DYNAMICAL BEHAVIOR OF FEAR EFFECT ON A DISEASED PREY-PREDATOR MODEL WITH REFUGE AND PREDATOR HARVESTING

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Abstract:
In this paper, we examine fear in an eco-epidemiological model that includes refuge harvesting in the population of predators and infection in the population of prey.As a Holling type II functional response, the predator eats its prey at various rates. The stability of all biologically viable equilibrium points, as well as the positivity and boundedness of the solutions, have to be examined.To analyse this, the interior equilibrium of the system's Hopf-bifurcation is obtained.Our analytical conclusions are supported by numerical simulations.

Keywords: Bifurcation,Stability, Refuge, Predator harvesting, Fear.

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1 Introduction

Predator-prey interactions have been included in the Lotka-Volterra model for a very long time, see [1-3]. In a similar vein, after the seminal work of Kermack and McKendrick [4,5].The interaction of the susceptible, infected,and recovered has been an interesting topic of study.The original predator-prey model was developed in large part by Vito Volterra and Alfred James Lotka. Ecology models and epidemiology models are the two basic categories into which mathematical models are often divided. In ecological models studying the interactions between populations of a particular community are studied. Epidemiology models mean studying the spread of diseases between animals and humans.It is increasingly crucial to do research on the dynamics of illness within ecological systems. On the one hand, several studies of prey-predator dynamics have been conducted in recent decades, taking into account the impact of a range of biological characteristics in [6-9].Many mathematical models have been created and investigated in the field of epidemiology, taking into consideration various incidence rates and illnesses;[10-14] Experts were particularly interested in their recommended ecological models since it is well accepted that species harvesting is necessary for species coexistence. Ecology models and epidemiology models are the two basic categories into which mathematical models are often divided.There are three different forms of harvesting: constant, proportional to density, nonlinear, and others. All of these have been proposed and investigated.There have been several suggestions harvesting methods,of research and including harvesting continuously and depends on density in proportional harvesting [15-20].

This piece is structured as follows: The prey-predator system's past is described as Section 1. In Section 2, the mathematical formulation is presented. We talk about the positivity and boundedness of solutions.

In Section 3, for the hypothetical sick system. The existence of equilibrium points is described in Section 4.Local stability analyses in Section 5.The Hope-Bifurcation Analysis is found in Section 6.Results are presented numerically in Section 7.Finally, this paper concludes with a few observations about the suggested system in Section 7.

2 Model formation

Table provides detailed biological meanings for the parameters

Parameters	living organisms
X	Susceptible Prey
Y	Infected Prey
Z	Predator
r	The Prey rate of growth
K	Carrying capacity for the environment
a_1	Semi-saturation constant
α_1	The amount of susceptible prey feeding
b_1	Proportion of infected prey to predator
c	Ratio of conversion from prey to predator.
d_1	Slaughter rate for Infected Prey
d_2	mortality rate among predators
λ	occurrence of infections
H_1	The rate at which the predator can be caught
E_1	laborious cropping

The system of Equation is:

$$\left.\begin{aligned}\frac{dX}{dT} &= \frac{r_1X}{1+fZ}\left(1-\frac{X+Y}{K}\right)-\lambda YX-\frac{\alpha_1XZ}{a_1+X},\\ \frac{dY}{dT} &= \lambda YX-d_1Y-\frac{b_1(1-m)YZ}{a_1+(1-m)Y},\\ \frac{dZ}{dT} &= -d_2Z+\frac{cb_1(1-m)YZ}{a_1+(1-m)Y}+\frac{c\alpha_1XZ}{a_1+X}-H_1E_1Z.\end{aligned}\right\} \quad (2.1)$$

Then the system change into the non-dimensional .

Here,

$$x = \frac{X}{K}, y = \frac{Y}{K}, z = \frac{Z}{K}.$$

Now the system becomes,

$$\left.\begin{aligned}\frac{dx}{dt} &= \frac{rx(1-x-y)}{1+fz} - xy - \frac{\alpha xy}{a+x}\\ \frac{dy}{dt} &= yx - dy - \frac{\theta y(1-m)z}{a+(1-m)y}\\ \frac{dz}{dt} &= -\delta z + \frac{c\theta y(1-m)z}{a+(1-m)y} + \frac{c\alpha yz}{a+x} - hz\end{aligned}\right\} \quad (2.2)$$

here the conditions are,

$$r = \frac{r_1}{\lambda K}, \alpha = \frac{\alpha_1}{\lambda K}, h = \frac{H_1 E_1}{\lambda K}$$
$$d = \frac{d_1}{\lambda K}, \theta = \frac{b_1}{\lambda K}$$
$$a = \frac{a_1}{K}, \delta = \frac{d_2}{\lambda K}, f = \frac{F}{K}$$

Assuming the initial values are not negative $x(0) \geq 0, y(0) \geq 0,$ and $z(0) \geq 0$ in $\mathbb{R}^3_+$

3 The Positive and boundaries of solutions

THEOREM 3.1 *All the solutions of (2.2) are efficacy in* $\mathbb{R}^3_+$

Proof. since $x(0)$, $y(0)$, and $z(0)$ are all greater than or equal to zero,Then the system (2.2) becomes

$$\begin{aligned}x(t) &= x(0)exp\left(\int_0^1 \left[\frac{r(1-x-y)}{1+fz} - y - \frac{\alpha z}{a+x}\right] dx\right) \geq 0,\\ y(t) &= y(0)exp\left(\int_0^1 \left[x - d - \frac{\theta(1-m)z}{a+(1-m)y}\right] dy\right) \geq 0,\\ z(t) &= z(0)exp\left(\int_0^1 \left[-\delta + \frac{c\theta(1-m)y}{a+(1-m)y} + \frac{c\alpha xz}{a+x} - h\right] dz\right) \geq 0\end{aligned}.$$

then the solutions of (2.2) are positive. □

THEOREM 3.2 *All the solutions of (2.2) are bounded in* $\mathbb{R}^3_+$

Proof. Any solution to the system (2.2) with positive starting conditions, let $x(t)$, y(t), and $z(t)$.Then $\frac{dx}{dt} \leq rx(1-y)$

$\lim\sup t \to \infty\ x(t) \leq 1$,

Let $u = x + y + z$

$$\begin{aligned}\frac{du}{dt} &= \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt}\\ &= \frac{(1-y-x)rx}{fz+1} - dx - \frac{\alpha xz(1-c)}{z+a} - hz\\ &\quad - \frac{\theta yz(1-c)}{y+a} - z\delta\\ &\leq \frac{(1-y-x)rx}{fz+1} - zh - xd - z\delta \quad \text{(while, } c < 1\text{)}\\ &\leq \frac{r}{4} - xh - yd - z\delta \left((Max(\frac{rx(1-x)}{1+fz} = \frac{r}{4})\right)\\ &\leq \frac{\ }{4} - \gamma w,\end{aligned}$$

$$r\ \frac{dw}{dt} + \gamma w \leq \frac{r}{4}. \quad \text{during which, } \gamma = min(\delta, h)$$

$$0 < w \leq \frac{r}{4\gamma}(1 - \exp^{-\gamma t}) + w(x_0, y_0, z_0)\exp^{-\gamma n}.$$

To $n \to \infty, 0 < w < \frac{r}{4\gamma}$.for $\epsilon > 0$ 2.2 is bounded for the region,then

$$\Omega = \left\{(z, x, y) \in \mathbb{R}^3_+; z + x + y \leq \frac{r}{4\gamma} + \epsilon\right\}$$

□

4 The Presence of equilibrium points

In this section the Possible equilibrium points (2.2) are investigated. Five equilibrium points for the system (2.2) were observed.

$$\frac{rx(1-x-y)}{1+fz} - xy - \frac{\alpha xz}{a+x} = 0 ,$$
$$xy - dy - \frac{\theta(1-m)yz}{a+(1-m)y} = 0 ,$$
$$-\delta z + \frac{c\theta(1-m)yz}{a+(1-m)y} + \frac{c\alpha xz}{a+x} - hz = 0 .$$

- The trivial equilibrium point is $E_0(0,0,0)$.
- Equilibrium with no diseased prey and no predator $E_1(1,0,0)$,
- The equilibrium state free from predators or predation $E_3(x,\hat{}\ y,\hat{}\ 0)$,

$\hat{x} = d, \hat{y} = \frac{r(1-d)}{r+1}$.

- interior equilibrium is endemic $E^*(x^*,y^*,z^*)$,

$y^* = \frac{a(a(\delta+h)+((\delta+h)-c\alpha)x^*)}{(1-m)c\alpha x^*+(c\theta-(\delta+h)(a+x^*))}$,

$z^* = \frac{ac(x^*-d)(a+x^*)}{(1-m)(c\alpha x^*+(c\theta-(\delta+h)(a+x^*))}$,

and the $x^*$is the one and only non negative quadratic root,

$R+QS+PS^2 = 0$,

$R = -a(((1-m)r(c\theta-(\delta+h)+(c\alpha d-a(\delta+h)(1+r)))$.

$P = r(1-m)(c\alpha+c\theta-(\delta+h))$

$Q = (1-m)(c\theta-(\delta+h))(-r+ar)-c\alpha r+a(\delta+h)+(\delta+h)-c\alpha)r)$.

5 Analyses of local stability

In order to determine the system trait for the regional stability equation (1),we identify the system's Jacobian matrix. $J(E) = \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{pmatrix}$

Where,

$$w_{11} = \frac{r(1-2x)}{1+fz} - y(\frac{r}{1+fz}+1) - \frac{\alpha az}{(a+x)^2}, w_{12} = -x(\frac{r}{1+fz}+1)$$
$$w_{13} = -\frac{rfx(1-x-y)}{(1+fp)^2} - \frac{\alpha x}{a+x}, w_{21} = y, w_{22} = x-d-\frac{a\theta(1-m)z}{(a+(1-m)y)^2},$$
$$w_{23} = \frac{-\theta(1-m)y}{(a+(1-m)y)}, w_{31} = \frac{ac\alpha z}{(a+x)^2}, w_{32} = \frac{ac\theta(1-m)z}{(a+(1-m)y)^2},$$
$$w_{33} = -\delta + \frac{c\theta(1-m)y}{a+(1-m)y} + \frac{\alpha cx}{a+x} - h.,$$

THEOREM 5.1 *The Point of trivial equilibrium It is unstable for* $E_0(0,0,0)$.

Proof. $J(E_0) = \begin{pmatrix} r & 0 & 0 \\ 0 & -d & 0 \\ 0 & 0 & -h-\delta \end{pmatrix}$,

$\lambda_{01} = r, \lambda_{02} = -d, \lambda_{03} = -h-\delta$,

$\lambda_{01} > 0, \lambda_{02} < 0, \lambda_{03} < 0$ then E_0 is an unstable equilibrium point.

THEOREM 5.2 *The equilibrium point without ill prey and without predators it is stable,* $E_1(1,0,0)$.*If* $d > 1$, *otherwise unstable.*

Proof. $J(E_1) = \begin{pmatrix} -r & -(r+1) & \frac{-\alpha}{a+1} \\ 0 & d-1 & 0 \\ 0 & 0 & \frac{c\alpha}{a+1}-h-\delta \end{pmatrix}$

The location of the Jacobian array described above has the following characteristic equation:
$(\lambda_{11}-(-r))(\lambda_{12}-(1-d)(\lambda_{13}-(\frac{c\alpha}{a+1}-\delta-h))=0$, $\lambda_{11}=-r, \lambda_{12}=1-d, \lambda_{13}=\frac{c\alpha}{a+1}-\delta-h$,

Hence $E_1(1,0,0)$ is stable if $d > 1$, otherwise unstable.

THEOREM 5.3 *Locally asymptotically stable equilibrium point E_3 without predators.if*

$$\textit{Proof.}\quad J(E_3)=\begin{pmatrix} f_{11} & f_{12} & f_{13}\\ f_{21} & f_{22} & f_{23}\\ f_{31} & f_{32} & f_{33}\end{pmatrix}$$

where,

$$f_{11}=-rd, f_{12}=(-1-r)\hat{x}, f_{13}=-rfx(1-x-y)-\frac{\alpha\hat{x}}{a+x},$$
$$f_{21}=y, f_{22}=0, f_{23}=\frac{-\theta(1-\hat{m})y}{a+(1-\hat{m})y}$$
$$f_{31}=0, f_{32}=0, f_{33}=\frac{c\alpha\hat{x}}{a+\hat{x}}-\delta+\frac{c\theta(1-\hat{m})y}{a+(1-m)y}-h.,$$
$$\lambda^3+L\lambda^2+M\lambda+N=0.$$

Here,
$L \quad = -f11 - f33,$
$M \quad = -f21f12 + f33f11,\ N = f12f21f33.$

According to the Routh-Hurwitz criterion, all of the aforementioned feature's zeros have negative real portions iff L,M and $LM-N$ are positive,

Now,
$LM-N=-f_{11}(-f_{12}f_{21}+f_{33}(f_{33}+f_{11}))$.For f_{33} to be negative,if $(h+\delta)>(\theta+\alpha)c$.

If the aforementioned condition in the theorem is met, the E_3 is locally asymptotically stable.

THEOREM 5.4 *Locally, the asymptotically stable positive equilibrium point $E*$.*

$$\textit{Proof.}\quad J(E^*)=\begin{pmatrix} l_{11} & l_{12} & l_{13}\\ l_{21} & l_{22} & l_{23}\\ l_{31} & l_{32} & l_{33}\end{pmatrix}$$

where,

$$l_{11}=-\frac{x*(-r+ar+(1+r)y^*+2rx^*)}{a+x^*}, l_{12}=-x^*(r+1), l_{13}=-\frac{\alpha x^*}{a+x^*}$$
$$l_{21}=y^*, l_{22}=\frac{a\theta z^*y^*(1-m)^2}{(a+(1-m)y^*)^2}, l_{23}=-\frac{\theta y^*(1-m)}{(a+y^*(1-m))}$$
$$l_{31}=\frac{ac\alpha z^*}{(a+x^*)^2}, l_{32}=\frac{ac\theta(1-m)y^*}{(a+(1-m)x^*)^2}, l_{33}=0.,$$

Then $\lambda^3+L\lambda^2+M\lambda+N=0$,
$L \quad = -l11 - l33,$
$M \quad = -l21l12 + l22l11 - l13l31 + l23l32,$
$N \quad = l13(-l22l31 + l21l32) + l23(l12l31 - l11l32).$

If$LM-N>0, M>0, L>0$,. Routh-Hurwitz criteria state that Every single one nothing in the aforementioned feature the actual unfavourable portions iff L,M and $LM-N$ are positive.
Asymptotically local stability exists for the $E*$. □

6 Hopf-Bifurcation Analysis

THEOREM 6.1 If the critical value for the bifurcation parameter q_1 is exceeded, the model (2.2) experiences the Hope-bifurcation. the existence of the following Hope-bifurcation criteria at $q_1 = q_1*$
$1.A_1(q_1*)A(q_1^*) - A_3(q_1^*) = 0$.

Proof. For $h_1 = q_1^*$,
$(\lambda^2(q_1^*) + A_2(q_1^*))(\lambda(q_1^*) + A_1(q_1^*)) = 0$. (6.1)
$\implies \pm i\sqrt{A_2(q_1^*)}$ and $-A_1(q_1^*)$ be the zeros of the above equation.The following transversality requirement must be satisfied in order to achieve the Hopf-bifurcation at $q_1 = q_1*$.
$\frac{d}{dq_1^*}(Re(\lambda(q_1^*)))| \neq 0$.

The generic roots of the aforementioned equation are (6.1) for all q_1.
$\lambda_1 = r(q_1) + is(q_1)$, $\lambda_2 = r(q_1) - is(q_1)$, $\lambda_3 = -A_1(q_1)$.

Now, we examine the situation. $\frac{d}{dq_1^*}(Re(\lambda(q_1^*)))| \neq 0$.
Let $\lambda_1 = r(q_1) + is(q_1)$ in the (6.1), we get
$A(q_1) + iB(q_1) = 0$.

Where,
$A(q_1) = r^3(q_1) + r^2(q_1)A_1(q_1) - 3r(q_1)s^2(q_1) - s^2(q_1)A_1V + A_2(q_1)r(q_1) + A_1(q_1)A_2(q_1)$, $B(q_1) = A_2(q_1)s(q_1) + 2r(q_1)s(q_1)A_1(q_1) + 3r^2(q_1)s(q_1) + s^3(q_1)$.

$$\frac{d\mathcal{A}}{dq_1} = \varsigma_1(q_1)r'(q_1) - \varsigma_2(q_1)s'(q_1) + \varsigma_3(q_1) = 0, \quad (6.2)$$

$$\frac{d\mathcal{B}}{dq_1} = \varsigma_2(q_1)r'(q_1) + \varsigma_1(q_1)s'(q_1) + \varsigma_4(q_1) = 0, \quad (6.3)$$

where,
$\varsigma_1 = 3r^2(q_1) + 2r(q_1)A_1(q_1) - 3s^2(q_1) + A_2(q_1)$, $\varsigma_2 = 6r(q_1)s(q_1) + 2s(q_1)a_1(q_1)$,
$\varsigma 3 = r2(q1)A'1(q1) + s2(q1)A1'(q1) + A'2(q1)r(q1)$,
$\varsigma_4 = A_2(q_1)s(q_1) + 2r(q_1)s(q_1)A_1(q_1)$.

On multiplying (6.2) by $\varsigma_1(q_1)$ and (6.3) by $\varsigma_2(q_1)$ respectively

$$r(q_1)' = -\frac{\varsigma_1(q_1)\varsigma_3(q_1) + \varsigma_2(q_1)\varsigma_4(q_1)}{\varsigma_1{}^2(q_1) + \varsigma_2{}^2(q_1)}. \quad (6.4)$$

Substituting $r(q_1) = 0$ and $s(q_1) = \sqrt{A_2(q_1)}$ at $q_1 = q_1^*$ on $\varsigma_1(q_1), \varsigma_2(q_1), \varsigma_3(q_1)$, and $\varsigma_4(q_1)$, we obtain

$$\varsigma_1(q_1^*) = -2A_2(q_2^*),$$
$$\varsigma_2(q_1^*) = 2A_1(q_1^*)\sqrt{A_2(q_1^*)}$$
$$\varsigma_3(q_1^*) = A_3'(q_1^*) - A_2(q_1^*)A_1'(q_1^*)$$
$$\varsigma_4(q_1^*) = A_2'(q_1^*)\sqrt{A_2 q_1^*}.$$

The equation (6.4), implies

$$r'(q_1^*) = \frac{A_3'(q_1^*) - (A_1(q_1^* A_2(q_1^*)))}{2(A_2(q_1^*) + A_1^2(q_1^*))}, \quad (6.5)$$

if $A_3'(q_1^*) - (A_1(q_1^*)A_2(q_1^*))' \neq 0 \implies \frac{d}{dq_1^*}(Re(\lambda(q_1^*)))| \neq 0$, and $\lambda_3(q_1^*) = -A_1(q_1^*) \neq 0$.
$A_3'(q_1^*) - (A_1(q_1^*)A_2(q_1^*))' \neq 0$ is ensured if the transversality criterion holds, and at this point, the model (2.2) enters the Hopf-bifurcation at $q_1 = q_1^*$ □

7 Numerical Simulations

The system is numerically simulated in this part to verify the theoretical findings (2.2).The extent of attack and extraction were examined in the present research. h will be the two key variables that serve as control parameters. For the specified set of variables for the computational simulation is performed using the MATLAB software package.

Parameters	Indicative number
m	Variable
β	Variable
α	Variable
h	0.1
a	0.2
d	0.6
r	0.3
δ	0.4
c	0.5
θ	0.7

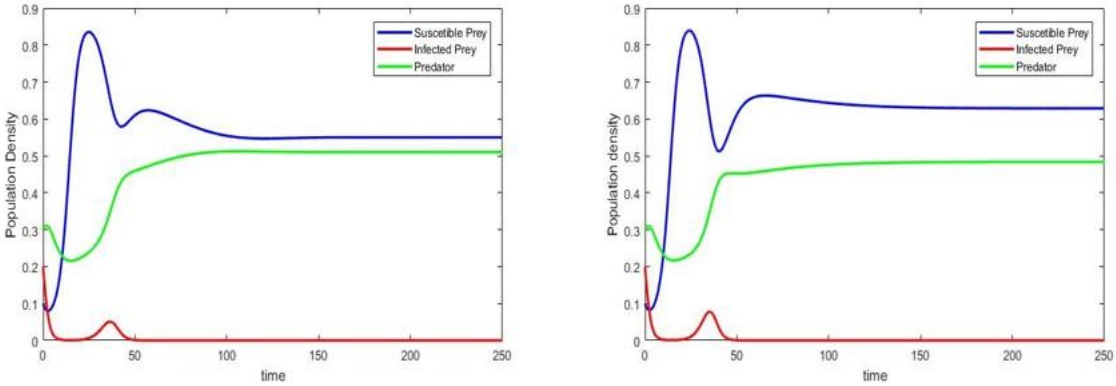

Figure 1: with the exception $h = 0.1$ and $\alpha = 0.3$,the parametric values in the table represent the time series of the system (2.2) at equilibrium point E_2.The time series below has the criteria displayed in the graph, the difference that exception $h = 0.1$ and $\alpha = 0.28$. are used in the vicinity of equilibrium point E_4

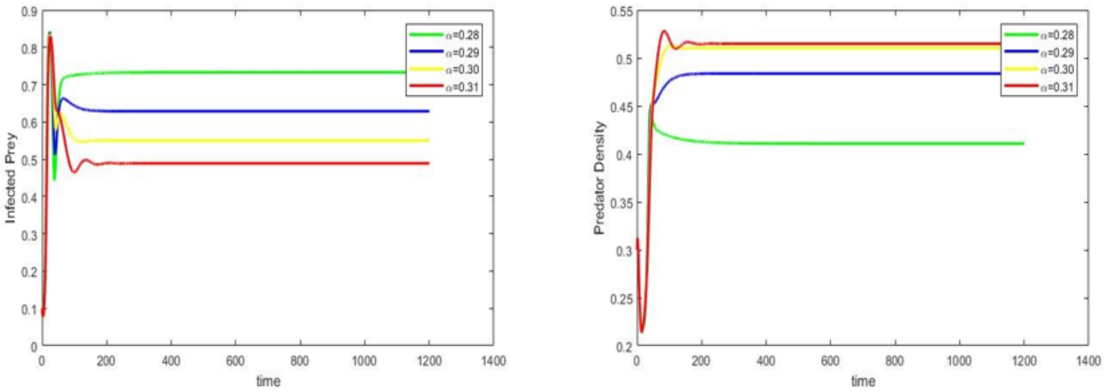

Figure 2: shows that a decrease in infected prey happens when the incidence of susceptible predation increases. shows that as the predator density rises, so do the rates of predation on susceptible prey.

7.1 Predation rate changes and their effects α

7.2 Changes to the harvesting rate Some impacts of h

The endemic equilibrium with regard to the parametric parameters indicated in the table with point E_4 and the balance with no sickness coexist with E_2 both exist with $\alpha = 0.25$. $h < 0.1$ and $0.0105264 < h < 0.773307$, respectively.

7.3 Changes the refuge constant m's effects

8 conclusion

This study used refuge predator harvesting, in which the predator eats both susceptible and diseased prey, to analyse the dynamical behaviour of the fear impact on a model of a sick prey predator. The created system (2) is physically well-behaved, per its boundaries, and optimism results. Three points of equilibrium are obtained.If all points are in harmony and the interior Local equilibrium points are asymptotically stable under specific conditions, with the exception of the unstable ill, prey-free, and predator-free equilibrium points as well as the trivial equilibrium point. Then, by choosing a bifurcation

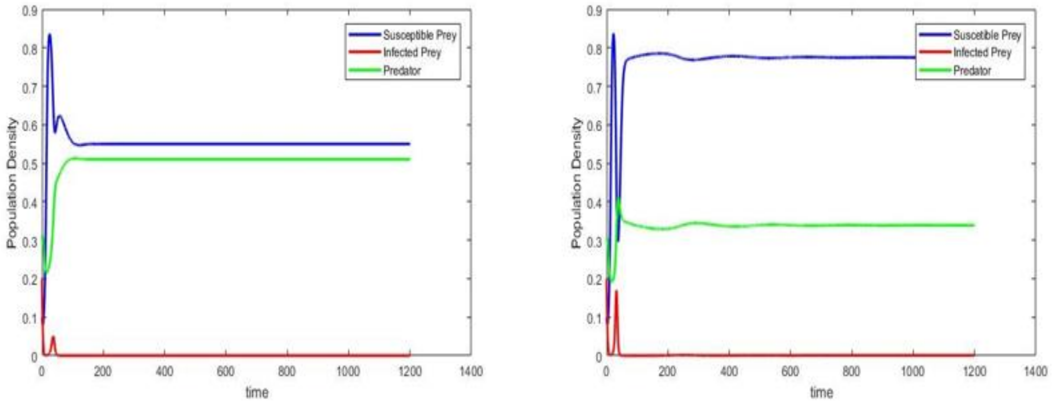

Figure 3: (*a*)Customizable values are included in the table. except *h* = 0.01 and *α* = 0.25.reflect the chronology of the system (2.2) at equilibrium point E_2,(*b*) With the exception of the equilibrium point, the following time series employs the identical parametric parameters as those in the table E_4 exception *h* = 0.07 and *α* = 0.23

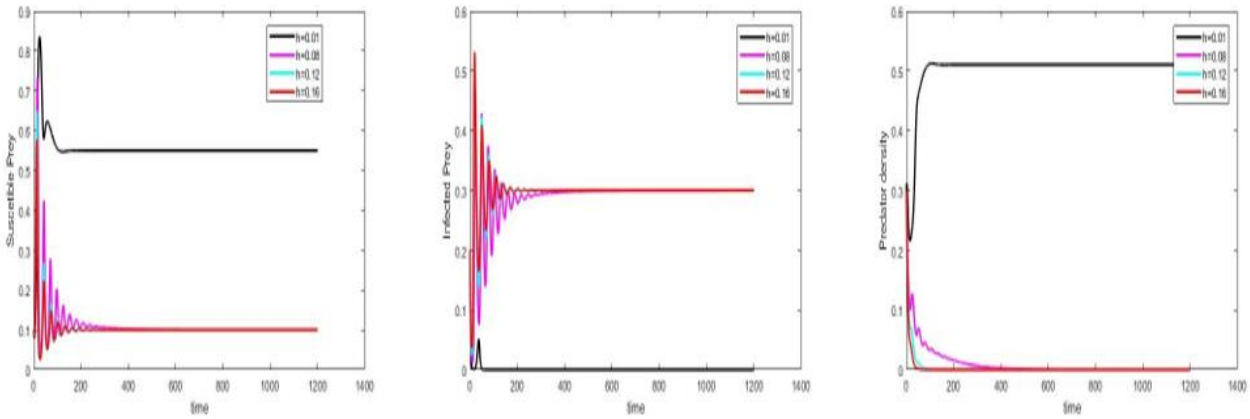

Figure 4: The density of(*a*) SPP(*b*)IPP(*c*)PP the table's parameter settings and h=0.01,0.14,0.16,0.18 *α* = 0.2

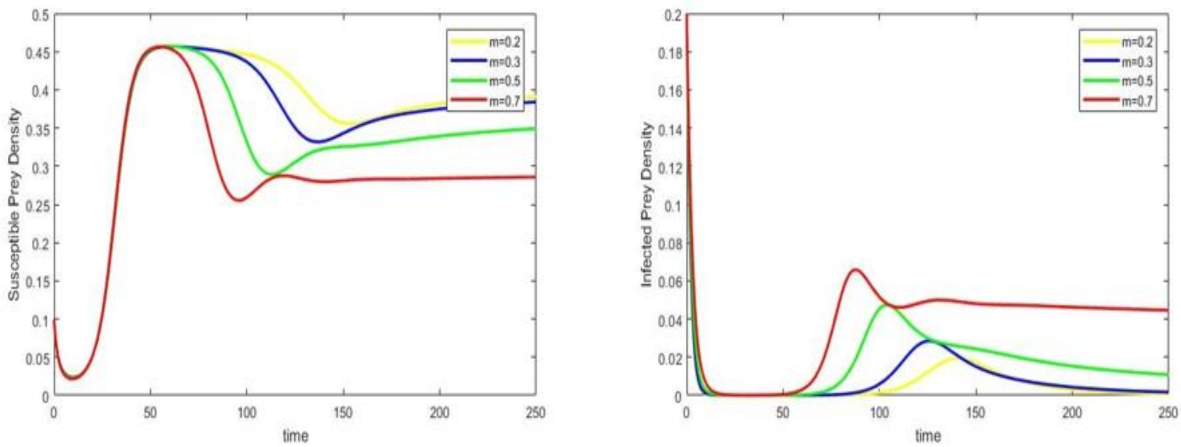

Figure 5: (a)We can see that when the refuge constant rises, the density of the sensitive prey population declines.(b)illustrates a rise in the number of the refuge constant m increases from 0.3 to 0.6, infected prey.. parameter from the constant q, we discover that the Hopf bifurcation takes place rather near to the interior equilibrium.

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