On $nIs_{\alpha}g-$ Homeomorphism

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Abstract

The purpose of this paper is to introduce a new class of nano ideal generalized homeomorphism namely, $nIs_{\alpha}g-$ homeomorphism (briefly, $nIs_{\alpha}g-$ Hompsm.) and $*nIs_{\alpha}g-$ Hompsm. Further, we have investigated certain characteristics and some equivalent conditions were discussed. Also, we have discussed its relationship with some of the existing mappings.

Keywords: $nIs_{\alpha}g-$ continuity, $nIs_{\alpha}g-$ irresolute function, $nIs_{\alpha}g-$ homeomorphism, $*nIs_{\alpha}g-$ homeomorphism

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1. Introduction

Parimala et.al[3] introduced and studied the notion of nano ideal generalized Cl.S.s in nano ideal topological spaces. Pasunkili Pandian et.al [6],[1] introduced $nIs_{\alpha}g-$ Cl. S.s and studied $nIs_{\alpha}g-$ Cl. Map., $nIs_{\alpha}g-$ Op. Map., $nIs_{\alpha}g-$ Cont.Fn. and $nIs_{\alpha}g-$ Irr.Fn. map in nano ideal topological spaces. In this paper, we introduce the concept of $nIs_{\alpha}g-$ Hompsm. and $*nIs_{\alpha}g-$ Hompsm. in nano ideal topological spaces and investigated its relationship with some of the existing Hompsm.s. Further, we have studied their characteristics.

2. Preliminaries

Definition 2.1 [4] A subset $\mathcal{H}$ of a nano topological space $(\Gamma, \mathcal{N})$ is said to be nano semi $\alpha-$ open set (briefly, $ns_{\alpha}-$ Op.S.) set if there exists a $n\alpha-$ Op. S. $\mathcal{P}$ in Γ such that $\mathcal{P} \subseteq \mathcal{H} \subseteq n-cl(\mathcal{P})$ or equivalently if $\mathcal{H} \subseteq n-cl(n\alpha-int(\mathcal{P}))$.

Definition 2.2 [2] Let $(\Gamma,\mathcal{N},\mathcal{I})$ be a nano ideal topological space with an ideal $\mathcal{I}$ on Γ where $\mathcal{N}=\tau_{\mathcal{R}}(X)$ and $(.)_n^*$ be a set operator from $\wp(\Gamma)$ to $\wp(\Gamma)$, ($\wp(\Gamma)$ the set of all subsets of Γ). For a subset $\mathcal{H}\subset\Gamma$, $\mathcal{H}_n^*(\mathcal{I},\mathcal{N})=\{x\in\Gamma: G_n\cap\mathcal{H}\notin\mathcal{I}$, for every $G_n\in G_n(x)\}$, where $G_n=\{G_n: x\in G_n, G_n\in\mathcal{N}\}$ is called the nano local function (briefly, $n-$ local function) of $\mathcal{H}$ with respect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write $\mathcal{H}_n^*$ for $\mathcal{H}_n^*(\mathcal{I},\mathcal{N})$.

Definition 2.3 [6] A subset $\mathcal{H}$ of a nano ideal topological space $(\Gamma,\mathcal{M},\mathcal{J})$ is said to be nano ideal semi α generalized Cl. S. (briefly, $nIs_{\alpha}g-$ Cl. S.) if $\mathcal{H}_n^*\subseteq\mathcal{K}$ whenever $\mathcal{H}\subseteq\mathcal{K}$ and $\mathcal{K}$ is nano semi $\alpha-$ open.

Definition 2.4 [1] Let $(\Gamma,\mathcal{M},\mathcal{J})$ and $(\mathcal{V},\mathcal{M}',\mathcal{J}')$ be nano ideal topological spaces. Then

(i) The mapping $\eta:(\Gamma,\mathcal{M},\mathcal{J})\to(\mathcal{V},\mathcal{M}',\mathcal{J}')$ is said to be $nIs_{\alpha}g-$ Cont.Fn. if the inverse image of every $n-$ Op. S. in $(\mathcal{V},\mathcal{M}',\mathcal{J}')$ is $nIs_{\alpha}g-$ open in $(\Gamma,\mathcal{M},\mathcal{J})$.

(ii) The mapping $\eta:(\Gamma,\mathcal{M},\mathcal{J})\to(\mathcal{V},\mathcal{M}',\mathcal{J}')$ is said to be $nIs_{\alpha}g-$ Irr.Fn. if the inverse image of every $nIs_{\alpha}g-$ Cl. S. in $(\mathcal{V},\mathcal{M}',\mathcal{J}')$ is $nIs_{\alpha}g-$ closed in $(\Gamma,\mathcal{M},\mathcal{J})$.

Definition 2.5 [6] A map $\eta:(\Gamma,\mathcal{M},\mathcal{J})\to(\mathcal{V},\mathcal{M}',\mathcal{J}')$ is said to be $nIs_{\alpha}g-$ Cl. Map. if for every $ns_{\alpha}-$ closed subset $\mathcal{H}$ of $(\Gamma,\mathcal{M},\mathcal{J})$, $\eta(\mathcal{H})$ is $nIs_{\alpha}g-$ Cl.S.
The complement of $nIs_{\alpha}g-$ Cl. Map. is $nIs_{\alpha}g-$ Op. Map.

Definition 2.6 [5] A map $\eta:(\Gamma,\mathcal{M},\mathcal{J})\to(\mathcal{V},\mathcal{M}',\mathcal{J}')$ is called $^*nIg-$ Op. Map. if for every $nIg-$ open subset $\mathcal{H}$ of $(\Gamma,\mathcal{M},\mathcal{J})$, $\eta(\mathcal{H})$ is $nIg-$ Op.S.

Definition 2.7 [5] (i) A map $f:(\Gamma,,\mathcal{N},\mathcal{I})\to(\Delta,\mathcal{N}',\mathcal{J})$ is called $n*-$ Hompsm., if both f and f^{-1} are $n*-$ Cont.Fn.

(ii) A map $f:(\Gamma,\mathcal{N},\mathcal{I})\to(\Delta,\mathcal{N}',\mathcal{J})$ is called $nIg-$ Hompsm., if both f and f^{-1} are $nIg-$ Cont.Fn.

(iii) A map $f:(\Gamma,\mathcal{N},\mathcal{I})\to(\Delta,\mathcal{N}',\mathcal{J})$ is called $^*nIg-$ Hompsm., if both f and f^{-1} are $nIg-$ Irr.Fn.

Theorem 2.1 [6] Every $nIs_{\alpha}g-$ Cl. S. is $nIg-$ closed but not conversely.

Theorem 2. 2 [6] Every $n*$-Cl. S. is $nIs_{\alpha}g-$ closed but not conversely.

Theorem 2.3 [1] Every $n*-$ Cont.Fn. is $nIs_{\alpha}g-$ Cont.Fn.but not conversely.

Theorem 2.4 [1] Every $nIs_{\alpha}g-$ Irr.Fn. function is $nIs_{\alpha}g-$ Cont.Fn.but not conversely.

Theorem 2.5 [1] Every $nIs_{\alpha}g-$ Cont.Fn. is $nIg-$ Cont.Fn.

3. $nIs_{\alpha}g -$ Homeomorphism

Definition 3.1 A bijection $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ is said to be $nIs_{\alpha}g -$ Hompsm. if both η and η^{-1} are $nIs_{\alpha}g -$ Cont.Fn.

Example 3.1 Let $\Gamma = \{u_1, u_2, u_3\}$; $\Gamma/\mathcal{R} = \{\{u_1, u_2\}, \{u_3\}\}$; $\mathcal{X} = \{u_1, u_3\}$; $\mathcal{J} = \{\emptyset, \{u_3\}\}$. $\mathcal{M} = \{\emptyset, \Gamma, \{u_3\}, \{u_1, u_2\}\}$. $nIs_{\alpha}g -$ Cl. S.s are $\wp(\Gamma)$. Let $\Delta = \{v_1, v_2, v_3\}$; $\Delta/\mathcal{R} = \{\{v_1\}, \{v_2, v_3\}\}$; $\mathcal{Y} = \{v_1, v_2\}$; $\mathcal{J}' = \{\emptyset, \{v_2\}\}$. $\mathcal{M}' = \{\emptyset, \Gamma, \{v_1\}, \{v_2, v_3\}\}$. $nIs_{\alpha}g -$ Cl. S.s are $\wp(\Delta)$. Define $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ as $\eta(u_1) = v_1; \eta(u_2) = v_2; \eta(u_3) = v_3$. Both η and η^{-1} are $nIs_{\alpha}g -$ Cont.Fn. Hence, η is $nIs_{\alpha}g -$ Hompsm.

Theorem 3.1

For any bijection $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$, the following axioms are equivalent.

(1) $\eta^{-1}: (\Delta, \mathcal{M}', \mathcal{J}') \to (\Gamma, \mathcal{M}, \mathcal{J})$ is $nIs_{\alpha}g -$ Cont.Fn.

(2) η is a $nIs_{\alpha}g -$ Op. Map.

(3) η is $nIs_{\alpha}g -$ Cl. Map.

Proof. (1) $\Rightarrow$ (2) : Let $\mathcal{H}$ be a $n -$ Op. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$. Since η^{-1} is $nIs_{\alpha}g -$ Cont.Fn., $(\eta^{-1})^{-1}(\mathcal{H}) = \eta(\mathcal{H})$ is $nIs_{\alpha}g -$ open in $(\Delta, \mathcal{M}', \mathcal{J}')$. Hence, η is $nIs_{\alpha}g -$ Op. Map.

(2) $\Rightarrow$ (3) : Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ be $nIs_{\alpha}g -$ Op. Map. Let $\mathcal{H}$ be a $n -$ Cl. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$.Then $\mathcal{V} - \mathcal{H}$ is $n -$ Op. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$.Since η is $nIs_{\alpha}g -$ Op. Map., $\eta(\Delta - \mathcal{H})$ is $nIs_{\alpha}g -$ Op. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$. This implies that $\eta(\Delta - \mathcal{H})$ is $nIs_{\alpha}g -$ Op. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$ so that $\eta(\mathcal{H})$ is $nIs_{\alpha}g -$ Cl. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$. Therefore, η is $nIs_{\alpha}g -$ Cl. Map.

(3) $\Rightarrow$ (1): Assume that $\mathcal{H}$ is a $n -$ Cl. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$. Then by hypothesis, $(\eta^{-1})^{-1}(\mathcal{H}) = \eta(\mathcal{H})$ is $nIs_{\alpha}g -$ Cl. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$ so that η^{-1} is $nIs_{\alpha}g -$ Cont.Fn.

Theorem 3.2 Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ be a bijective and $nIs_{\alpha}g -$ Cont.Fn. Then the following statements are equivalent.

(1) η is a $nIs_{\alpha}g -$ Op. Map.

(2) η is a $nIs_{\alpha}g -$ Hompsm.

(3) η is a $nIs_{\alpha}g -$ Cl. Map.

Proof. (1) $\Rightarrow$ (2):Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ be $nIs_{\alpha}g -$ Op. Map. Let $\mathcal{H}$ be a $n -$ Cl. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$.Then $\Delta - \mathcal{H}$ is $n -$ Op. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$.Since η is $nIs_{\alpha}g -$ Op. Map., $\eta(\Delta - \mathcal{H})$ is $nIs_{\alpha}g -$ Op. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$. This implies that $\eta(\Delta - \mathcal{H})$ is $nIs_{\alpha}g -$ Op. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$ so that $\eta(\mathcal{H})$ is $nIs_{\alpha}g -$ Cl. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$. Therefore, η is a $nIs_{\alpha}g -$ Cl. Map. By Theorem 3.1, $\eta^{-1}: (\Delta, \mathcal{M}', \mathcal{J}') \to (\Gamma, \mathcal{M}, \mathcal{J})$ is $nIs_{\alpha}g -$ Cont.Fn. By hypothesis, η is $nIs_{\alpha}g -$ Cont.Fn.so that η is $nIs_{\alpha}g -$ Hompsm.

(2) $\Rightarrow$ (3) : Assume that η is $nIs_{\alpha}g -$ Hompsm. Then η and η^{-1} are $nIs_{\alpha}g -$ Cont.Fn. By Theorem 3.1, η is a $nIs_{\alpha}g -$ Cl. Map.

(3) ⇒ (1) : The result is trivial.
Remark 3.1 Let $\eta:(\Gamma,\mathcal{M},\mathcal{J})\to(\Delta,\mathcal{M}',\mathcal{J}')$ be bijevtive. η is said to be $nIs_\alpha g-$ Hompsm. if η is both $nIs_\alpha g-$ Cont.Fn.and $nIs_\alpha g-$ Op. Map.
Theorem 3.3 Let $\eta:(\Gamma,\mathcal{M},\mathcal{J})\to(\Delta,\mathcal{M}',\mathcal{J}')$ be a n^*- Hompsm. Then η is $nIs_\alpha g-$ Hompsm.
Proof. From the hypothesis, both η and η^{-1} are n^*- Cont.Fn. Since every n^*- Cont.Fn. is $nIs_\alpha g-$ Cont.Fn., the result follows.
Remark 3.2 The reverse implication of the previous need not be true. This is shown in the following example.
Example 3.2 Let $\Gamma=\{u_1,u_2,u_3\}$; $\Gamma/\mathcal{R}=\{\{u_1,u_2\},\{u_3\}\}$; $\mathcal{X}=\{u_1,u_3\}$; $\mathcal{J}=\{\emptyset,\{u_3\}\}$. $\mathcal{M}=\{\emptyset,\Gamma,\{u_3\},\{u_1,u_2\}\}$. $nIs_\alpha g-$ Cl. S.s are $\wp(\Gamma)$. n^*- Cl. S.s are $\emptyset,\Gamma,\{u_3\},\{u_1,u_2\}$. Let $\Delta=\{v_1,v_2,v_3\}$; $\Delta/\mathcal{R}=\{\{v_1\},\{v_2,v_3\}\}$; $\mathcal{Y}=\{v_1,v_2\}$; $\mathcal{J}'=\{\emptyset,\{v_2\}\}$. $\mathcal{M}'=\{\emptyset,\Gamma,\{v_1\},\{v_2,v_3\}\}$. $nIs_\alpha g-$ Cl. S.s are $\wp(\Delta)$. n^*- Cl. S.s are $\emptyset,\Delta,\{v_1\},\{v_2\},\{v_1,v_2\},\{v_2,v_3\}$. Define η as in the Example 3.1. Here, $\eta^{-1}(\{v_2,v_3\})=\{u_2,u_3\}$ is not n^*- Cl. S. in $(\Gamma,\mathcal{M},\mathcal{J})$ for the $n-$ Cl. S. $\{v_2,v_3\}$ in $(\Delta,\mathcal{M}',\mathcal{J}')$. Therefore, η is $nIs_\alpha g-$ Hompsm. but not n^*- Cont.Fn., hence η is not n^*- Hompsm.
Theorem 3.4 Every $nIs_\alpha g-$ Hompsm. is a $nIg-$ Hompsm.
Proof. Let $\eta:(\Gamma,\mathcal{M},\mathcal{J})\to(\Delta,\mathcal{M}',\mathcal{J}')$ be a $nIs_\alpha g-$ Hompsm. Then η and η^{-1} are $nIs_\alpha g-$ Cont.Fn.and η is a bijection. By Theorem 2.5, every $nIs_\alpha g-$ Cont.Fn. is $nIg-$ Cont.Fn., the result follows.
Remark 3.3 The reverse implication of the preceding theorem is not valid as shown in the successive example.
Example 3.3 Let $\Gamma=\{u_1,u_2,u_3,u_4\}$; $\Gamma/\mathcal{R}=\{\{u_1\},\{u_2,u_3\},\{u_4\}\}$; $\mathcal{X}=\{u_1,u_4\}$; $\mathcal{J}=\{\emptyset,\{u_1\}\}$. $\mathcal{M}=\emptyset,\Gamma,\{u_1,u_4\}$. Here, $nIs_\alpha g-$ Cl. S.s are $\emptyset,\Gamma,\{u_1\},\{u_2,u_3\}$, $\{u_1,u_2,u_3\},\{u_2,u_3,u_4\}$ and $nIg-$ Cl. S.s are $\emptyset,\Gamma,\{u_1\},\{u_2\},\{u_3\},\{u_1,u_2\},\{u_1,u_3\}$, $\{u_2,u_3\},\{u_2,u_4\},\{u_3,u_4\},\{u_1,u_2,u_3\},\{u_1,u_2,u_4\},\{u_1,u_3,u_4\},\{u_2,u_3,u_4\}$.
Let $\Delta=\{v_1,v_2,v_3,v_4\}$; $\Delta/\mathcal{R}=\{\{v_1\},\{v_2,v_4\},\{v_3\}\}$; $\mathcal{Y}=\{v_1,v_2\}$; $\mathcal{J}'=\{\emptyset,\{v_2\},\{v_3\},\{v_2,v_3\}\}$. $\mathcal{M}'=\emptyset,\Delta,\{v_1\},\{v_2,v_4\},\{v_1,v_2,v_4\}$. Here $\emptyset,\Delta,\{v_2\},\{v_3\}$, $\{v_1,v_3\},\{v_2,v_3\},\{v_3,v_4\},\{v_1,v_2,v_3\},\{v_1,v_3,v_4\},\{v_2,v_3,v_4\}$ are both $nIs_\alpha g-$ Cl. S.s and $nIg-$ Cl. S.s. Define $\eta:(\Gamma,\mathcal{M},\mathcal{J})\to(\Delta,\mathcal{M}',\mathcal{J}')$ as $\eta(u_1)=v_4;\eta(u_2)=v_3;\eta(u_3)=v_1;\eta(u_4)=v_2$. η^{-1} is both $nIg-$ Cont.Fn.and $nIs_\alpha g-$ Cont.Fn.
For the $n-$ Cl. S. $\{v_3\}$ in $(\Delta,\mathcal{M}',\mathcal{J}')$, $\eta^{-1}(\{v_3\})=\{u_2\}$ is $nIg-$ closed but not $nIs_\alpha g-$ closed in $(\Gamma,\mathcal{M},\mathcal{J})$. Therefore, η is $nIg-$ Cont.Fn. but not $nIs_\alpha g-$ Cont.Fn. Hence, η is $nIg-$ homeomorphsim but not $nIs_\alpha g-$ Hompsm.
Remark 3.4 Composition of two $nIs_\alpha g-$ Hompsm. need not be $nIs_\alpha g-$ Hompsm.
Example 3.4 Let $\Gamma=\{u_1,u_2,u_3\}$; $\Gamma/\mathcal{R}=\{\{u_1\},\{u_2,u_3\}\}$; $\mathcal{X}=\{u_2\}$; $\mathcal{J}=\{\emptyset,\{u_2\}\}$. $\mathcal{M}=\{\emptyset,\Gamma,\{u_2,u_3\}\}$. $nIs_\alpha g-$ Cl. S.s are $\emptyset,\Gamma,\{u_1\},\{u_2\},\{u_1,u_2\},\{u_1,u_3\}$. Let $\Delta=\{v_1,v_2,v_3\}$; $\Delta/\mathcal{R}=\{\{v_1,v_3\},\{v_2\}\}$; $\mathcal{Y}=\{v_3\}$; $\mathcal{J}'=\{\emptyset,\{v_1\}\}$. $\mathcal{M}'=\{\emptyset,\Gamma,\{v_1,v_3\}\}$.

$nIs_{\alpha}g -$ Cl. S.s are $\emptyset, \Delta, \{v_1\}, \{v_2\}, \{v_1, v_2\}, \{v_2, v_3\}$. Let $\Lambda = \{w_1, w_2, w_3\}$; $\Lambda/\mathcal{R} = \{\{w_1, w_2\}, \{w_3\}\}$; $\mathcal{Z} = \{w_2\}$; $\mathcal{J}'' = \{\emptyset, \{w_2\}\}$. $\mathcal{M}'' = \{\emptyset, \Lambda, \{w_1, w_2\}\}$. $nIs_{\alpha}g -$ Cl. S.s are $\emptyset, \Lambda, \{w_2\}, \{w_3\}, \{w_1, w_3\}, \{w_2, w_3\}$. Define $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ as $\eta(u_1) = v_1; \eta(u_2) = v_2; \eta(u_3) = v_3$. Define $\zeta: (\Delta, \mathcal{M}', \mathcal{J}') \to (\Lambda, \mathcal{M}'', \mathcal{J}'')$ as $\zeta(v_1) = w_1; \zeta(v_2) = w_3; \zeta(v_3) = w_2$. Both η and ζ are $nIs_{\alpha}g -$ Hompsm. As $\zeta \circ \eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Lambda, \mathcal{M}'', \mathcal{J}'')$, $(\zeta \circ \eta)(\{u_2, u_3\}) = \zeta(\eta(\{u_2, u_3\})) = \zeta(\{v_2, v_3\}) = \{w_2, w_3\}$ which is not $nIs_{\alpha}g -$ open in $(\Lambda, \mathcal{M}'', \mathcal{J}'')$ for $n -$ Op. S. $\{u_2, u_3\}$ of $(\Gamma, \mathcal{M}, \mathcal{J})$. Therefore, $\zeta \circ \eta$ is not $nIs_{\alpha}g -$ Hompsm.

4. *$nIs_{\alpha}g$ − Closed Maps

Definition 4.1 A map $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ is said to be *$nIs_{\alpha}g -$ Cl. Map. if for every $nIs_{\alpha}g -$ closed subset $\mathcal{H}$ of $(\Gamma, \mathcal{M}, \mathcal{J}$, $\eta(\mathcal{H})$ is $nIs_{\alpha}g -$ closed. The complement of *$nIs_{\alpha}g -$ Cl. Map. is *$nIs_{\alpha}g -$ Op. Map.

Example 4.1 Let $\Gamma = \{u_1, u_2, u_3, u_4\}$; $\Gamma/R = \{\{u_1\}, \{u_2, u_3\}, \{u_4\}\}$; $\mathcal{X} = \{u_1, u_3\}$ and $\mathcal{J} = \{\emptyset, \{u_2\}\}$. $\mathcal{M} = \{\emptyset, \Gamma, \{u_1\}, \{u_1, u_2, u_3\}, \{u_2, u_3\}\}$. $nIs_{\alpha}g -$ Cl. S.s are $\emptyset, \Gamma, \{u_2\}, \{u_4\}, \{u_1, u_4\}, \{u_2, u_4\}, \{u_3, u_4\}, \{u_1, u_2, u_4\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}$. Let $\Delta = \{v_1, v_2, v_3, v_4\}$; $\Delta/R = \{\{v_1\}, \{v_2, v_4\}, \{v_3\}\}$; $\mathcal{Y} = \{v_1, v_2\}$ and $\mathcal{J}' = \{\emptyset, \{v_2\}, \{v_3\}, \{v_2, v_3\}\}$. $nIs_{\alpha}g -$ Cl. S.s are $\emptyset, \Gamma, \{v_2\}, \{v_3\}, \{v_1, v_3\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4\}$. Define $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ by $\eta(u_1) = v_4, \eta(u_2) = v_2, \eta(u_3) = v_1, \eta(u_4) = v_4$. Here, η is *$nIs_{\alpha}g -$ Cl. Map.

Theorem 4.1 Every *$nIs_{\alpha}g -$ Cl. Map. is $nIs_{\alpha}g -$ Cl. Map.

Proof. Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ is *$nIs_{\alpha}g -$ Cl. Map. Let $\mathcal{H}$ be a $n -$ closed seubset of $(\Gamma, \mathcal{M}, \mathcal{J})$. Since every $n -$ Cl. S. is $nIs_{\alpha}g -$ Cl. S., $\mathcal{H}$ is $nIs_{\alpha}g -$ Cl. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$. Also, since η is *$nIs_{\alpha}g -$ Cl. Map. $\eta(\mathcal{H})$ is $nIs_{\alpha}g -$ Cl. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$ so that η is $nIs_{\alpha}g -$ Cl. Map.

Remark 4.1 A $nIs_{\alpha}g -$ Cl. Map. Need not be *$nIs_{\alpha}g -$ Cl. Map.

Example 4.2 Consider $(\Gamma, \mathcal{M}, \mathcal{J})$ and $(\Delta, \mathcal{M}', \mathcal{J}')$ of Example 4.1. Define $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ by $\eta(u_1) = v_4, \eta(u_2) = v_1, \eta(u_3) = v_2, \eta(u_4) = v_3$ which is $nIs_{\alpha}g -$ Cl. Map. For the $nIs_{\alpha}g -$ Cl. S. $\{u_2\}$ of $(\Gamma, \mathcal{M}, \mathcal{J})$, $\eta(\{u_2\}) = \{v_1\}$ is not $nIs_{\alpha}g -$ Cl. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$. Therefore, η is not *$nIs_{\alpha}g -$ Cl. Map.

Theorem 4.2 A map $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ is *$nIs_{\alpha}g -$ Cl. Map. if and only if for every $nIs_{\alpha}g -$ open subset $\mathcal{H}$ containing $\eta^{-1}(\mathcal{S})$, there is a $nIs_{\alpha}g -$ Op. S. $\mathcal{K}$ of $(\Delta, \mathcal{M}', \mathcal{J}')$, $\eta(\mathcal{H})$ such that $\mathcal{S} \subseteq \mathcal{K}$ and $\eta^{-1}(\mathcal{K}) \subseteq \mathcal{H}$.

Proof. Necessity: Let $\mathcal{H}$ be a $nIs_{\alpha}g -$ Op. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$. Then $\mathcal{H}^c$ is $nIs_{\alpha}g -$ Cl. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$. Since η is *$nIs_{\alpha}g -$ Cl. Map., $\eta(\mathcal{H}^c)$ is $nIs_{\alpha}g -$ Cl. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$.

Thus, $\Gamma - \eta(\mathcal{H}^c)$ is $nIs_{\alpha}g -$ Op. S., say $\mathcal{K}$ containing $\mathcal{S}$ such that $\eta^{-1}(\mathcal{K}) \subseteq \eta^{-1}\big(\Delta - \eta(\mathcal{H}^c)\big) = \Gamma - \mathcal{H}^c = \mathcal{H}$.
Sufficient: Let $\mathcal{H}$ be $nIs_{\alpha}g -$ Cl. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$. Then $\mathcal{H}^c$ is $nIs_{\alpha}g -$ Op. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$. By hypothesis, there exists a $nIs_{\alpha}g -$ Op. S. $\mathcal{K}$ of $(\Delta, \mathcal{M}', \mathcal{J}')$ such that $\mathcal{S} \subseteq \mathcal{K}$ and $\eta^{-1}(\mathcal{K}) \subseteq \mathcal{H}$ and so $\mathcal{H} \subseteq \big(\eta^{-1}(\mathcal{K})\big)^c = \eta^{-1}(\mathcal{K}^c)$ which implies $\eta(\mathcal{H}) = \mathcal{K}^c$. Since $\mathcal{K}^c$ is $*nIs_{\alpha}g -$ closed, then $\eta(\mathcal{H})$ is $*nIs_{\alpha}g -$ closed in $(\Delta, \mathcal{M}', \mathcal{J}')$. Hence, η is $*nIs_{\alpha}g -$ closed.

5. $*nIs_{\alpha}g -$ Homeomorphism

Definition 5.1 A bijection $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \rightarrow (\Delta, \mathcal{M}', \mathcal{J}')$ is said to be $*nIs_{\alpha}g -$ Hompsm. if both η and η^{-1} are $nIs_{\alpha}g -$ Irr.Fn.
Example 5.1 Let $\Gamma = \{u_1, u_2, u_3\}$; $\Gamma/\mathcal{R} = \{\{u_1, u_2\}, \{u_3\}\}$; $\mathcal{X} = \{u_1, u_3\}$; $\mathcal{J} = \{\emptyset, \{u_3\}\}$. $\mathcal{M} = \{\emptyset, \Gamma, \{u_3\}, \{u_1, u_2\}\}$. $\wp(\Gamma)$ is the $nIs_{\alpha}g -$ Cl. S.
Let $\Delta = \{v_1, v_2, v_3\}$; $\Delta/\mathcal{R} = \{\{v_1\}, \{v_2, v_3\}\}$; $\mathcal{Y} = \{v_1, v_2\}$; $\mathcal{J}' = \{\emptyset, \{v_2\}\}$.
$\mathcal{M}' = \{\emptyset, \Delta, \{v_1\}, \{v_2, v_3\}\}$. $\wp(\Delta)$ is the $nIs_{\alpha}g -$ Cl. S.
Define $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \rightarrow (\Delta, \mathcal{M}', \mathcal{J}')$ as $\eta(u_1) = v_1; \eta(u_2) = v_2; \eta(u_3) = v_3$. Both η and η^{-1} are $nIs_{\alpha}g -$ Irr.Fn. Hence, η is $*nIs_{\alpha}g -$ Hompsm.
Theorem 5.1 For any bijection $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \rightarrow (\Delta, \mathcal{M}', \mathcal{J}')$, the following axioms are equivalent.

(1) $\eta^{-1}: (\Delta, \mathcal{M}', \mathcal{J}') \rightarrow (\Gamma, \mathcal{M}, \mathcal{J})$ is $nIs_{\alpha}g -$ Irr.Fn.
(2) η is a $*nIs_{\alpha}g -$ Op. Map.
(3) η is $*nIs_{\alpha}g -$ Cl. Map.

Proof. (1) ⇒ (2) : Let $\mathcal{H}$ be a $nIs_{\alpha}g -$ Op. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$. Since η^{-1} is $nIs_{\alpha}g -$ Irr.Fn., $(\eta^{-1})^{-1}(\mathcal{H}) = \eta(\mathcal{H})$ is $nIs_{\alpha}g -$ open in $(\Delta, \mathcal{M}', \mathcal{J}')$. Hence, η is $*nIs_{\alpha}g -$ Op. Map.
(2) ⇒ (3) : Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \rightarrow (\Delta, \mathcal{M}', \mathcal{J}')$ be $*nIs_{\alpha}g -$ Op. Map. Let $\mathcal{H}$ be a $nIs_{\alpha}g -$ Cl. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$.Then $\Gamma - \mathcal{H}$ is $nIs_{\alpha}g -$ Op. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$.Since η is $*nIs_{\alpha}g -$ Op. Map., $\eta(\Gamma - \mathcal{H})$ is $nIs_{\alpha}g -$ Op. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$. This implies that $\eta(\mathcal{H})^c$ is $nIs_{\alpha}g -$ Op. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$ so that $\eta(\mathcal{H})$ is $nIs_{\alpha}g -$ Cl. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$. Therefore, η is $*nIs_{\alpha}g -$ Cl. Map.
(3) ⇒ (1): Assume that $\mathcal{H}$ is a $nIs_{\alpha}g -$ Cl. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$. Then by hypothesis, $(\eta^{-1})^{-1}(\mathcal{H}) = \eta(\mathcal{H})$ is $nIs_{\alpha}g -$ Cl. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$ so that η^{-1} is $nIs_{\alpha}g -$ Irr.Fn. map.
Remark 5.1 Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \rightarrow (\Delta, \mathcal{M}', \mathcal{J}')$ be bijevtive. η is said to be $*nIs_{\alpha}g -$ Hompsm. if η is both $nIs_{\alpha}g -$ Irr.Fn. and $*nIs_{\alpha}g -$ Op. Map.

Theorem 5.2 Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ be a bijective and \$$nIs$_\$alpha\ g -$ \$ Irr.Fn. map. Then the following statements are equivalent.

(1) η is a $*nIs_\alpha g -$ Op. Map.
(2) η is a $*nIs_\alpha g -$ Hompsm.
(3) η is a $*nIs_\alpha g -$ Cl. Map.

Proof. (1) ⇒ (2):Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ be $*nIs_\alpha g -$ Op. Map. Let $\mathcal{H}$ be a $nIs_\alpha g -$ Cl. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$.Then its complement $\mathcal{H}^c$ is $nIs_\alpha g -$ Op. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$.Since η is $*nIs_\alpha g -$ Op. Map., $\eta(\mathcal{H}^c)$ is $nIs_\alpha g -$ Op. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$. This implies that $\left(\eta(\mathcal{H})\right)^c$ is $nIs_\alpha g -$ Op. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$ so that $\left(\eta(\mathcal{H})\right)$ is $nIs_\alpha g -$ Cl. S. in $(\Delta, \mathcal{M}', \mathcal{J}')$. Therefore, η is $*nIs_\alpha g -$ Cl. Map. By Theorem 4.3, $\eta^{-1}: (\Delta, \mathcal{M}', \mathcal{J}') \to (\Gamma, \mathcal{M}, \mathcal{J})$ is $nIs_\alpha g -$ Irr.Fn. By hypothesis, η is $nIs_\alpha g -$ Irr.Fn. so that η is $*nIs_\alpha g -$ Hompsm.

(2) ⇒ (3) : Assume that η is $*nIs_\alpha g -$ Hompsm. Then η and η^{-1} are $nIs_\alpha g -$ Irr.Fn. By Theorem 4.3, η is $*nIs_\alpha g -$ Cl. Map.

(3) ⇒ (1) : The result is trivial.

Theorem 5.3 Every $*nIs_\alpha g -$ Hompsm. is $nIs_\alpha g -$ Hompsm.

Proof. Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ be $*nIs_\alpha g -$ Hompsm. Then η and η^{-1} are $nIs_\alpha g -$ Irr.Fn. and η is bijective.

Since every $nIs_\alpha g -$ Irr.Fn. function is $nIs_\alpha g -$ Cont.Fn., both η and η^{-1} are $nIs_\alpha g -$ Cont.Fn. Therefore, η is $nIs_\alpha g -$ Hompsm.

Remark 5.2 The reverse implication of the preceding theorem is not valid as shown in the successive example.

Example 5.2 Let $\Gamma = \{u_1, u_2, u_3, u_4\}$; $\Gamma/\mathcal{R} = \{\{u_1\}, \{u_2, u_3\}, \{u_4\}\}$; $\mathcal{X} = \{u_1, u_3\}$; $\mathcal{J} = \{\emptyset, \{u_3\}\}$. $\mathcal{M} = \{\emptyset, \Gamma, \{u_1\}, \{u_2, u_3\}, \{u_1, u_2, u_3\}\}$. $nIs_\alpha g -$ Cl. S.s are $\emptyset, \Gamma, \{u_3\}$, $\{u_4\}, \{u_1, u_4\}, \{u_2, u_4\}, \{u_3, u_4\}, \{u_1, u_2, u_4\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}$.

Let $\Delta = \{v_1, v_2, v_3, v_4\}$; $\Delta/\mathcal{R} = \{\{v_1, v_3\}, \{v_2\}, \{v_4\}\}$; $\mathcal{Y} = \{v_2, v_3\}$; $\mathcal{J}' = \{\emptyset, \{v_1\}\}$. $\mathcal{M}' = \{\emptyset, \Delta, \{v_2\}, \{v_1, v_3\}, \{v_1, v_2, v_3\}$. $nIs_\alpha g -$ Cl. S.s are $\emptyset, \Delta, \{v_1\}, \{v_4\}, \{v_1, v_4\}$, $\{v_2, v_4\}, \{v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4\}$.

Define $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ as $\eta(u_1) = v_1; \eta(u_2) = v_2; \eta(u_3) = v_3; \eta(u_4) = v_4$ which is $nIs_\alpha g -$ Hompsm. For the $nIs_\alpha g -$ Cl. S. $\{v_1\}$ in $(\Delta, \mathcal{M}', \mathcal{J}')$, $\eta^{-1}(\{v_1\}) = \{u_1\}$ is not $nIs_\alpha g -$ Cl. S. in $(\Gamma, \mathcal{M}, \mathcal{J})$ hence, η^{-1} is not $nIs_\alpha g -$ Irr.Fn. Therefore, η is $nIs_\alpha g -$ Hompsm. but not $*nIs_\alpha g -$Hompsm.

Theorem 5.4 Composition of two $*nIs_\alpha g -$ Hompsm. is $*nIs_\alpha g -$ Hompsm.

Proof. Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Delta, \mathcal{M}', \mathcal{J}')$ and $\zeta: (\Delta, \mathcal{M}', \mathcal{J}') \to (\Lambda, \mathcal{N}', \mathcal{I}')$ be $*nIs_\alpha g -$ Hompsm. respectively. Then $\zeta \circ \eta: (\Gamma, \mathcal{M}, \mathcal{J}) \to (\Lambda, \mathcal{N}', \mathcal{I}')$. Let $\mathcal{H}$ be $nIs_\alpha g -$ Op. S. in $(\Lambda, \mathcal{N}', \mathcal{I}')$. Since ζ is $nIs_\alpha g -$ Irr.Fn., $\zeta^{-1}(\mathcal{H})$ is $nIs_\alpha g -$ open in $(\Delta, \mathcal{M}', \mathcal{J}')$. Since η is $nIs_\alpha g -$ Irr.Fn., $\eta^{-1}(\mathcal{H})$ is $nIs_\alpha g -$ open in $(\Gamma, \mathcal{M}, \mathcal{J})$. Therefore, $\zeta \circ \eta$ is $nIs_\alpha g -$

Irr.Fn. Also, for the $nIs_{\alpha}g -$ Op. S. $\mathcal{H}$ in $(\Gamma, \mathcal{M}, \mathcal{J})$, $\eta(\mathcal{H})$ is $nIs_{\alpha}g -$ open in $(\Delta, \mathcal{M}', \mathcal{J}')$, since η^{-1} is $nIs_{\alpha}g -$ Irr.Fn. Since ζ is $nIs_{\alpha}g -$ Irr.Fn., $(\zeta \circ \eta)(\mathcal{H}) = \zeta(\eta(\mathcal{H}))$ is $nIs_{\alpha}g -$ open in $(\Lambda, \mathcal{N}', \mathcal{I}')$. Therefore, $(\zeta \circ \eta)^{-1}$ is $nIs_{\alpha}g -$ Irr.Fn. Hence, $\zeta \circ \eta$ is $*nIs_{\alpha}g -$Hompsm.

Theorem 5.5 The set $s^{*}nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$ is a group under the composition of mapping.

Proof. Define a binary operation $*$: $*nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J}) \times *nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J}) \rightarrow *nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$ by $\eta * \zeta = \eta \circ \zeta$ for all $\eta, \zeta \in^{*} nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$ and $\circ$ is the usual operation of map. Then by Theorem 4.9, $\eta \circ \zeta \in^{*} nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$. We know that the composition of maps associative. The identity map $I: (\Gamma, \mathcal{M}, \mathcal{J}) \rightarrow (\Gamma, \mathcal{M}, \mathcal{J})$ belonging to $*nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$ serves as the identity element. For any $\eta \in^{*} nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$, $\eta \circ \eta^{-1} = \eta \circ \eta^{-1} = I$. Hence, inverse exists for each element of $*nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$. $*nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$ forms a group under the composition of maps.

Theorem 5.6 Let $\eta: (\Gamma, \mathcal{M}, \mathcal{J}) \rightarrow (\Delta, \mathcal{M}', \mathcal{J}')$ be an $*nIs_{\alpha}g -$ Hompsm. Then η induces an isomorphism from the group $*nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$ onto the group $*nIs_{\alpha}g - h\left(\Delta, \mathcal{M}', \mathcal{J}'\right)$.

Proof. Let $\eta \in {}^{*}nIs_{\alpha}g - h(\Delta, \mathcal{M}', \mathcal{J}')$. Define a map $\Omega_{\eta}: {}^{*}nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J}) \rightarrow {}^{*}nIs_{\alpha}g - h(\Delta, \mathcal{M}', \mathcal{J}')$ by $\Omega_{\eta}(\sigma) = \eta \circ \sigma \circ \eta^{-1}$ for every $\sigma \in {}^{*}nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$. Then σ is a bijection. Now, for all $\zeta, \sigma \in {}^{*}nIs_{\alpha}g - h(\Gamma, \mathcal{M}, \mathcal{J})$,
$\Omega_{\eta}(\zeta \circ \sigma) = \eta \circ (\zeta \circ \sigma)\eta^{-1} = (\eta \circ \eta^{-1}) \circ (\eta\sigma\eta^{-1}) = \Omega_{\eta}(\zeta) \circ \Omega_{\eta}(\sigma)$.

6. Conclusion

In this paper, we introduce Homeomorphism using $nIs_{\alpha}g -$ closed sets and discussed some of its characteristics. Further, we investigated some of the equivalent conditions.

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