied. $F_c(20,4)$ at 10% level of significance is 3.844. Calculated value is $F=1.669$. So, H_0 is eliminated. Hence testing the condition is satisfied. Result of F test is 1.669. Now comparing with 1% ,5% and 10% with the level of significance we can get the value is 14.020, 5.8025 and 3.844. Therefore calculation value is greater than table value so we agree condition. Based on the Chemical substance & Duplication we plot the graph using R studio for completely randomized design in the Figure 1.2.

Figure 1.2 Completely randomized design



4.3 Problems based on different approaches: Five varieties L,M,N,O,P of a chemical substance are tested in with a five duplication the ground gives in pound. Analyze the experimental yield.

Table:1.9 chemical substance for RBD

Chemical substance(yields)					
Duplication(paddy)	L90	O80	M134	K112	N92
	O85	N84	L70	M141	K82
	M110	K90	N87	L84	O69
	K81	M125	O85	N76	L72
	N82	L60	K94	O85	M88

Solution: H_0 : Testing the condition is satisfied. H_1 : Testing the condition is not satisfied

Table: 1.10 Rearrange RBD

Duplication (paddy)	Chemical substance(yields)					Row total	$(Y1)^2$	$(Y2)^2$	$(Y3)^2$	$(Y4)^2$	$(Y5)^2$
	Y1	Y2	Y3	Y4	Y5						
L	81	90	94	112	82	$\sum L = 461$	6561	8100	8836	12544	6724
M	90	60	70	84	72	$\sum M = 376$	8100	3600	4900	7056	5184
N	110	125	134	141	88	$\sum N = 598$	12100	15625	17956	19881	7744
O	82	84	87	76	92	$\sum O = 421$	6724	7056	7569	5776	8464
P	85	80	85	85	69	$\sum P = 404$	7225	6400	7225	7225	4761
Column total	$\sum y_1 = 448$	$\sum y_2 = 439$	$\sum y_3 = 470$	$\sum y_4 = 498$	$\sum y_5 = 405$	2260	$\sum (Y1)^2 = 40710$	$\sum (Y2)^2 = 40781$	$\sum (Y3)^2 = 46486$	$\sum (Y4)^2 = 52482$	$\sum (Y5)^2 = 32877$

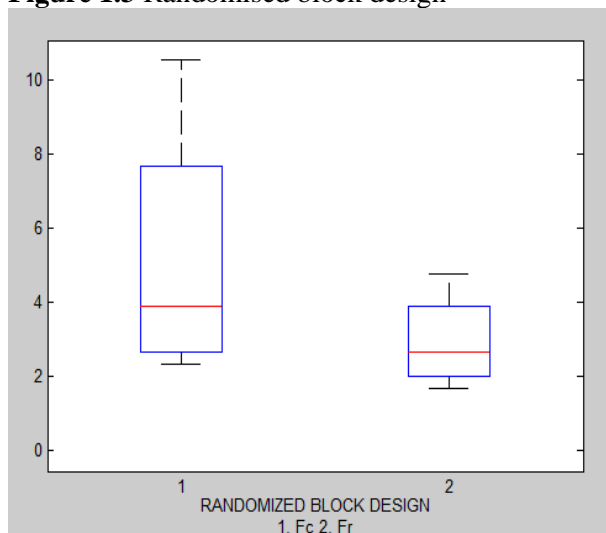
Table:1.11 ANOVA table

Source of Variation	Sums of Squares SS	Degrees of freedom DF	Mean Squares MS	F	p-value		
					1%	5%	10%
Between duplication	SSR=6087.6	r-1=4	MSR=6087.64 =1521.9	10.5431	4.773	3.0069	2.332
Between chemical substance	SSC=966.8	c-1=4	MSC=966.84 =241.7	1.6744	4.773	3.0069	2.332
Error (residual)	SSE=2309.6	(r-1)(c-1)=16	MSE=2309.61 / 6 =144.35				
Total	SST=9364	rc-1=24					

Result: Fr is greater than F tab at 1%, 5% and 10% level of significance is eliminated. Fc is greater than F tab at 1%, 5% and 10% level of significance is agree. Result of chemical substance is 1.6744 but compare with the optimal cost of assignment problem is 390units. Now we can get chemical substance as 1.6744 is the minimum cost.

Therefore Randomized block design is better that assignment problem at 1%, 5% and 10% level of significance in the chemical substance. Based on the Chemical substance & Duplication we plot the graph using R studio for Randomized block design in the Figure 1.3

Figure 1.3 Randomised block design



4.4 Problems based on different approaches: Five varieties L,M,N,O and P of a chemical substance are tested in with a five duplication the ground gives in pound. Following table are carryout an analysis of variance.

Table: 1.12 Latin square

L90	O80	M134	K112	N92
O85	N84	L70	M141	K82
M110	K90	N87	L84	O69
K81	M125	O85	N76	L72
N82	L60	K94	O85	M88

Solution: **Ho:** Testing the condition is satisfied. **H1:** Testing the condition is not satisfied Subtract the value to 120.

Table 1.13 reduced value

L-30	O-40	M14	K-8	N-28
O-35	N-36	L-50	M21	K-38
M-10	K-30	N-33	L-36	O-51
K-39	M5	O-35	N-44	L-48
N-38	L-60	K-26	O-35	M-32

Table: 1.16 Arrange the element treatment

	X1	X2	X3	X4	X5
K	-10	-20	34	12	-8
L	-15	-16	-30	41	-18
M	10	-10	-13	-16	-31
N	-19	25	-15	-24	-28
O	-18	-40	-6	-15	-12

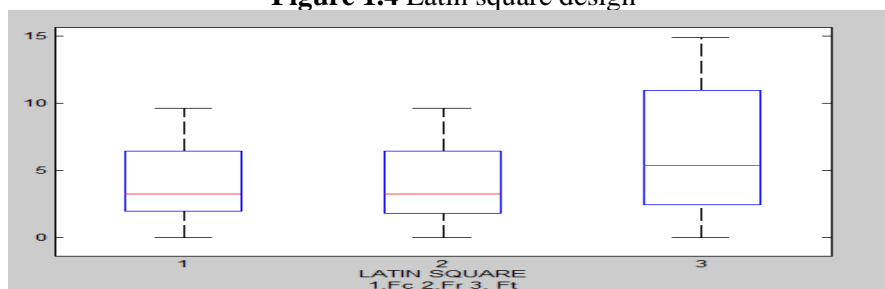
Table:1.15 ANOVA table

Source of Variation	Sums of Squares SS	Degrees of freedom DF	Mean Squares MS	F-test	p-value		
					1%	5%	10%
Between duplication	SSR=1079.44	r-1=4	MSR=269.86	2.64	5.41	3.26	9.63
Between chemical substance	SSC=1005.04	c-1=4	MSC=251.26	2.46	5.41	3.26	9.63
Between treatment	SSt=6081.04	t-1=4	MSt=1520.26	14.90	5.41	3.26	9.63
Error (residual)	SSE=1227.92.6	(r-1)(c-1)=12	MSE=102.33				

Result: Fr is greater than F tab at 1%, 5% and 10% level of significance is eliminated. Fc is greater than F tab at 1%, 5% and 10% level of significance is agree. Ft is greater than F tab at 1%, 5% and 10% level of significance is eliminated. Result of duplication is 2.64 and chemical substance is 2.46 compare with these optimal cost of assignment problem is

390units. Now we can get duplication & chemical substance as 2.64 and 2.46 is the minimum cost. Therefore Latin square is better that assignment problem at 1%, 5% and 10% level of significance in the duplication & chemical substance. . Based on the Chemical substance & Duplication we plot the graph using R studio for Latin square design in the Figure 1.4

Figure 1.4 Latin square design



4.5 Problems based on different approaches: Five varieties L,M,N,O and P of a chemical substance are tested in with a five duplication the ground gives in pound. Using the Game theory we can find the result.

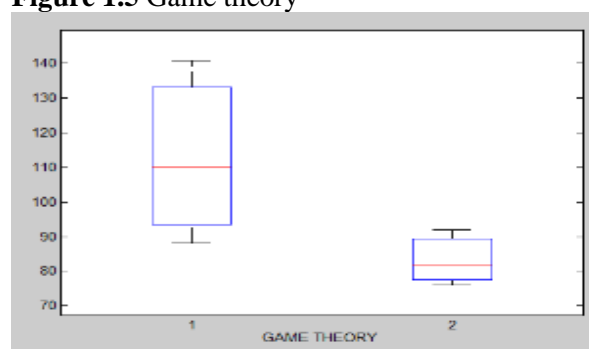
Based on the Chemical substance & Duplication we plot the graph using R studio for Game theory in the Figure 1.5

Solution:

Table:1.16 Game theory

K	81	90	94	112	82
L	90	60	70	84	72
M	110	125	134	141	88
N	82	84	87	76	92
O	85	80	85	85	69

Figure 1.5 Game theory



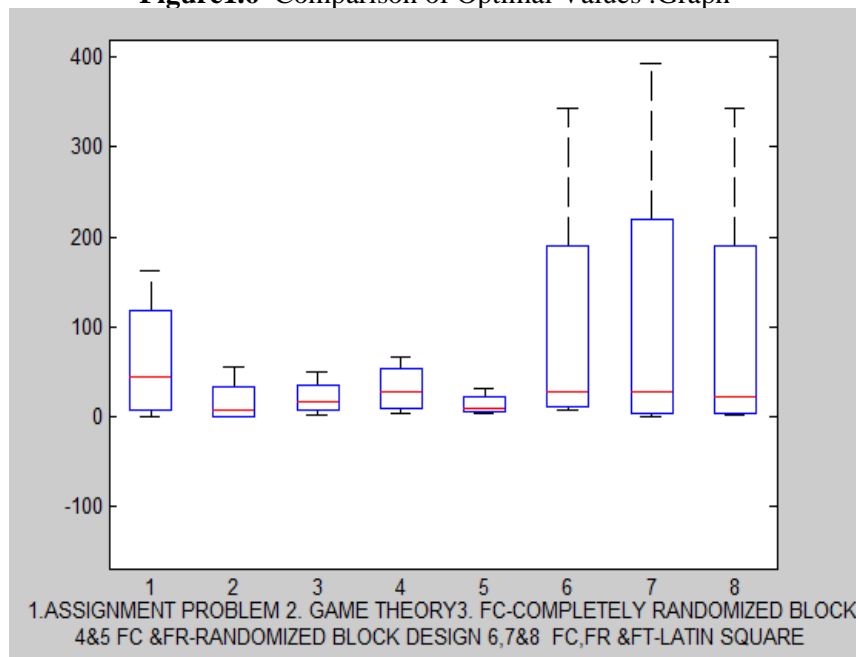
Using the dominant rule we the get, Value of the game =90.75.

Result:

Table1.17 comparative table we get R coding graph.[elim-eliminated, agree-agre]

EXAMPLE	salesmen				Treatment					Detergent					
	states	I	II	III	IV	Doctors	I	II	III	IV	V	watertemp.	I	II	III
	A	6	5	3	8	A	10	14	23	19	20	A	57	55	67
	B	8	9	6	5	B	11	15	24	17	21	B	49	52	68
	C	10	7	8	7	C	9	12	20	16	19	C	54	46	58
ASSIGNMENT PROBLEM	A=3,B=5,C=7,D=0 Total=15units				A=10,B=21,C=12,D=17,E=15 Total=75units					A=55,B=49,C=58,Total=162units					
GAMETHEORY	Value of the game =2				Value of the game =12					Value of the game =55					
COMPLETELY RANDOMIZED DESIGN	F tab	1%	5%	10%	F tab	1%	5%	10%	F tab	1%	5%	10%			
	F cal				F cal				F cal						
	Fc=4.07	27.49	0.67	130.62	Fc=31.83,	4.43	2.866	7.10	Fc=6.74,	27.49	5.14	10.92			
Result	agree	agree	agree	Result	elim	elim	elim	Result	agree	elim	agree				
RANDOMIZED BLOCK DESIGN	F tab	1%	5%	10%	F tab	1%	5%	10%	F tab	1%	5%	10%			
	F cal				F cal				F cal						
	Fc=2.78	99.33	0.81	132.32	Fc=44.58	4.77	3.01	7.94	Fc=2.38	18.00	6.94	61.25			
	Result	agree	elim	agree	Result	elim	elim	elim	Result	agree	agree	agree			
	Fr=6.33	10.92	1.84	27	Fr=3.002	4.77	3.01	7.94	Fr=9.84	18.00	6.94	61.25			
Result	agree	elim	agree	Result	agree	agree	agree	Result	agree	agree	agree				
LATIN SQUARE	F tab	1%	5%	10%	F tab	1%	5%	10%	F tab	1%	5%	10%			
	F cal				F cal				F cal						
	Fc=0.7740	9.78	0.56	23.70	fc=42.28	5.14	3.26	9.63	Fc=7.99	99.00	19	999			
	Result	agre	elim	agre	Result	elim	elim	elim	Result	agre	agre	agre			
	Fr=0.759	27.61	0.263	132.6	Fr=0.18	14.37	5.91	47.41	Fr=0.245	99.00	19	999			
	Result	agre	elim	agre	Result	agre	agre	agre	Result	agre	agre	agre			
Ft=1.764	9.78	0.56	23.70	ft=2.84	5.14	3.26	9.63	Ft=1.93	99	19	999				
Result	agre	elim	agre	Result	agre	agre	agre	Result	agre	agre	agre				

Figure1.6 Comparison of Optimal Values :Graph



IV. CONCLUSIONS:

If a large number of treatments are to be compared, then a large number of experimental units are required. The agricultural sciences where the designing of an experiment is an inevitable component of research. Comparing the solution of Assignment problem with One way classification, two way classification and Latin square we get the optimal value is less than comparative study. Using the Analysis of proposed algorithm is systematic than the existing methods. In future assignment problem we can solve in Statistics based on completely randomized design, randomized block design Latin square and Game Theory using R Language we can get value very easy. We can test the assignment problem using Statistics. The test result is agree in future we can find different way to the optimal solution.

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