INSTABILITY OF SUSPENDED PARTICLES ON VISCO-ELASTIC ROTATING FLUID FLOW WITH THERMAL CONVECTION

Devilal Kumawat[1*]

Abstract –
In this paper, the effect of suspended particles and rotation on viscoelastic fluid layer heated from below saturating a porous medium in the presence of uniform vertical magnetic field has been studied. The dispersion relation was obtained using the normal mode and the problem has been numerically analyzed by MATLAB. The effect of medium permeability, magnetic field, suspended particles and rotation have been obtained and the effect of suspended particle on the system is very important result. The condition of over stability is also obtained.
2020 Mathematics Subject Classification: 76A10, 76D05, 76D10.

Keywords - Thermal Convection, Visco-elastic Fluid, Suspended Particle, Porous Medium and Rotation.

[1]*Department of Mathematics and Statistics, Jai Narain Vyas University, Jodhpur, India
E-mail: devilalkumawat225@gmail.com

***Corresponding Author: -** Devilal Kumawat
*Department of Mathematics and Statistics, Jai Narain Vyas University, Jodhpur, India
E-mail: devilalkumawat225@gmail.com

DOI: 10.53555/ecb/2023.12.si10.00469

1 Introduction

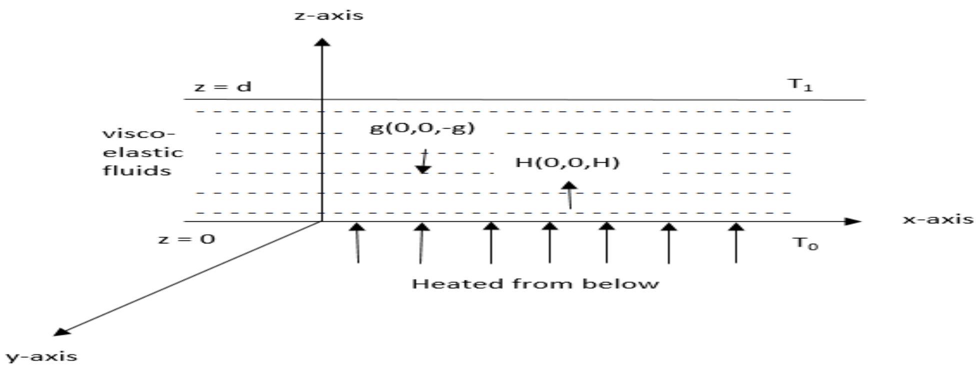
In order to study of visco-elastic fluids, the general theory of viscoelastic fluid was developed by Oldroyd [10]. Beard and Walters [1] investigated the viscous elastic boundary layer flow. Sharma [11] discussed the thermal convection on visco-elastic fluid. Kumar and Kumar [7] analyzed the dust particles on viscoelastic fluid with thermal convection. Singh and Gupta [13] investigated the thermal instability of dust particles on viscoelastic fluid flow. Choudhary and Das [2] discussed the unsteady MHD of visco-elastic fluid flow between two parallel plates.

Kumar and Mehta [6] studied the hall effect of visco-elastic fluid flow. The theory of micro polar fluid and viscous fluid in a vertical channel was introduced Kumar et al. [5]. Wooding [14] investigated the Rayleigh instability in a porous medium. Kumar and Mohan [4] analyzed the hall effect of oldroydian viscoelastic fluid in porous medium. Khanduri and Sharma [12] discussed the entropy analysis for MHD flow with thermal conductivity. Khan and Sasmal [3] studied the electro-elastic instability of visco-elastic fluid in a porous medium. Moatimid et al. [9] analyzed the instability of two viscoelastic fluid flow with heat transfer.

In view of the above discussion, application of the viscoelastic fluid in geophysics, film lubrication, chemical technology and industry. In this paper, I attempt to study the instability of suspended particles on visco-elastic rotating fluid flow with thermal convection. To the best of my knowledge, Darcy's generalized model has not yet been used to study this problem.

2 Mathematical Formulation

A horizontal, infinite and incompressible viscoelastic fluid layer of thickness d is assumed. The lower boundary at $z=0$ and upper boundary at $z=d$ are continued at constant but variable temperatures T_0 and T_1 such that a study adverse temperature gradient $\beta=\left|\frac{dT}{dz}\right|$ has been maintained. The uniform vertical magnetic field, rotation $\Omega=(0,0,\Omega)$ and gravity $g=(0,0,-g)$ are applied along z-axis.



The equation of continuity and motion are

$$\nabla.\vec{q}=0 \tag{2.1}$$

$$\frac{\rho_0}{\in}\left[\frac{\partial\vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right]=-\nabla P-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}+\mu\nabla^2\vec{q}-\rho g\hat{e}_z+\frac{2\rho_0}{\in}(\vec{q}\times\Omega)+\frac{KN}{\in}(\vec{q}_d-\vec{q})+\frac{\mu_e}{4\pi}(\nabla\times\vec{H})\times\vec{H} \tag{2.2}$$

where, P – Pressure, ρ – Fluid density, ρ_0 – Reference density, μ - Viscosity, μ' - Viscoelasticity, $\vec{q}$ – Velocity, $\vec{q}_d(x,t)$ – Velocity of suspended particles, μ_e – Magnetic permeability, $\hat{e}_z$ – Unit vector in z-direction, t – time, $N(x,t)$ – Number density of suspended particles, $x(x,y,z)$ and $K=6\pi\mu r$, r - being the particle radius, is the stokes drag coefficient.

The equation of energy, Maxwell's equation and basic's state are

$$\left[\in \rho_0 C_v + (1-\in)\rho_s C_s\right]\frac{\partial T}{\partial t} + \rho_0 C_v (\vec{q}.\nabla)T + mNC_{pt}\left(\in \frac{\partial T}{\partial t} + \vec{q}_d.\nabla T\right) = \chi \nabla^2 T \quad (2.3)$$

$$\in \frac{\partial \vec{H}}{\partial t} = \nabla \times (\vec{q}\times\vec{H}) + \in \eta \nabla^2 \vec{H} \quad (2.4)$$

$$\nabla.\vec{H} = 0 \quad (2.5)$$

$$\rho = \rho_0\left[1-\alpha(T-T_a)\right] \quad (2.6)$$

where, $C_v -$ Specific heat at constant volume, $\vec{H} = (0,0,H_z)$, $H_z -$ Constant, $C_s -$ Specific heat of solid (Porous Material Matrix), $C_{Pt} -$ Specific heat of suspended particles, $\eta -$ Magnetic viscosity, $\rho_s -$ Density of solid matrix, T – Temperature, $\chi -$ Thermal conductivity, $\alpha -$ Coefficient of thermal expansion, $T_a -$ Average temperature is given by $T_a = \frac{(T_0+T_1)}{2}$.

The equations of motion and continuity for the particles are

$$mN\left[\frac{\partial \vec{q}_d}{\partial t} + \frac{1}{\in}(\vec{q}_d.\nabla)\vec{q}_d\right] = KN(\vec{q}-\vec{q}_d) \quad (2.7)$$

$$\in \frac{\partial N}{\partial t} + \nabla.(N\vec{q}_d) = 0 \quad (2.8)$$

where, $mN -$ Mass of suspended particles pre unit volume, $KN(\vec{q}-\vec{q}_d) -$ Extra body force per unit volume.

3 Basic State of Problem

The basic state is

$$\vec{q} = \vec{q}(0,0,0),\ \vec{H} = (0,0,H_z), \vec{q}_d = (\vec{q}_d)(0,0,0),$$

$$\rho = \rho(z) \text{ and } P = P(z)$$

Using this condition, equations (2.1) to (2.8), becomes

$$\frac{dP_b}{dz} + \rho_b\, g = 0 \quad (3.1)$$

$$T = -\beta z + T_a = T_b(z), \text{ where } \beta = \frac{(T_1 - T_0)}{d} \quad (3.2)$$

$$\rho_b = \rho_0 + \alpha\beta z \rho_0 \quad (3.3)$$

4 Linearize Perturbation Equations

Now, linearize the equation of (2.1) to (2.8), we have

$$\nabla.\vec{q}\,' = 0 \quad (4.1)$$

$$L\left[\frac{\rho_0}{\in}\frac{\partial \vec{q}\,'}{\partial t}\right] = L\left[-\nabla P' - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\vec{q}\,'\right] + L\left[\mu\nabla^2\vec{q}\,' + \rho_0\alpha\theta g + \frac{2\rho_0}{\in}(\vec{q}\,'\times\Omega)\right] + L\left[\frac{\mu_e H_z}{4\pi}(\nabla\times\vec{h})\times\hat{e}_z\right] - \frac{mN_0}{\in}\frac{\partial \vec{q}\,'}{\partial t} \quad (4.2)$$

$$L\left[E + h_T \in\right]\frac{\partial \theta}{\partial t} = L\left[k_T\nabla^2\theta + \beta(\vec{q}\,')_z\right] + h_T\beta(\vec{q}\,')_z \quad (4.3)$$

$$\in \frac{\partial \vec{h}}{\partial t} = H_z \times (\vec{q}\,'\times\hat{e}_z) + \in \eta\nabla^2\vec{h} \quad (4.4)$$

$$\nabla.\vec{h} = 0 \quad (4.5)$$

$$\rho' = -\rho_0\alpha\theta \quad (4.6)$$

where, $k_T = \frac{\chi}{\rho_0 C_v}$, $h_T = \frac{mN_0 C_{pt}}{\rho_0 C_v}$ are the thermal diffusivity and $L = \left[\frac{m}{K}\frac{\partial}{\partial t}+1\right]$, $E = \in + \frac{(1-\in)\rho_s C_s}{\rho_0 C_v}$.

Converting equation (4.1) to (4.6) by the following transformation $\vec{q}\,' = \frac{k_T}{d}\vec{q}*$, $\nabla = \frac{\nabla *}{d}$, $\Omega = \frac{\mu}{\rho_0 d^2}\Omega*$, $P' = \frac{\mu k_T}{d^2}P*$, $\vec{h} = H_z h*$, $t = \frac{\rho_0 d^2}{\mu}t*$, $L = \tau\frac{\partial}{\partial t*}+1$, $\tau = \frac{m\mu}{\rho_0 d^2}$ and $\theta = \beta d\,\theta*$, we have

$$\nabla.\vec{q} = 0 \quad (4.7)$$

$$L\frac{1}{\in}\frac{\partial \vec{q}}{\partial t} = L\left[-\nabla P - \frac{1}{K_1}\left(1 + F\frac{\partial}{\partial t}\right)\vec{q} + R\theta\hat{e}_z\right] + L\left[\nabla^2\vec{q} + \frac{2}{\in}(\vec{q}\times\Omega) + Q(\nabla\times\vec{h})\times\hat{e}_z\right] - \frac{f}{\in}\frac{\partial \vec{q}}{\partial t} \quad (4.8)$$

$$LP_rE_r\frac{\partial\theta}{\partial t}=L\left[\nabla^2\theta+(\vec{q})_z\right]+h_T(\vec{q})_z \quad (4.9)$$

$$\in P_r\frac{\partial\vec{h}}{\partial t}=\frac{\partial\vec{q}}{\partial z}+\frac{\in P_r}{P_m}\nabla^2\vec{h} \quad (4.10)$$

where $Q=\frac{\mu_e H_z^2 d^2}{4\pi\mu k_T}-$ Chandrasekhar number, $R=\frac{\rho_0\alpha\beta g d^4}{k_T\mu}-$ Thermal Rayleigh number, $F=\frac{\mu'}{\rho_0 d^2}-$ Viscoelastic Parameter, $P_r=\frac{\mu}{k_T\rho_0}-$ Prandtl number, $P_m=\frac{\mu}{\rho_0\eta}-$ Magnetic Prandtl number, $f=\frac{mN_0}{\rho_0}$, $E=\in+\frac{(1-\in)\rho_sC_s}{\rho_0C_v}$, $K_1=\frac{k_1}{d^2}$, $E_r=E+h_T\in$, and $W=\vec{q}.\hat{e}_z$.

5 Boundary Condition

The boundary condition is

$$W=\frac{d^2W}{dz^2}=0,\theta=0\, at\, z=0\, and\, z=d. \quad (5.1)$$

6 Dispersion Relation

Taking curl on both side equation (4.8), we have

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}\left(1+F\frac{\partial}{\partial t}\right)-\nabla^2\right\}L+\frac{f}{\in}\frac{\partial}{\partial t}\right](\nabla\times\vec{q})$$
$$=L\left[R\left(\frac{\partial\theta}{\partial y}\hat{e}_x+\frac{\partial\theta}{\partial x}\hat{e}_y\right)+\frac{2}{\in}\nabla\times(\vec{q}\times\Omega)+Q\frac{\partial}{\partial z}\left(\nabla\times\vec{h}\right)\right] \quad (6.1)$$

Let $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$, $D=\frac{\partial}{\partial z},\zeta_z=(\nabla\times\vec{q})_z$, $m_z=\left(\nabla\times\vec{h}\right)\hat{e}_z$ and $\vec{h}_z=\vec{h}$, $\hat{e}_z$ is the z-component of vorticity.

Again, applying curl on both sides of equation (6.1) and (4.10), taking z-component on both side, we have

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}\left(1+F\frac{\partial}{\partial t}\right)-\nabla^2\right\}L+\frac{f}{\in}\frac{\partial}{\partial t}\right]\nabla^2W$$
$$=L\left[R\nabla_1^2\theta-\frac{2}{\in}\Omega(D\zeta_z)+QD\left(\nabla^2\vec{h}_z\right)\right] \quad (6.2)$$

$$\in P_r\frac{\partial m_z}{\partial t}=D\zeta_z+\frac{\in P_r}{P_m}\nabla^2 m_z \quad (6.3)$$

Taking z-component on the both side of equation (6.1), (4.9) and (4.10), we have

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}\left(1+F\frac{\partial}{\partial t}\right)-\nabla^2\right\}L+\frac{f}{\in}\frac{\partial}{\partial t}\right]\zeta_z$$
$$=L\left[\frac{2\Omega}{\in}DW+QDm_z\right] \quad (6.4)$$

$$LP_rE_r\frac{\partial\theta}{\partial t}=L\left[\nabla^2\theta+W\right]+h_TW \quad (6.5)$$

$$\in P_r\frac{\partial\vec{h}_z}{\partial t}=DW+\frac{\in P_r}{P_m}\nabla^2\vec{h}_z \quad (6.6)$$

Now, the boundary condition (5.1) becomes

$$W=D^2W=Dm_z=h_z=\zeta_z=D\zeta_z=\theta \quad at\, z=0\, to\, z=1 \quad (6.7)$$

Normal Mode Analysis

Let $[W,m_z,\zeta_z,h_z,\theta]=\begin{bmatrix}W(z),M(z),X(z),\\ B(z),\Theta(z)\end{bmatrix}\exp.\left[ik_xx+ik_yy+\sigma t\right]$

Applying above normal mode of the equation (6.2) to (6.6), we have

$$\left[\left\{\frac{\sigma}{\in}+\frac{1}{K_1}(1+F\sigma)-\left(D^2-a^2\right)\right\}(1+\tau\sigma)+\frac{f}{\in}\sigma\right]$$
$$\left(D^2-a^2\right)W=(1+\tau\sigma)\begin{bmatrix}-Ra^2\Theta-\frac{2}{\in}\Omega DX\\ +QD\left(D^2-a^2\right)B\end{bmatrix} \quad (6.8)$$

$$\left[\left\{\frac{\sigma}{\in}+\frac{1}{K_1}(1+F\sigma)-\left(D^2-a^2\right)\right\}(1+\tau\sigma)+\frac{f}{\in}\sigma\right]X$$
$$=(1+\tau\sigma)\left[\frac{2}{\in}\Omega DW+QDM\right] \quad (6.9)$$

$$\left[\left\{E_rP_r\sigma-\left(D^2-a^2\right)\right\}(1+\tau\sigma)\right]\Theta=(1+\tau\sigma)W+h_TW \quad (6.10)$$

$$\left[\in P_r\sigma-\frac{\in P_r}{P_m}\left(D^2-a^2\right)\right]M=DX \quad (6.11)$$

$$\left[\in P_r\sigma-\frac{\in P_r}{P_m}\left(D^2-a^2\right)\right]B=DW \quad (6.12)$$

where $\sigma=\sigma_r+i\sigma_r$ is the stability parameter and $a^2=k_x^2+k_y^2$ is the wave number.

Now, the boundary condition becomes

$$W = D^2W = 0 = X = DX = M = DM, \Theta = 0$$
$$at\ z = 0\ to\ z = 1 \qquad (6.13)$$
$$D^{2n}W = 0\ at\ z = 0\ to\ z = 1, \quad n > 0.$$

The proper solution of equation (6.13)

$W = W_0 \sin \pi z$, W_0 - Constant.

Eliminating $\Theta, B\ and\ X$ from equation (6.8) to (6.12), putting the value of $W\ and\ b = \pi^2 + a^2$, we have

$$b\left[\begin{matrix}\left\{\left(\frac{\sigma}{\in} + \frac{1}{K_1}(1+F\sigma) + b\right)(1+\tau\sigma) + \frac{f}{\in}\sigma\right\}\left[\left(P_rE_r\sigma + b\right)(1+\tau\sigma)\right]\left[P_r\sigma + \frac{P_r}{P_m}b\right] \\ \left[\left\{\left(\frac{\sigma}{\in} + \frac{1}{K_1}(1+F\sigma) + b\right)(1+\tau\sigma) + \frac{f}{\in}\sigma\right\}\left\{\in P_r\sigma + \frac{\in P_r}{P_m}b\right\} + (1+\tau\sigma)Q\pi^2\right]\end{matrix}\right]$$

$$= Ra^2\left[\begin{matrix}\left[\left\{\left(\frac{\sigma}{\in} + \frac{1}{K_1}(1+F\sigma) + b\right)(1+\tau\sigma) + \frac{f}{\in}\sigma\right\}\left\{\in P_r\sigma + \frac{\in P_r}{P_m}b\right\} + (1+\tau\sigma)Q\pi^2\right] \\ (1+\tau\sigma)\{1+\tau\sigma + h_T\}\left\{P_r\sigma + \frac{P_r}{P_m}b\right\}\end{matrix}\right]$$

$$-\frac{4\Omega^2\pi^2}{\in}(1+\tau\sigma)\left[P_r\sigma + \frac{P_r}{P_m}b\right]^2\left[\left(P_rE_r\sigma + b\right)(1+\tau\sigma)\right]$$

$$-\frac{Qb\pi^2}{\in}\left[\begin{matrix}\left[\left(P_rE_r\sigma + b\right)(1+\tau\sigma)\right](1+\tau\sigma) \\ \left[\left\{\left(\frac{\sigma}{\in} + \frac{1}{K_1}(1+F\sigma) + b\right)(1+\tau\sigma) + \frac{f}{\in}\sigma\right\}\left\{\in P_r\sigma + \frac{\in P_r}{P_m}b\right\} + (1+\tau\sigma)Q\pi^2\right]\end{matrix}\right] \qquad (6.14)$$

7 Stationary Convection

Putting $\sigma = 0$ in equation (6.14), we have

$$R = \frac{1}{a^2(1+h_T)}\left[b^2\left(\frac{1}{K_1} + b\right) + \frac{Q\pi^2b}{\in}\frac{P_m}{P_r} + \frac{\frac{4\Omega^2\pi^2b^2}{\in}\frac{P_r}{P_m}}{\left[\left\{\left(\frac{1}{K_1} + b\right)\frac{\in P_rb}{P_m} + Q\pi^2\right\}\right]}\right] \qquad (7.1)$$

Neglecting the magnetic field, suspended particles and rotation, we have

$$R = \frac{b^2}{a^2}\left(\frac{1}{K_1} + b\right) \qquad (7.2)$$

Again, we take non-porous medium $(K_1 \to \infty)$ then equation (7.2), reduce

$$R = \frac{b^3}{a^2}$$

It is derived by G. Lebon and C. Perez-Garcia.

Now discuss the behavior of medium permeability, magnetic field, suspended particles, rotation, and find the nature of $\frac{dR}{dK_1}, \frac{dR}{dQ}, \frac{dR}{dh_T}\ and\ \frac{dR}{d\Omega}$ respectively.

$$\frac{dR}{dK_1}=\frac{-b^2}{a^2K_1^2\left(1+h_T\right)}\left[1-\frac{4\Omega^2\pi^2 b\left(\frac{P_r}{P_m}\right)^2}{\left[\left\{\left(\frac{1}{K_1}+b\right)\frac{\in P_r b}{P_m}+Q\pi^2\right\}\right]^2}\right] \tag{7.3}$$

$$\frac{dR}{dK_1}<0 \text{ if } Q>\frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m}$$

Now, ignoring the magnetic field, suspended particles and rotation, we have

$$\frac{dR}{dK_1}=\frac{-b^2}{a^2K_1^2} \tag{7.4}$$

Thus, the effect of medium permeability is destabilizing and is the same destabilizing effect when the magnetic field, suspended particles and rotation are neglected.

$$\frac{dR}{dQ}=\frac{\frac{\pi^2 b}{\in}\frac{P_m}{P_r}}{a^2\left(1+h_T\right)}\left[1-\frac{4\Omega^2\pi^2 b\left(\frac{P_r}{P_m}\right)^2}{\left[\left\{\left(\frac{1}{K_1}+b\right)\frac{\in P_r b}{P_m}+Q\pi^2\right\}\right]^2}\right] \tag{7.5}$$

$$\frac{dR}{dQ}>0 \text{ if } \Omega<\frac{\in\sqrt{b}}{2\pi}$$

It is clear that the magnetic field has stabilizing effect.

$$\frac{dR}{dh_T}=\frac{-b}{a^2\left(1+h_T\right)^2}\left[b\left(\frac{1}{K_1}+b\right)+\frac{Q\pi^2}{\in}\frac{P_m}{P_r}+\frac{\frac{4\Omega^2\pi^2 b}{\in}\frac{P_r}{P_m}}{\left[\left\{\left(\frac{1}{K_1}+b\right)\frac{\in P_r b}{P_m}+Q\pi^2\right\}\right]}\right] \tag{7.6}$$

$$\frac{dR}{dh_T}<0$$

Thus, the suspended particles have destabilizing effect.

$$\frac{dR}{d\Omega}=\frac{1}{a^2\left(1+h_T\right)}\left[\frac{\frac{8\Omega\pi^2 b^2}{\in}\frac{P_r}{P_m}}{\left[\left\{\left(\frac{1}{K_1}+b\right)\frac{\in P_r b}{P_m}+Q\pi^2\right\}\right]}\right] \tag{7.7}$$

$$\frac{dR}{d\Omega}>0$$

Now, we can say that the rotation has stabilizing effect.

8 Oscillatory convection

Substituting $\sigma = i\,\sigma_i$ in equation (6.14), we get real and imaginary parts, eliminating R between them, we have

$$f_0\sigma_i^{12}+f_1\sigma_i^{10}+f_2\sigma_i^{8}+f_3\sigma_i^{6}+f_4\sigma_i^{4}+f_5\sigma_i^{2}+f_6=0$$

Putting $s=\sigma_i^2$, we have

$$f_0 s^6 + f_1 s^5 + f_2 s^4 + f_3 s^3 + f_4 s^2 + f_5 s + f_6 = 0 \tag{8.1}$$

where ,

$$f_0 = a_1 q_1 - p_1 b_1,\ f_1 = a_2 q_1 + a_1 q_2 - p_2 b_1 - p_1 b_2,\ f_2 = a_3 q_1 + a_2 q_2 + a_1 q_3 - p_3 b_1 - p_2 b_2 - p_1 b_3,$$

$$f_3 = a_4 q_1 + a_3 q_2 + a_2 q_3 - p_4 b_1 - p_3 b_2 - p_2 b_3 - p_1 b_4,\ f_4 = a_5 q_1 + a_4 q_2 + a_3 q_3 - p_4 b_2 - p_3 b_3 - p_2 b_4,$$

$$f_5 = a_5 q_2 + a_4 q_3 - p_4 b_3 - p_3 b_4,\ f_6 = a_5 q_3 - p_4 b_4,$$

$$a_1 = b\tau \in E_r P_r^3 \left(\frac{\tau}{\in} + \frac{F\tau}{K_1}\right)^2,\ b_1 = -a^2\tau^2 P_r \left\{\in P_r \left(\frac{\tau}{\in} + \frac{F\tau}{K_1}\right)\right\},$$

,

$$a_2 = -b\left[\begin{aligned}
&\tau E_r P_r^2\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\left\{\in P_r\left(\frac{1}{K_1}+b\right)+\frac{\in P_r}{P_m}\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\tau Q\pi^2\right\}+\\
&\left\{\tau E_r P_r^2\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\left(b\tau P_r+E_r P_r^2+\frac{b\tau E_r P_r^2}{P_m}\right)\right\}\\
&\left\{\in P_r\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\frac{\in P_r}{P_m}\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\right\}+\left\{\in P_r\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\right\}\\
&\left[\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\left\{P_r b+\frac{P_r b}{P_m}\left(b\tau+E_r P_r\right)\right\}+\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)\left(b\tau P_r+E_r P_r^2+\frac{b\tau E_r P_r^2}{P_m}\right)\right]
\end{aligned}\right]$$

$$a_3 = b\left[\begin{aligned}
&\left\{\tau E_r P_r^2\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\left(b\tau P_r+E_r P_r^2+\frac{b\tau E_r P_r^2}{P_m}\right)\right\}\left\{\frac{\in P_r}{P_m}\left(\frac{1}{K_1}+b\right)+Q\pi^2\right\}\\
&+\left[\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\left\{P_r b+\frac{P_r b}{P_m}\left(b\tau+E_r P_r\right)\right\}+\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)\left\{P_r b+\frac{P_r b}{P_m}\left(b\tau+E_r P_r\right)\right\}+\tau E_r P_r^2\right]\\
&\left\{\in P_r\left(\frac{1}{K_1}+b\right)+\frac{\in P_r}{P_m}\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\tau Q\pi^2\right\}\\
&+\left[\begin{aligned}&\frac{P_r b^2}{P_m}\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)+\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)\left\{P_r b+\frac{P_r b}{P_m}\left(b\tau+E_r P_r\right)\right\}\\&+\left(\frac{1}{K_1}+b\right)\left(b\tau P_r+E_r P_r^2+\frac{b\tau E_r P_r^2}{P_m}\right)\end{aligned}\right]\\
&\left\{\in P_r\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\frac{\in P_r}{P_m}\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\right\}+\\
&\left\{\in P_r\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\right\}\left[\frac{P_r b^2}{P_m}\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\left(\frac{1}{K_1}+b\right)\left\{P_r b+\frac{P_r b}{P_m}\left(b\tau+E_r P_r\right)\right\}\right]+\frac{4\Omega^2\pi^2}{\in}\tau E_r P_r^3\\
&+\frac{Q\pi^2}{\in}\left[\tau E_r P_r\left\{\in P_r\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\frac{\in P_r}{P_m}\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\right\}+\left(E_r P_r+b\tau\right)\left\{\in P_r\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\right\}\right]
\end{aligned}\right]$$

$$b_2 = a^2 \left[\begin{aligned} & \tau^2 P_r \left\{ \in P_r \left(\frac{1}{K_1} + b \right) + \frac{\in P_r}{P_m} \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) + \tau Q \pi^2 \right\} + \left(2\tau P_r + \frac{\tau^2 P_r b}{P_m} \right) \\ & \left\{ \in P_r \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) + \frac{\in P_r}{P_m} \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \right\} + \left\{ \frac{2\tau P_r b}{P_m} + P_r (1 + h_T) \right\} \left\{ \in P_r \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \right\} \end{aligned} \right],$$

$$b_3 = -a^2 \left[\begin{aligned} & \left(2\tau P_r + \frac{\tau^2 P_r b}{P_m} \right) \left\{ \frac{\in P_r}{P_m} \left(\frac{1}{K_1} + b \right) + Q\pi^2 \right\} \\ & + \left\{ \frac{2\tau P_r b}{P_m} + P_r (1 + h_T) \right\} \left\{ \in P_r \left(\frac{1}{K_1} + b \right) + \frac{\in P_r}{P_m} \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) + \tau Q \pi^2 \right\} \\ & + \frac{P_r b}{P_m} (1 + h_T) \left\{ \in P_r \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) + \frac{\in P_r}{P_m} \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \right\} \end{aligned} \right],$$

$$b_4 = a^2 \left[\frac{P_r b}{P_m} (1 + h_T) \left\{ \frac{\in P_r}{P_m} \left(\frac{1}{K_1} + b \right) + Q\pi^2 \right\} \right],$$

$$p_1 = -b \left[\begin{aligned} & \left\{ \tau E_r P_r^2 \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) + \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \left(b\tau P_r + E_r P_r^2 + \frac{b\tau E_r P_r^2}{P_m} \right) \right\} \\ & \left\{ \in P_r \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \right\} + \tau E_r P_r^2 \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \left\{ \in P_r \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) + \frac{\in P_r}{P_m} \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \right\} \end{aligned} \right],$$

$$p_2 = b \left[\begin{aligned} & \tau E_r P_r^2 \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \left\{ \frac{\in P_r b}{P_m} \left(\frac{1}{K_1} + b \right) + Q\pi^2 \right\} + \left\{ \in P_r \left(\frac{1}{K_1} + b \right) + \frac{\in P_r}{P_m} \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) + \tau Q\pi^2 \right\} \\ & \left\{ \tau E_r P_r^2 \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) + \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \left(b\tau P_r + E_r P_r^2 + \frac{b\tau E_r P_r^2}{P_m} \right) + \tau E_r P_r^2 \right\} + \\ & \left[\left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \left\{ P_r b + \frac{P_r b}{P_m} (b\tau + E_r P_r) \right\} + \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) \left(b\tau P_r + E_r P_r^2 + \frac{b\tau E_r P_r^2}{P_m} \right) + \tau E_r P_r^2 \right] \\ & \left\{ \in P_r \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) + \frac{\in P_r}{P_m} \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \right\} + \left\{ \in P_r \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \right\} \\ & \left[\begin{aligned} & \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \frac{P_r b^2}{P_m} + \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) \left\{ P_r b + \frac{P_r b}{P_m} (b\tau + E_r P_r) \right\} \\ & + \left(\frac{1}{K_1} + b \right) \left(b\tau P_r + E_r P_r^2 + \frac{b\tau E_r P_r^2}{P_m} \right) \end{aligned} \right] \\ & + \frac{Q\pi^2}{\in} \left[\tau E_r P_r \left\{ \in P_r \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \right\} \right] \end{aligned} \right]$$

$$q_1 = -a^2 \left[\tau^2 P_r \left\{ \in P_r \left(b\tau + \frac{F}{K_1} + \frac{\tau}{K_1} + \frac{1}{\in} + \frac{f}{\in} \right) + \frac{\in P_r}{P_m} \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \right\} + \left(2\tau P_r + \frac{\tau^2 P_r b}{P_m} \right) \left\{ \in P_r \left(\frac{\tau}{\in} + \frac{F\tau}{K_1} \right) \right\} \right],$$

$$q_2=-a^2\left[\begin{array}{c}\tau^2 P_r\left\{\frac{\in P_r}{P_m}\left(\frac{1}{K_1}+b\right)+Q\pi^2\right\}+\left(2\tau P_r+\frac{\tau^2 P_r b}{P_m}\right)\\ \left\{\in P_r\left(\frac{1}{K_1}+b\right)+\frac{\in P_r}{P_m}\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\tau Q\pi^2\right\}+\left\{\frac{2\tau P_r b}{P_m}+P_r\left(1+h_T\right)\right\}\\ \left\{\in P_r\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\frac{\in P_r}{P_m}\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\right\}+\frac{P_r b}{P_m}\left(1+h_T\right)\left\{\in P_r\left(\frac{\tau}{\in}+\frac{F\tau}{K_1}\right)\right\}\end{array}\right],$$

$$q_3=a^2\left[\begin{array}{c}\left\{\frac{2\tau P_r b}{P_m}+P_r\left(1+h_T\right)\right\}\left\{\frac{\in P_r}{P_m}\left(\frac{1}{K_1}+b\right)+Q\pi^2\right\}+\frac{P_r b}{P_m}\left(1+h_T\right)\\ \left\{\in P_r\left(\frac{1}{K_1}+b\right)+\frac{\in P_r}{P_m}\left(b\tau+\frac{F}{K_1}+\frac{\tau}{K_1}+\frac{1}{\in}+\frac{f}{\in}\right)+\tau Q\pi^2\right\}\end{array}\right]$$

From equation (8.1), we see that $s=\sigma_i^2$ is always positive, so the sum of roots of equation (8.1) is positive but this is impossible if $f_0>0$ and $f_1>0$, sum of roots of equation (8.1) is $-{f_1}/{f_0}$. Thus, $f_0>0$ and $f_1>0$ are the sufficient condition for the non-existence of over stability. Now $f_0>0$ and $f_1>0$ when

$$Q>\frac{\in b^2}{\pi^2 P_m}\, and\, \frac{E_r P_r^2}{\in^2}<\frac{1}{K_1}.$$

9 Numerical Computation

In this section, we discussed the plotted variation of Rayleigh number R with K_1, Q, h_T and Ω from equation (7.1).

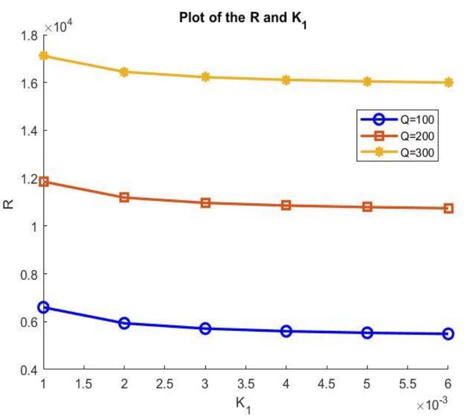


Figure 9.1

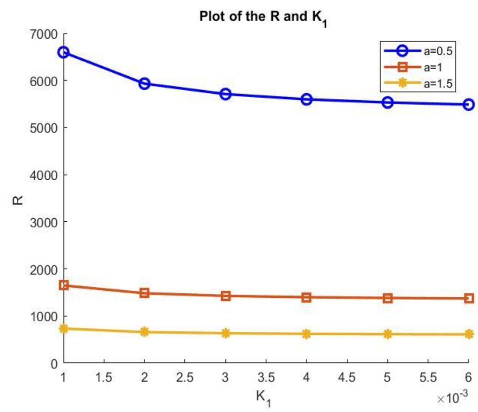


Figure 9.2

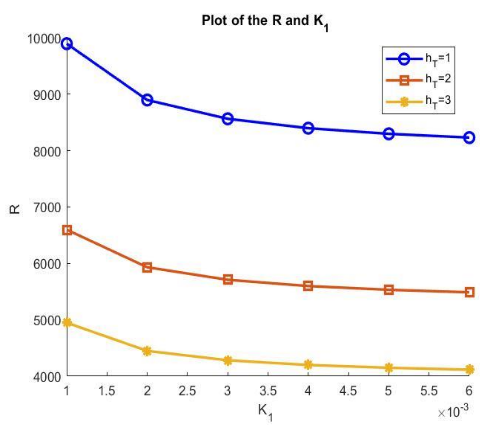


Figure 9.3

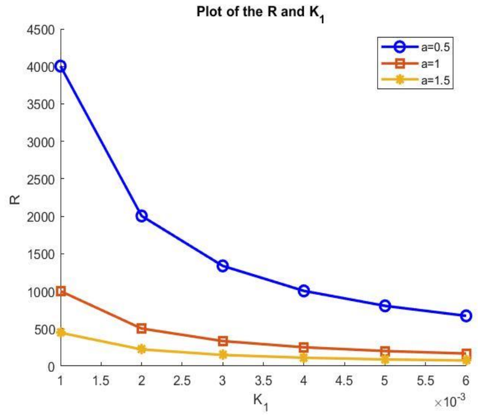


Figure 9.4

Figure 9.1, 9.2 and 9.3 shows the plots of Rayleigh number R versus medium permeability K_1 i.e. medium permeability increase then Rayleigh number decrease with magnetic field $Q=(100,200,300)$, wave number $a=(0.5,1.0,1.5)$ and suspended particles $h_T=(1,2,3)$ when $P_r=2$, $\Omega=10$, $\in=0.5$ and $P_r=4$. Now, neglecting the magnetic field, suspended particles and rotation in Figure 10.4, the Rayleigh number decrease when medium permeability increase.

It is clear that, the effect of medium permeability is destabilizing. In the absence of magnetic field, suspended particles and rotation, the effect of medium permeability is always destabilizing.

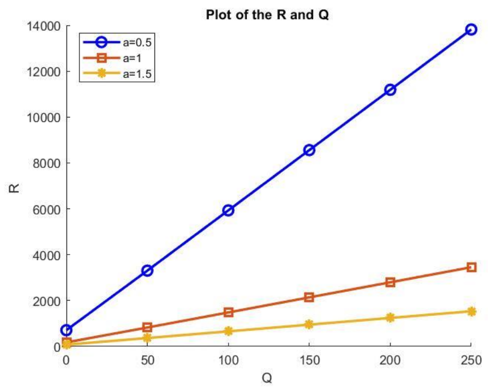


Figure 9.5

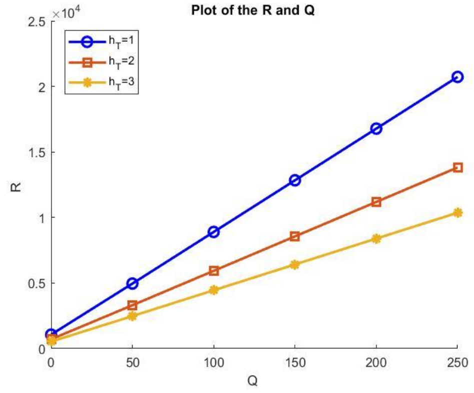


Figure 9.6

Figure 9.5 and 9.6, represent the plot of Rayleigh number R versus magnetic field Q i.e. magnetic field increase then Rayleigh number increase with the wave number $a=(0.5,1.0,1.5)$ and suspended particles $h_T=(1,2,3)$ when $P_r=2$, $\Omega=10$, $\in=0.5$ and $P_r=4$. Thus, we can say that the effect of magnetic field is stabilizing.

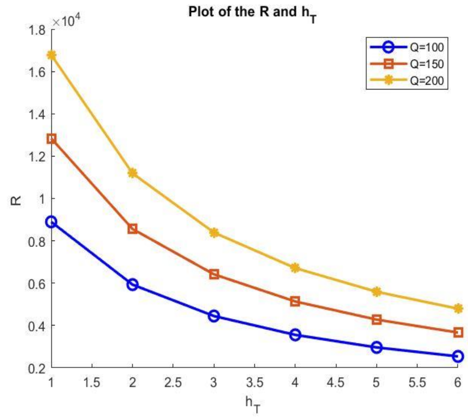


Figure 9.7

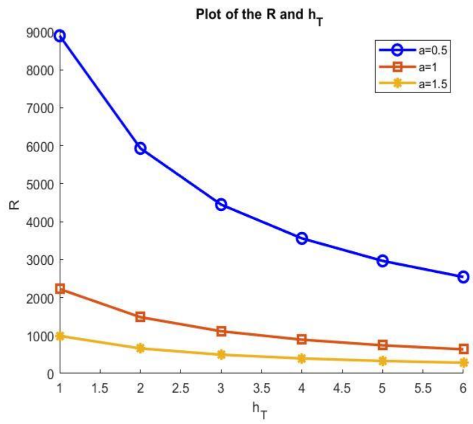


Figure 9.8

Figure 9.7 and 9.8, plot between Rayleigh number R and suspended particles h_T i.e. suspended particles increase then Rayleigh number decrease with magnetic field $Q=(100,200,300)$ and wave number $a=(0.5,1.0,1.5)$ when $P_r=2$, $\Omega=10$, $\in=0.5$ and $P_r=4$. Hence, the effect of suspended particles destabilizing.

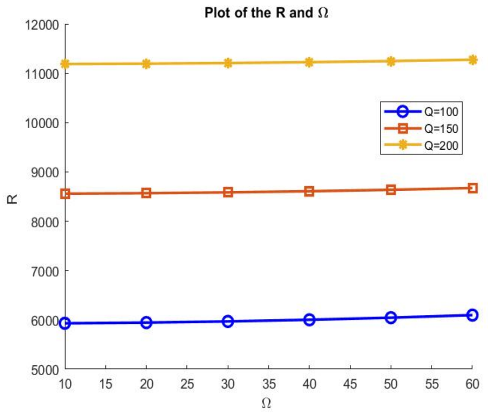


Figure 9.9

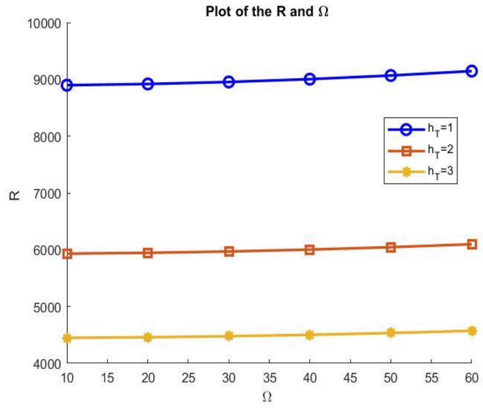


Figure 9.10

Figure 9.9 and 9.10, shows the variation of Rayleigh number R versus rotation Ω i.e. rotation increase then Rayleigh number increase with magnetic field $Q=(100,200,300)$, and suspended particles $h_T=(1,2,3)$ when $P_r=2$, $\in=0.5$ and $P_r=4$.

10 Conclusions

According to sign of the derivatives and numerically discussion, I found that the effect of medium permeability and suspended particles are destabilizing. The effect of medium permeability is always destabilizing in the absence of magnetic field, suspended particles and rotation. The effect of magnetic field and rotation are stabilizing. Among them the most important result that the effect of suspended particles destabilize on the system.

$$Q>\frac{\in b^2}{\pi^2 P_m}\ and\ \frac{E_r P_r^2}{\in^2}<\frac{1}{K_1}$$

It is sufficient condition for the non-existence of over stability.

11 References

1. Beard D.W. and Walters K., Elastico-viscous boundary layer flow I two-dimensional flow near a stagnation point, *Mathematical Proceedings of the Cambridge Philosophical Society*, 1964; **60**(3): 667-674.
2. Choudhary R. and Das K.S., Visco-elastic unsteady MHD flow between two horizontal parallel plates with hall current, *IOSR Journal of Mathematics*, 2013; **5**(1): 20-28.
3. Khan M.B. and Sasmal C., Electro-elastic instability in electroosmotic flows of viscoelastic fluids through a model porous system, *European Journal of Mechanics-B/Fluids*, 2023; **97**: 173-186.
4. Kumar P. and Mohan H., Hall current effect on Thermosolutal instability in a viscoelastic fluid through porous medium, *Journal of Engineering Science and Technology*, 2012; **7**(2): 219-231.
5. Kumar J.P., Umavathi J.C., Chamkha A.J. and Pop I., Fully-developed free-convective flow of micropolar and viscous fluids in a vertical channel, *Applied Mathematics Modelling*, 2010; **34**(5): 1175-1186.
6. Kumar R. and Mehta V., Hall Effect on MHD flow of visco-elastic fluid layer heated for below saturating a porous medium, *International Journal of Statistics and Applied Mathematics*, 2017; **2**(6):250-260.
7. Kumar V. and Kumar P., Thermosolutal convection in a viscoelastic dusty fluid with hall currents in porous medium, *Egyptian Journal of Basic and Applied Science*, 2015; **2**(3): 221-228.

8. Lebon G, Perez-Garcia C., Convective instability of a micro fluid layer by the method of energy, *International Journal of Engineering Science*, 1981; **19**(10): 13211329.
9. Moatimid G.M., Zekry M.H. and Ibrahim D.A., Nonlinear EHD instability of two viscoelastic fluids under the influence of mass and heat transfer, *Scientific Reports*, 2023; **13**(1): 357.
10. Oldroyd J.G., Non-Newtonian effects in steady motion of some idealized elastico-viscous liquids, *Proceedings of the Royal Society of London Series A Mathematical and Physical Science*, 1958; **245**(1241): 278-297.
11. Sharma R.C., Thermal instability in a viscoelastic fluid in hydromagnetics, *Acta Physica Academiae Scientiarum Hungaricae*, 1976; **38**(4): 293-298.
12. Khanduri U. and Sharma B.K., Entropy analysis for MHD flow subject to temperature dependent viscosity and thermal conductivity, *Nonlinear Dynamics and Applications*, 2022; 457-471.
13. Singh M. and Gupta R.K., Hall effect on thermal instability of viscoelastic dusty fluid in porous medium, *International Journal of Applied Mechanics and Engineering*, 2013; **18**(3): 871-886.
14. Wooding R., Rayleigh instability of a thermal boundary layer in flow through a porous medium, *Journal of Fluid Mechanics*, 1960; **9**(2):183-192.