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Strongly $\xi_{\Im}$-Continuous Maps in $I\xi_T S$

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Abstract: In this paper, we introduced the several generalized forms of continuous maps such as strongly $\xi_{\Im}$-continuous maps in generalized binary ideal topological spaces and investigate various relationships of these maps by demonstrating the use of some counter examples.

Keywords: *Stronglyξ-continuous, strongly$\xi_{\Im}$-continuous maps, strongly$\xi_{\Im}$-continuous maps*

Mathematics Subject Classification: 54C05-54C08

1. Introduction

The observations have shown several impacts of topological space in computer In recent study, topology in data mining plays a significant role Pawlak, (1991), Kırbas and Aslım (2009). Information systems are fundamental instruments for generating information understanding in any real-life sector, and topological information collection structures are appropriate mathematical models for both quantitative as well as the qualitative information mathematics. This has an impact in digital topology and computer science Khalimsky, Kopperman and Meyer (1990), Kong, Kopperman and Meyer (1991), Kovalesky and Kopperman (1994) and Moore and Peters (2005), quantum, high-energy, particle physics and superstring theory Landi, (1997) and Svozil, (1987).

Initially, Kuratowski (1930) introduced the concept of ideal topological space. The concepts of semi-open and semi-continuous maps is introduced and studied by Levine (1963). Further, the same author Levin (1970) studied generalized closed sets in topology and investigates various properties. Jankovic and Hamlett (1992) studied the concept of a local function and obtained the significant properties of these functions. Meanwhile, Dontchev (1996) verified the certain properties of pre-I-open sets. Initially the notion of generalized topology was introduced by Csaszar (2001

Hatir and Noiri (2006) studied d–b-continuous functions and obtained several results related to continuity. Levine (1961) introduced weakly continuous functions and established some new results.

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Further, Son, Park and Lim (2007) introduced weakly clopen and almost clopen functions and investigate various properties of almost clopen functions. Nithyanantha and Thangavelu (2011) studied binary topology and investigate various characterizations.

In this paper we developed the concept of strongly $\xi_{\Im}$-continuous maps and the significance of results have been shown by several counter examples. Some require basic definitions, concepts of ξ-topological, $I\xi_T S$and notations arediscussed in Section 2. The concept ofstrongly $\xi_{\Im}$-continuous maps is demonstrated in Section 3. At last, the conclusion is given in Section 4.

2. Preliminaries

In this portion, we discussed few require and important definitions, concepts of ξ-topological, $I\xi_T S$ and some notations.

Definition 2.1: SupposeY_1 and Y_2 are any two non-void sets. Then ξ_T from Y_1 to $Y_2$is a binary structure $\xi \subseteq \wp(Y_1) \times \wp(Y_2)$ satisfying the conditions i.e., $(\emptyset, \emptyset)$, $(Y_1, Y_2) \in \xi$ and If $\{(L_\alpha, M_\alpha)\ ;\ \alpha \in \Gamma\}$ is a family of elements of ξ, then $(\bigcup_{\alpha\in\Gamma} L_\alpha, \bigcup_{\alpha\in\Gamma} M_\alpha) \in \xi$. If ξ is ξ_T from $Y_1$to Y_2, then (Y_1, Y_2, ξ) is known as ξ-topological space ($\xi_T S$) and the elements of ξ are known as the ξ-open subsets of (Y_1, Y_2, ξ). The elements of $Y_1 \times Y_2$ are known as ξ-points.

Definition 2.2: LetY_1 and Y_2 be any two non-void set and (L_1, M_1), (L_2, M_2) are the elements of $\wp(Y_1) \times \wp(Y_2)$. Then $(L_1, M_1) \subseteq (L_2, M_2)$ only if $L_1 \subseteq L_2$and $M_1 \subseteq M_2$.

Remark 2.1: Let $\{T_\alpha\ ;\ \alpha \in \Lambda\}$ be the family of ξ_T from Y_1 to Y_2. Then, $\bigcap_{\alpha\in\Lambda} T_\alpha$ is also ξ_T from Y_1 to Y_2. Further $\cup_{\alpha\in\Lambda} T_\alpha$ need not be a ξ_T.

Definition 2.3: Let (Y_1, Y_2, ξ) be a $\xi_T S$ and $L \subseteq Y_1, M \subseteq Y_2$. Then (L, M) is called ξ-closed in (Y_1, Y_2, ξ) if $(Y_1 \backslash L, Y_2 \backslash M) \in \xi$.

Proposition 2.1: Let(Y_1, Y_2, ξ) is $\xi_T S$. Then (Y_1, Y_2) and $(\emptyset, \emptyset)$ are ξ-closed sets. Similarly if $\{(L_\alpha, M_\alpha): \alpha \in \Gamma\}$ is a family of ξ-closed sets, then $(\bigcap_{\alpha\in\Gamma} L_\alpha, \bigcap_{\alpha\in\Gamma} M_\alpha)$ is ξ-closed.

Proposition 2.2: Let$(L, M) \subseteq (Y_1, Y_2)$. Then (L, M) is ξ-open in (Y_1, Y_2, ξ) iff $(L, M) = I_\xi(L, M)$ and (L, M) is ξ-closed in (Y_1, Y_2, ξ) iff $(L, M) = Cl_\xi(L, M)$.

Definition 2.4: Any non-empty collection $\Im$ of subsets of $Y_1 \times Y_2$ is an ideal only if it satisfies the two important axioms, i.e. if $(L, M) \in \Im$ and $(P, Q) \subseteq (L, M)$ then$(P, Q) \in \Im$ and If $(L, M) \in \Im$ and $(P, Q) \in \Im$ then $(L \cup P,\ M \cup Q) \in \Im$. Let ξ be ξ_T and $\Im$ be an ideal, then $(Y_1, Y_2, \xi, \Im)$ is said to be an ideal ξ-topological space ($I\xi_T S$).

Example 2.2: Let (Y_1, Y_2, ξ) be $\xi_T S$. The collection $\Im = \emptyset$ and $\Im = \wp(Y_1) \times \wp(Y_2)$ are also ideals on $Y_1 \times Y_2$.

Definition 2.5: Let $(Y_1, Y_2, \xi, \Im)$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then the set $(L, M)^*(\Im) = \{(x, y) \in Y_1 \times Y_2 / (U \cap L, V \cap M) \notin \Im$ for every nbd (U, V) of $(x, y)\}$ is known as the local function of (L, M) in the respect of $\Im$ and ξ. We normally denote $(L, M)^*$ instead of $(L, M)^*(\Im)$ to avoid any confusion.

Definition 2.6: Let $(Y_1, Y_2, \xi, \mathfrak{I})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then (L, M) is known as $\xi_{\mathfrak{I}}$-semi-open if for any ξ-open set (U, V), $(L, M) \backslash Cl_{\xi}(U, V) \in \mathfrak{I}$ whenever, $(U, V) \backslash (L, M) \in \mathfrak{I}$. Likewise (L, M) is known as $\xi_{\mathfrak{I}}$-α-openif for any ξ-open set (U, V), $(L, M) \backslash I_{\xi}(Cl_{\xi}(U, V)) \in \mathfrak{I}$ whenever, $(U, V) \backslash (L, M) \in \mathfrak{I}$.

Definition 2.7: Let $(Y_1, Y_2, \xi, \mathfrak{I})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then (L, M) is known as $\xi_{\mathfrak{I}}$-pre-open if for any ξ-open set (U, V), $(U, V) \backslash Cl_{\xi}(L, M) \in \mathfrak{I}$ whenever, $(L, M) \backslash (U, V) \in \mathfrak{I}$. Likewise (L, M) is known as $\xi_{\mathfrak{I}}$-β-openif for any ξ-open set (U, V) such that $(U, V) \backslash I_{\xi}(Cl_{\xi}(L, M)) \in \mathfrak{I}$ whenever, $(L, M) \backslash (U, V) \in \mathfrak{I}$.

Definition 2.8: Let $(Y_1, Y_2, \xi, \mathfrak{I})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then (L, M) is called ${\xi^*}_{\mathfrak{I}}$-open set if $(U, V) \backslash (L, M) \in \mathfrak{I}$ whenever, $(L, M) \subseteq (U, V)$, where (U, V) is ξ-open set.

Definition 2.9: Let $(Y_1, Y_2, \xi, \mathfrak{I})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then (L, M) is called ${\xi^*}_{\mathfrak{I}}$-semi-open set if $(U, V) \backslash I_{\xi}(L, M) \in \mathfrak{I}$ whenever, $(L, M) \subseteq (U, V)$, where (U, V) is ξ-open set. Similarly (L, M) is called ${\xi^*}_{\mathfrak{I}}$-α-open set if $(U, V) \backslash Cl_{\xi}(I_{\xi}(L, M)) \in \mathfrak{I}$ whenever, $(L, M) \subseteq (U, V)$, where (U, V) is ξ-open set.

Definition 2.10: Let $(Y_1, Y_2, \xi, \mathfrak{I})$ be $I\xi_T S$ and $(L, M) \subseteq Y_1 \times Y_2$. Then (L, M) is called ${\xi^*}_{\mathfrak{I}}$-pre-open set if $(U, V) \backslash Cl_{\xi}(L, M) \in \mathfrak{I}$ whenever, $(L, M) \subseteq (U, V)$, where (U, V) is ξ-open set. Similarly (L, M) is called ${\xi^*}_{\mathfrak{I}}$-β-open set if $(U, V) \backslash I_{\xi}(Cl_{\xi}(L, M)) \in \mathfrak{I}$ whenever, $(L, M) \subseteq (U, V)$, where (U, V) is ξ-open set.

3. Strongly $\xi_{\mathfrak{I}}$-Continuous Maps

We introduced in this section the concept of strongly $\xi_{\mathfrak{I}}$-continuous in $I\xi_T S$ and established the relationships between these maps and some other maps. The results have been shown by making the use of counter examples.

Definition 3.1: If $(Z, \mathcal{T})$ be $G_T$and $(Y_1, Y_2, \xi, \mathfrak{I})isI\xi_T S$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \to Y_1 \times Y_2$ is known as **$\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map** if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-$\alpha$) open in $(Z, \mathcal{T})$ $\forall$ ξ-open sets $(L, M) \in (Y_1, Y_2, \xi)$.

Definition 3.2: If $(Z, \mathcal{T})$ be G_T and $(Y_1, Y_2, \xi, \mathfrak{I})$ is $I\xi_T S$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \to Y_1 \times Y_2$ is known as **strongly $\xi_{\mathfrak{I}}$-semi (strongly $\xi_{\mathfrak{I}}$-α) continuous map** if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-$\alpha$) clopen in $(Z, \mathcal{T})$ $\forall$ ξ-set $(L, M) \in (Y_1, Y_2, \xi)$.

Example 3.1: If $Z = \{1, 2, 3\}$, $Y_1 = \{a_1, a_2\}$ and $Y_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1, 3\}, \{2, 3\}, Z\}$ is G_T, $\mathfrak{I} = \{\emptyset, \{3\}, \{2, 3\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{a_1\}, \{b_2\}), (\{a_2\}, \{Y_2\}), (Y_1, Y_2)\}$ is ξ_T from Y_1 to Y_2. Consider$\mathcal{F}: (Z, \mathcal{T}) \to Y_1 \times Y_2$ defined as $\mathcal{F}(1) = (a_1, b_1)$ and $\mathcal{F}(2) = (a_2, Y_2) = \mathcal{F}(3)$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{b_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{a_1\}, \{Y_2\}) = \{\emptyset\}$, $f^{-1}(\{a_2\}, \{Y_2\}) = \{2,3\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{b_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{Y_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{b_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \emptyset) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \{b_2\}) = \{2,3\}$, $\mathcal{F}^{-1}(\{Y_1\}, \{\emptyset\}) = \{\emptyset\}$,

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$\mathcal{F}^{-1}(\{\Upsilon_1\},\{b_1\}) = \{2\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{b_2\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\Upsilon_2) = \{Z\}$. Hence we see inverse image of every ξ-set is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) clopen set in $(Z,\mathcal{T})$. Thus $\mathcal{F}$ is strongly $\xi_{\mathfrak{I}}$-semi (strongly $\xi_{\mathfrak{I}}$-α) continuous map.

Remark 3.1: ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map ⇒⇍ strongly $\xi_{\mathfrak{I}}$-semi (strongly $\xi_{\mathfrak{I}}$-α) continuous map

Proof: Quite easy while the converse can be illustrated as follows.

Example 3.2: If $Z = \{1, 2, 3\}$, $\Upsilon_1 = \{a_1, a_2\}$ and $\Upsilon_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, Z\}$ is G_T, $\mathfrak{I} = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ is an ideal on Z and $\xi = \{(\emptyset,\emptyset), (\{a_2\},\{b_2\}), (\{a_1\},\{\Upsilon_2\}), (\Upsilon_1,\Upsilon_2)\}$ is ξ_T from Υ_1 to Υ_2. Consider the mapping $\mathcal{F}: (Z, \tau_g) \to \Upsilon_1 \times \Upsilon_2$ defined as $\mathcal{F}(1) = (a_2, b_2) = \mathcal{F}(3)$ and $\mathcal{F}(2) = (a_1, \Upsilon_2)$. Therefore $\mathcal{F}^{-1}(\emptyset,\emptyset) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\},\{b_1\}) = \{2\}$, $\mathcal{F}^{-1}(\{a_1\}, \{\Upsilon_2\}) = \{2\}$, $f^{-1}(\{a_2\}, \{\Upsilon\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\},\{b_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{\Upsilon_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_1\}, \{b_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\},\emptyset) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \{b_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{a_2\}, \{b_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{\emptyset\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{b_1\}) = \{2\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{b_2\}) = \{\emptyset\}$and $\mathcal{F}^{-1}(\Upsilon_1,\Upsilon_2) = \{Z\}$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}_{\mathfrak{I}}$-semi-clopen set in $(Z,\mathcal{T})$. Thus $\mathcal{F}$ is strongly$\xi_{\mathfrak{I}}$-semi (strongly$\xi_{\mathfrak{I}}$-α) continuous map but not ξ-continuous (ξ-semi, ξ-α, ξ-pre, ξ-β) map.

Proposition 3.1: Strongly $\xi_{\mathfrak{I}}$-semi (strongly $\xi_{\mathfrak{I}}$-α) continuous map⇒⇍ $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map.

Proof: Suppose(L, M) be ξ-set and $\mathcal{F}$ be strongly $\xi_{\mathfrak{I}}$-semi (strongly $\xi_{\mathfrak{I}}$-α) continuous map. Therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) clopen in $(Z,\mathcal{T})$. Since $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) clopen is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) open. Hence $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-semi ($\mathcal{T}_{\mathfrak{I}}$-α) open in $(Z,\mathcal{T})$. Thus $\mathcal{F}$ is $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map.

The converse can be illustrated as follows.

Example 3.3: If $Z = \{1, 2, 3\}$, $\Upsilon_1 = \{a_1, a_2\}$ and $\Upsilon_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1, 2\}, \{2, 3\}, Z\}$ is G_T, $\mathfrak{I} = \{\emptyset, \{2,3\}\}$ is an ideal on Z and $\xi = \{(\emptyset,\emptyset), (\{a_1\},\{b_1\}), (\{a_2\},\{\Upsilon_2\}), (\Upsilon_1,\Upsilon_2)\}$ is ξ_T from $\Upsilon_1$to Υ_2. Consider $\mathcal{F}: (Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ defined as $\mathcal{F}(1) = (a_1, b_1) = \mathcal{F}(3)$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) open set in $(Z,\mathcal{T})$. Thus $\mathcal{F}$ is $\xi_{\mathfrak{I}}$-semi-continuous map while not strongly$\xi_{\mathfrak{I}}$-semi (strongly $\xi_{\mathfrak{I}}$-α) continuous map because $\mathcal{F}^{-1}(\{a_1\},\{b_1\}) = \{1,3\}$ where $\{1,3\}$ is not $\mathcal{T}_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) clopen set in $(Z,\mathcal{T})$.

Proposition 3.2: Strongly $\xi_{\mathfrak{I}}$-α-continuous map ⇒⇍ strongly $\xi_{\mathfrak{I}}$-semi-continuous map

Proof: Suppose(L, M) be a ξ-set and$\mathcal{F}$ is strongly$\xi_{\mathfrak{I}}$-α-continuous map. Therefore$\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-α-clopen set in $(Z,\mathcal{T})$. Since every $\mathcal{T}_{\mathfrak{I}}$-α-clopen set is $\mathcal{T}_{\mathfrak{I}}$-semi-clopen, therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-semi-clopen set in $(Z,\mathcal{T})$. Hence$\mathcal{F}$ is strongly $\xi_{\mathfrak{I}}$-semi-continuous map.

The converse can be is illustrated as follows.

Example 3.4: Let $Z = \{1, 2, 3,4\}$,$\Upsilon_1 = \{a_1, a_2\}$ and $\Upsilon_2 = \{b_1, b_2\}$. Then $\mathcal{T} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{1,2,4\}, \{2,3,4\}, Z\}$ is G_T,$\mathfrak{I} = \{\emptyset, \{2\}\}$ is an ideal on Z and

$\xi = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{b_2\}), (\{a_1\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$ is ξ_T from Υ_1 to Υ_2. Consider $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ defined as $\mathcal{F}(2) = (a_2, b_2)$ and $\mathcal{F}(1) = \mathcal{F}(3) = (a_1, b_1) = \mathcal{F}(4)$. Hence we see that the inverse image of every ξ-set is $\mathcal{T}_{\mathfrak{I}}$-semi-clopen set but not $\mathcal{T}_{\mathfrak{I}}$- α-clopen set in $(Z, \mathcal{T})$. Thus $\mathcal{F}$ is strongly $\xi_{\mathfrak{I}}$-semi-continuousbut strongly $\xi_{\mathfrak{I}}$-α-continuous map.

Definition 3.3: If $(Z, \mathcal{T})$ be $G_T S$ and $(\Upsilon_1, \Upsilon_2, \xi, \mathfrak{I})$ is $I\xi_T S$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is known as **$\xi_{\mathfrak{I}}$-pre ($\xi_{\mathfrak{I}}$-β) continuous map** if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-pre ($\mathcal{T}_{\mathfrak{I}}$-β) open in $(Z, \mathcal{T})$ $\forall$ ξ-open sets $(L, M) \in (\Upsilon_1, \Upsilon_2, \xi)$.

Definition 3.4: If $(Z, \mathcal{T})$ be $G_T S$ and $(\Upsilon_1, \Upsilon_2, \xi, \mathfrak{I})$ is $I\xi_T S$. Then the mapping $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is known as **strongly $\xi_{\mathfrak{I}}$-pre (strongly$\xi_{\mathfrak{I}}$-β) continuous map** if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-pre ($\mathcal{T}_{\mathfrak{I}}$-β) clopen in $(Z, \mathcal{T})$ $\forall$ ξ-set $(L, M) \in (\Upsilon_1, \Upsilon_2, \xi)$.

Example 3.5: Let $Z = \{1, 2, 3\}$, $\Upsilon_1 = \{a_1, a_2\}$ and $\Upsilon_2 = \{b_1, b_2\}$. Then clearly $\mathcal{T} = \{\emptyset, \{1, 2\}, \{2, 3\}, Z\}$ is $G_T S$, $\mathfrak{I} = \{\emptyset, \{2\}, \{2,3\}\}$ is an ideal on Z and $\xi = \{(\emptyset, \emptyset), (\{a_2\}, \{b_2\}), (\{a_1\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$ is ξ_T from Υ_1 to Υ_2. Consider $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ defined as $\mathcal{F}(1) = (a_1, \Upsilon_2) = \mathcal{F}(2)$ and $\mathcal{F}(3) = (a_2, b_2)$. Hence we see that the inverse image of every ξ-set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}_{\mathfrak{I}}$-pre ($\mathcal{T}_{\mathfrak{I}}$-β) clopen set in $(Z, \mathcal{T})$. Thus $\mathcal{F}$ is strongly $\xi_{\mathfrak{I}}$-pre (strongly $\xi_{\mathfrak{I}}$-β) continuous map.

Remark 3.2: ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map $\Rightarrow\not\Leftarrow$ strongly $\xi_{\mathfrak{I}}$-pre (strongly $\xi_{\mathfrak{I}}$-β) continuous map

Proof is quite easy while the converse can be is illustrated as follows.

Example 3.6: In Example 3.5, $\mathcal{F}$ is strongly$\xi_{\mathfrak{I}}$-pre (strongly$\xi_{\mathfrak{I}}$-β) continuous map but not ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map.

Proposition 3.3: Strongly $\xi_{\mathfrak{I}}$-β-continuous map $\Rightarrow\not\Leftarrow$ strongly$\xi_{\mathfrak{I}}$-pre-continuous map

Proof: Suppose(L, M) beξ-set and $\mathcal{F}$ be strongly $\xi_{\mathfrak{I}}$-β-continuous map. Therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-β-clopen set in $(Z, \mathcal{T})$. Since every $\mathcal{T}_{\mathfrak{I}}$-β-clopen set is $\mathcal{T}_{\mathfrak{I}}$-pre-clopen, therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}_{\mathfrak{I}}$-pre-clopen set in $(Z, \mathcal{T})$. Thus $\mathcal{F}$ is strongly $\xi_{\mathfrak{I}}$-pre-continuous map.

The converse can be is illustrated as follows.

Example 3.7: Let $\mathfrak{I} = \{\emptyset, \{2\}, \{2,3\}\}$ in Example 3.4, then $\mathcal{F}$ is strongly $\xi_{\mathfrak{I}}$-pre-continuous map but not strongly $\xi_{\mathfrak{I}}$-β-continuous map.

4. Conclusion

We introduced and study useful concept of strongly $\xi_{\mathfrak{I}}$-semi-continuous and established the relationship between these maps with several other maps like ξ, ξ-semi, ξ-pre, ξ-α, ξ-β-continuous maps etc. The significant of results have been shown by several counter examples. In the present direction, we have categorised maps in generalized binary ideal topological spaces and investigated the behaviour of presented maps by utilizing ideal. We conclude the following relationships in this paper

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ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map $\Rightarrow\not\Leftarrow$ strongly $\xi_{\mathfrak{I}}$-semi (strongly $\xi_{\mathfrak{I}}$-α) continuous map

Strongly $\xi_{\mathfrak{I}}$-semi (strongly $\xi_{\mathfrak{I}}$-α) continuous map$\Rightarrow\not\Leftarrow$ $\xi_{\mathfrak{I}}$-semi ($\xi_{\mathfrak{I}}$-α) continuous map.

Strongly $\xi_{\mathfrak{I}}$-α-continuous map $\Rightarrow\not\Leftarrow$ strongly $\xi_{\mathfrak{I}}$-semi-continuous map

ξ (ξ-semi, ξ-α, ξ-pre, ξ-β) continuous map $\Rightarrow\not\Leftarrow$ strongly$\xi_{\mathfrak{I}}$-pre (strongly$\xi_{\mathfrak{I}}$-β) continuous map

Strongly $\xi_{\mathfrak{I}}$-β-continuous map $\Rightarrow\not\Leftarrow$ strongly$\xi_{\mathfrak{I}}$-pre-continuous map

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