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Application of passive techniques for controlling vibration in a dynamically oscillating beam

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Abstract

This research paper demonstrates an enhancement in the damping of a dynamic system by utilizing the dissipation of energy resulting from repeated collisions between a free mass and the base structure. The study involves both theoretical and experimental investigations conducted on a base-excited cantilever beam. The experiments were performed under two conditions: without an impact mass and with an impact mass, both at the fundamental frequency of the system and in close proximity to it. The mathematical model employed a multi-degree-of-freedom (MDOF) system, which experiences momentum transfer due to the impact of the free mass on the main mass.

MATLAB codes were developed to simulate the dynamic response of the MDOF system for the base-excited cantilever beam, both with and without the impact mass. The influence of the impact mass on the response of the base-excited cantilever beam was analyzed using the finite element method and the constant average acceleration method of the Newmark's family. The theoretical results were compared to the experimental findings and showed good agreement. The frequencies predicted by the theoretical model matched the frequencies obtained from the experiments for both cases (with and without impact mass). Additionally, the damping value predicted by the theoretical model with the impact mass aligned with the damping value obtained from the experiments. The model also estimated the contact force between the colliding bodies at the tip of the beam.

Keywords: Base-excited cantilever beam, Finite element method, Newmark method, Impact mass.

1. Introduction

Active and passive techniques are commonly employed to reduce resonant vibrations in structures. While active damping techniques may not always be suitable due to factors such as power requirements, cost, and environmental constraints, passive damping techniques offer a viable alternative. Passive damping encompasses various forms, including viscous damping, viscoelastic damping, friction damping, and impact damping. Viscous and viscoelastic damping are typically affected by temperature variations, while friction dampers may degrade over time due to wear. Consequently, attention has turned to impact dampers, especially for applications in cryogenic or high-temperature environments. These dampers utilize repeated impacts between a small moving mass and the main structure to effectively control random vibrations. The resulting momentum exchange between the main mass and impact mass leads to substantial damping. Vibration control systems can employ passive, active, or hybrid approaches to manage vibrations. Passive systems are generally considered the simplest, most cost-effective, and reliable among all control systems. While equipping a body with an absorber can be challenging from a construction standpoint, impact dampers can be easily installed.

The objective of the study was to investigate the dynamic response of beams when subjected to moving point loads. The researchers utilized the finite element method and a numerical time integration technique known as the Newmark method for the vibration analysis [1]. The purpose of the research was to examine the characteristics of a vibration system that is mitigated using an impact damper. The authors derived precise formulas for determining the optimal values of the impact damper's damping and initial displacement in a single-degree-of-freedom structure [2]. The study investigated methods for enhancing the damping capacity of long and slender cutting tools, such as boring tools or drills, by utilizing impact dampers [3].

The study analyzed the performance of particle dampers (vertical and horizontal) when attached to a primary system (with single degree of freedom (SDOF) and multi degree of freedom (MDOF)) under varying dynamic loads, including free

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vibration and random excitation [4]. The study investigated the free vibration behavior of a vibratory system equipped with a resilient impact damper, taking into account the contact time. The findings revealed that the effectiveness of reducing vibration response depends not on the number of impacts, but primarily on the type of collision between the impact mass and the main mass, specifically when they collide face-to-face [5]. The performance of a single particle vertical impact damper was examined across different forcing oscillation amplitudes and frequencies, mass ratios, structural damping ratios, impact damper lid heights, and damper/structure coefficients of restitution [6].

The paper discusses the findings of experimental and analytical investigations regarding the performance of a multi-unit particle damper in a horizontally vibrating system. The study reveals that the response of the primary system is influenced by the number of cavities and the dimensions of these cavities [7]. An analysis is conducted on a system with two degrees of freedom that can function as a vibration impact damper. The results demonstrate that a decrease in the coefficient of restitution leads to a reduction in vibration amplitude. It is also observed that the impact damper performs more effectively compared to a traditional dynamic vibration damper [8]. The study focuses on investigating the performances of impact dampers that incorporate a free mass and a damping mechanism. Several key findings were obtained:

(a) The consumption of vibratory energy through collisions plays a crucial role in determining the damping capability of impact dampers.

(b) The frequency of a vibratory system equipped with an impact damper can be estimated based on the natural frequency of the main vibratory system and the mass ratio.

(c) The damping capability of the impact damper increases with both the amplitude of the main vibratory system and the mass ratio.

Furthermore, the study reveals that the optimal damping effect of the impact damper can be achieved through a combination of the mass ratio and the clearance [9].

Despite the existing studies on the dynamics of impact dampers, there is still a need to further understand their performance across a wide range of conditions. This paper addresses this gap by employing the finite element method along with the Newmark method to analyze the dynamic response of a multi-degree-of-freedom (MDOF) system.

The Newmark integration method assumes that acceleration varies linearly between two time instants. Among the methods in the Newmark family, the constant average acceleration method is utilized to obtain the solution for the structural dynamic problems. The theoretical model developed in this paper provides predictions for frequencies and damping values that are in good agreement with the experimental results. Additionally, the theoretical model is capable of predicting the contact force between the colliding bodies. This comprehensive approach using the finite element method and the Newmark method contributes to a better understanding of the behavior of impact dampers under various operating conditions.

NOMENCLATURE

Symbol	*Description*
A	Area of beam, m^2
cc	Damping matrix without impact mass
cc_1	Damping matrix at non- contact
cc_2	Damping matrix at contact
(d_1+d_2)	Total clearance, m
E	Young’s modulus, $\mathrm{N/m}^2$
F	Force vector at time ‘t’
$\hat{F}^{t+\Delta t}$	Effective force vector
$F_I^{\,t+\Delta t}$	Impact force
$F^{t+\Delta t}$	Force vector at time‘ $t+\Delta t$ ’
f_{d_n}	Damped fundamental frequencies, Hz
f_{n_n}	Fundamental frequencies , Hz
I_e	Moment of inertia of beam m^4
kk	Stiffness matrix of beam
kk_1	Stiffness matrix at no contact
kk_2	Stiffness matrix at contact
k_{I0}	Spring stiffness at no contact, N/m
k_I	Spring stiffness at contact, N/m
$\hat{k}$	Effective stiffness matrix
L_e	Length of beam, m
m_b	Mass of box, kg
m_I	Impact mass, kg
m_a	Mass of box plus impact mass, kg
mm	Mass matrix of beam including box
mm_f	Mass matrix beam, box plus impact mass
mm_1	Mass matrix for contact & non-contact
th	Thickness of beam, m
Δt	Time step, sec
$U^t, \dot{U}^t, \ddot{U}^t$	Displacement, velocity, acceleration vectors
$U^{t+\Delta t}, \dot{U}^{t+\Delta t}, \ddot{U}^{t+\Delta t}$	Displacement, velocity, acceleration vectors at time ‘ $t+\Delta t$ ’
U_{n+1}	Displacement of impact mass

U_{n-1}	Displacement at tip
w	Width of beam, m
ρ	Density of beam, kg/m^3
δ	Input excitation level , m
ωn_n	Natural frequencies, $rad/\sec$
ξ	Damping ratio
β_1, β_2	Proportional damping constants
α, β, γ	Stability & accuracy constants for Constant average acceleration method
$\begin{pmatrix} a_1, a_2, a_3, a_4 \\ a_5, a_6, a_7, a_8 \end{pmatrix}$	Newmark methods constants

2. Theoretical model

2.1 Cantilever beam without impact mass

Fig.1 shows the cantilever beam (with box). Fig.2 shows the mathematical model of MDOF system for base excited cantilever beam without impact mass.

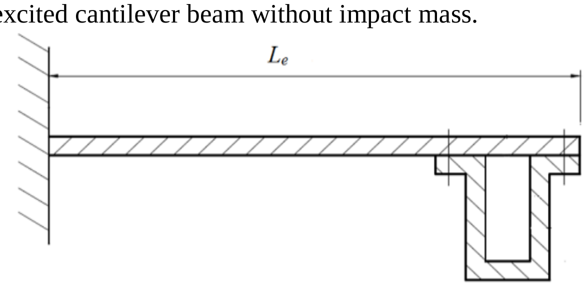

Fig.1 Cantilever beam without impact mass

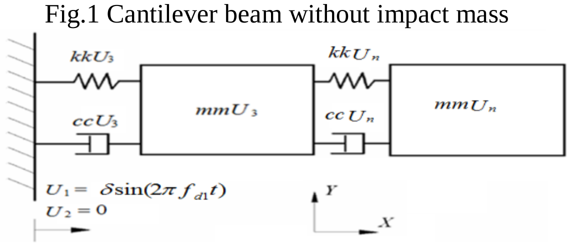

Fig.2 Schematic diagram of MDOF system

The assumptions are made in the mathematical formulation:

i. Vibratory system is considered as MDOF system (with box).
ii. Rotary inertia of beam is neglected.
iii. Shear modulus has not been considered.
iv. Euler-Bernoulli beam element is used.
v. Beam with box considered as a main system

Mass and stiffness matrix for a cantilever beam element at time 't' are given below,

$$mm = \frac{(\rho A L_e)}{420}\begin{bmatrix} 156 & -22L_e & 54 & 13L_e \\ -22L_e & 4L_e^2 & -13L_e & -3L_e^2 \\ 54 & -13L_e & 156 & 22L_e \\ 13L_e & -3L_e^2 & 22L_e & 4L_e^2 \end{bmatrix} \quad (1)$$

To get the fundamental frequencies including mass of box (m_b) the corresponding diagonal element of mass matrix is modified and equation(1) becomes,

$$mm = \frac{(\rho A L_e)}{420}\begin{bmatrix} 156 & -22L_e & 54 & 13L_e \\ -22L_e & 4L_e^2 & -13L_e & -3L_e^2 \\ 54 & -13L_e & 156 + m_b C_b & 22L_e \\ 13L_e & -3L_e^2 & 22L_e & 4L_e^2 \end{bmatrix} \quad (2)$$

where, $C_b = \left(\frac{420}{(\rho A L_e)}\right)$

The expression of mass matrix with mass of box (m_b) and impact mass$(m_I)$is given by:

$$mm_f = \frac{(\rho A L_e)}{420}\begin{bmatrix} 156 & -22L_e & 54 & 13L_e \\ -22L_e & 4L_e^2 & -13L_e & -3L_e^2 \\ 54 & -13L_e & 156 + m_a C_b & 22L_e \\ 13L_e & -3L_e^2 & 22L_e & 4L_e^2 \end{bmatrix} \quad (3)$$

$$kk = \frac{(2EI_e)}{L_e^3}\begin{bmatrix} 6 & -3L_e & -6 & -3L_e \\ -3L_e & 2L_e^2 & 3L_e & L_e^2 \\ -6 & 3L_e & 6 & 3L_e \\ -3L_e & L_e^2 & 3L_e & 2L_e^2 \end{bmatrix} \quad (4)$$

The general form of differential equation for undamped forced vibration of MDOF system without impact mass is obtained by substituting equation (2) and (4) in the following equation

$$([kk] - \omega_n^2 [mm])[U^t] = [F^t] \quad (5)$$

Applying boundary conditions for cantilever beam (left end of beam is assumed to be fixed (i.e. displacement and rotation are zero here) and neglecting the forcing function,

where, the natural frequencies of the system is:

$$\omega n_n = \sqrt{\frac{kk}{mm}} \qquad n = 1,2..... \quad (6)$$

$$f_{n_n} = \frac{\omega n_n}{2\pi} \quad (7)$$

The damped natural frequency of the system is:

$$f_{d_n} = \sqrt{(1-\xi^2)}\, f_n \quad (8)$$

The expression for proportional damping [cc] of the system is obtained by using equations (2), (4) and (7):

$$[cc]=\beta_1[mm]+\beta_2[kk] \tag{9}$$

where, $$\beta_1=2\xi\left(\frac{f_{n1}+f_{n2}}{f_{n1}f_{n2}}\right) \tag{10}$$

$$\beta_2=\frac{2\xi}{(f_{n1}f_{n2})} \tag{11}$$

Linear dynamic equilibrium equation of motion, at time 't' with damping is:

$$[mm][\ddot{U}^t]+[cc][\dot{U}^t]+[kk][U^t]=[F^t] \tag{12}$$

Constant average acceleration method is used to solve above expression.

$$U^t=\begin{bmatrix}U_1\\U_2\\U_3\\U_n\end{bmatrix},\dot{U}^t=\begin{bmatrix}\dot{U}_1\\\dot{U}_2\\\dot{U}_3\\\dot{U}_n\end{bmatrix},F^t=\begin{bmatrix}F_1\\F_2\\F_3\\F_n\end{bmatrix},\ddot{U}^t=([F^t]-[kk][U^t])/[mm]=\begin{bmatrix}\ddot{U}_1\\\ddot{U}_2\\\ddot{U}_3\\\ddot{U}_n\end{bmatrix} \tag{13}$$

where n = 4, 5, 6, ……

Initially at time t=0, above displacement, velocity and acceleration vectors are zero.

We have linear dynamic equilibrium equation of motion, at time '$t+\Delta t$' with damping is:

$$[mm][\ddot{U}^{t+\Delta t}]+[cc][\dot{U}^{t+\Delta t}]+[kk][U^{t+\Delta t}]=[F^{t+\Delta t}] \tag{14}$$

Where, $\Delta t=1/(20\,f_{n1})$ (15) At time $t+\Delta t$, for base excited system, we have

$$\begin{bmatrix}U_1\\U_2\end{bmatrix}=\begin{bmatrix}\delta\sin(2\pi\, f_{d1}t)\\0\end{bmatrix} \tag{16}$$

$$\begin{bmatrix}\ddot{U}_1\\\ddot{U}_2\end{bmatrix}=\begin{bmatrix}-\delta(2\pi\, f_{d1}t)^2\sin(2\pi\, f_{d1}t)\\0\end{bmatrix} \tag{17}$$

To calculate displacement, velocity, acceleration at time' $t+\Delta t$', following procedure is used. By using equation (14) for known displacement and rotation.

Effective stiffness matrix is given by,

$\hat{k}=kk+a_3mm+a_6cc$ (18) For each time step, calculate effective loads at time '$t+\Delta t$',

$$\hat{F}^{t+\Delta t}=F^{t+\Delta t}+mm(a_3U^t+a_4\dot{U}^t+a_5\ddot{U}^t)$$
$$+cc(a_6U^t+a_7\dot{U}^t+a_8\ddot{U}^t) \tag{19}$$

$$\hat{k}U^{t+\Delta t}=\hat{F}^{t+\Delta t} \tag{20}$$

$$\ddot{U}^{t+\Delta t}=a_3(U^{t+\Delta t}-U^t)-a_4\dot{U}^t-a_5\ddot{U}^t \tag{21}$$

$$\dot{U}^{t+\Delta t}=\dot{U}^t+a_2\ddot{U}^t+a_1\ddot{U}^{t+\Delta t} \tag{22}$$

For Constant average acceleration method we have, $\alpha=1/2,\beta=1/4,\gamma=1/2$

$$a_1=\alpha\Delta t,a_2=\Delta t(1-\alpha),a_3=1/(\beta\,\Delta t^2),a_4=a_3\Delta t,a_5=1/\gamma-1,$$
$$a_6=\alpha/\beta\Delta t,a_7=\alpha/\beta-1,a_8=(\alpha/\gamma-1)\Delta t \tag{23}$$

2.2 Cantilever beam with impact mass

Fig.3 shows the cantilever beam with impact mass at non-contact condition. Fig.4 shows mathematical model of MDOF system for base excited cantilever beam with impact mass.

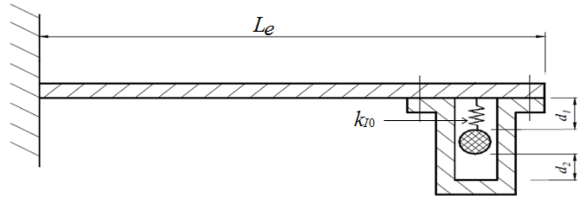

Fig.3. Cantilever beam with impact mass

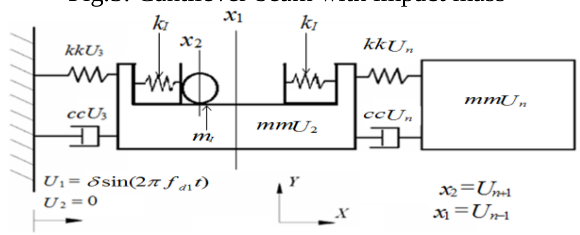

Fig.4. Schematic diagram of MDOF system with impact mass

Assumptions:-

i. Linear motion of impact mass (ball)
ii. Impact mass attached to the main system by a spring.
iii. At contact and non-contact condition spring stiffness have different values
iv. Friction between main mass & impact mass is neglected.
v. Vibratory system is considered as a MDOF system.
vi. Rotary inertia of beam was neglected.
vii. Shear modulus has not been considered.
viii. Euler-Bernoulli beam element is used.
ix. Normal contact of colliding bodies considered.

Natural frequencies for cantilever beam including mass of box and mass of impact mass are calculated by putting equation (3) and (4) in equation (5).

For forced vibration system, equation [2] is modified as given in [24], while the stiffness matrix varies for non- contact and contact conditions for an impact system as discussed in case 1 and 2.

$$mm_1=\frac{(\rho AL_e)}{420}\begin{bmatrix}156 & -22L_e & 54 & 13L_e & 0\\ -22L_e & 4L_e^2 & -13L_e & -3L_e^2 & 0\\ 54 & -13L_e & 156+m_bC_b & 22L_e & 0\\ 13L_e & -3L_e^2 & 22L_e & 4L_e^2 & 0\\ 0 & 0 & 0 & 0 & m_I\end{bmatrix} \tag{24}$$

Case1: $-d_2\le(x_2-x_1)\le d_1$, impact mass moves freely at a constant speed without any collision inside the cavity of box $(d_1=d_2=0.004\,\text{m})$ attached to main mass. So, stiffness of system for non-contact condition is given as,

$$kk_1 = \frac{(2E\,I_e)}{L_e^{\,3}} \begin{bmatrix} 6 & -3L_e & -6 & -3L_e & 0 \\ -3L_e & 2L_e^{\,2} & 3L_e & L_e^{\,2} & 0 \\ -6 & 3L_e & 6+k_{I0} & 3L_e & -k_{I0} \\ -3L_e & L_e^{\,2} & 3L_e & 2L_e^{\,2} & 0 \\ 0 & 0 & -k_{I0} & 0 & k_{I0} \end{bmatrix} \quad (25)$$

The expression for proportional damping [cc1] of the system including impact mass and for non-contact condition is obtained by using equations (24), (25) and (7):

$$[cc_1] = \beta_1[mm_1] + \beta_2[kk_1] \quad (26)$$

Linear dynamic equilibrium equation of motion for non-contact condition and known displacement and rotation,

$$[mm_1][\ddot{U}^{t+\Delta t}] + [cc_1][\dot{U}^{t+\Delta t}] + [kk_1][U^{t+\Delta t}] = [F^{t+\Delta t}] \quad (27)$$

Constant average acceleration method is used to solve above expression to obtain displacement, velocity and acceleration of the system at each time increment.

Case2: $d_1 < (x_2 - x_1)$ or $-d_2 > (x_2 - x_1)$, impact mass collides with left or right side of main mass. So, stiffness of system including contact condition is shown below,

$$kk_2 = \frac{(2E\,I_e)}{L_e^{\,3}} \begin{bmatrix} 6 & -3L_e & -6 & -3L_e & 0 \\ -3L_e & 2L_e^{\,2} & 3L_e & L_e^{\,2} & 0 \\ -6 & 3L_e & 6+k_I & 3L_e & -k_I \\ -3L_e & L_e^{\,2} & 3L_e & 2L_e^{\,2} & 0 \\ 0 & 0 & -k_I & 0 & k_I \end{bmatrix} \quad (28)$$

The expression for proportional damping [cc2] of the system with impact mass and for contact condition is obtained by using equations (24), (28) and (7):

$$[cc_2] = \beta_1[mm_1] + \beta_2[kk_2] \quad (29)$$

Linear dynamic equilibrium equation of motion for contact condition and known displacement and rotation,

$$[mm_1][\ddot{U}^{t+\Delta t}] + [cc_2][\dot{U}^{t+\Delta t}] + [kk_2][U^{t+\Delta t}] = [F^{t+\Delta t}] \quad (30)$$

Constant average acceleration method is used to solve above expression to obtain displacement, velocity and acceleration of the system at each time increment.

2.3 Contact force between colliding bodies

Contact force between colliding bodies is estimated using following equation,

$$m_I \ddot{U}_{n+1}^{\;t+\Delta t} + k_I (U_{n+1}^{\;t+\Delta t} - U_{n-1}^{\;t+\Delta t}) = F_I^{\;t+\Delta t} \quad (31)$$

3. EXPERIMENTAL SETUP

A cantilever beam with tip mass (box) is considered as the main mass. The spherical impact mass, which has a mass of 0.00343 *kg* and a diameter of 9.49 mm, is positioned inside the box so that it only moves perpendicular to the beam within the clearance ($d_1 = d_2 =$ 0.004 m).

Fig. 5 depicts a schematic of the experimental setup used for impact damper tests. Aside from the test article, the test setup consists of an excitation (shaker) system, control system, and data acquisition system.

The test object is given harmonic excitation using a hydraulic shaker system and a load cell.

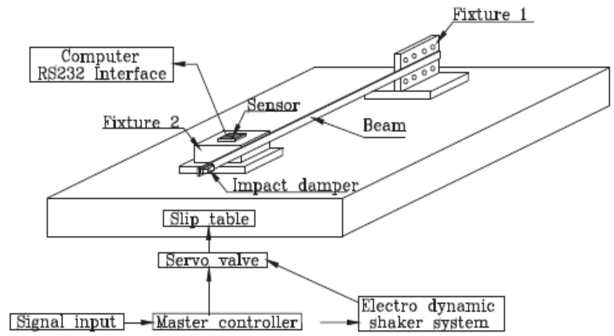

Fig.5. Schematic diagram of experimental set up

One end of the beam is fastened to fixture 1 that is set up on the shaker table in order to simulate harmonic vibration conditions experimentally. The vibration signals from experiments are measured using non-contact type sensors (laser sensors) near the middle of the beam.

Due to the sensor limitation (± 20 mm), the cantilever beam's middle was measured and test data was extrapolated to the beam's tip based on the first mode shape function. Both experiments with and without impact mass are run.

4. METHODOLOGY

At the end of cantilever the impact mass is kept for forced (base) excitation tests. Repeated theoretical and experimental runs are conducted at various excitation intensities. The system's damping varies for lower levels of excitation whereas the variation is almost nonexistent for higher levels of excitation, according to the results of testing and theoretical models for various excitation levels. Thus, the basic excitation level of 0.2037mm is used. The following scenarios are tested both with and without an impact damper:

I. For a base excitation level of 0.2037 mm, without impact mass. Here, the sine sweep levels are provided from 4Hz to 6Hz in 21 steps, with the base excitation level remaining at 0.2037 mm. 0.1 Hz more was added to each stage.

II. With an impact mass for a 0.2037 mm base excitation level. Here, the sine sweep levels are given from 4Hz to 6Hz in 21 steps with an increase in step frequency of 0.1 Hz, while the base excitation level is fixed at 0.2037 mm. To determine the frequencies and damping of the system, the time domain data from the aforementioned tests was processed and analysed.

5. RESULTS AND DISCUSSION

It is discovered that the experimentally measured frequencies coincide with those predicted by theory. Table 1 compares the system's natural frequency as determined by experiment and theoretical model.

Table 1 : Comparison of natural frequencies

Description	Theoretical estimate (Hz)	Experiment (Hz)
Without impact mass (beam + box)	5.30	5.05
With impact mass (beam + box & ball)	4.98	4.95

Using forced vibration with a base excitation of 0.2037 mm and various sine levels, experiments were conducted without and with an impact mass (ball). The results are shown in Fig. 6.

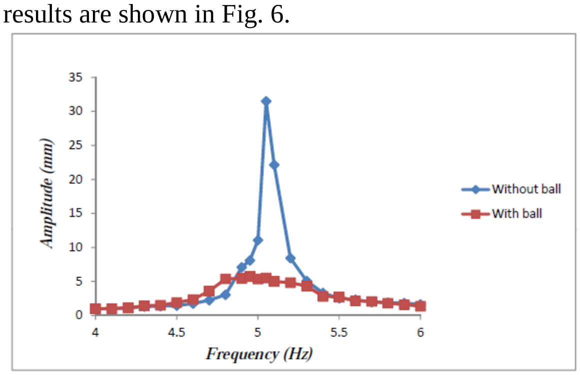

Fig.6. Response of beam without and with ball for 0.2037 mm base excitation -Experiment

When the impact mass is introduced, the response obtained from the aforementioned figure is utilised to calculate the damping of the system, and it is discovered that this damping increases from 0.29% to 1.4%.

Fig. 7, illustrates the theoretical behaviour of a cantilever beam without and with an impact mass (ball) for a forced vibration system with a base excitation of 0.2037mm at various sine levels.

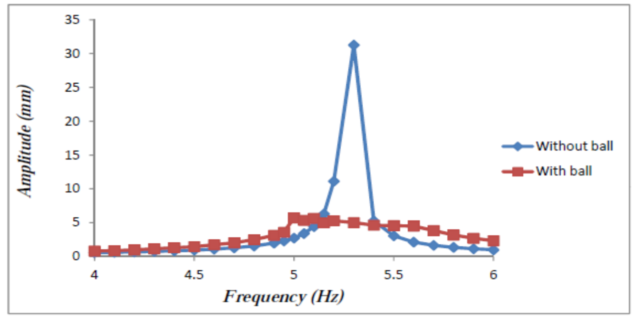

Fig.7. Response of beam without and with ball for 0.2037 mm base excitation –Theoretical

Following the determination of the system's damping value from an experiment without an impact mass, the theoretical model's prediction of the damping value with an impact mass is tested, and it is discovered to be a good match to the experimental value. As observed in Fig. 7, the addition of impact mass causes the system's damping to increase by almost five times.

The contact force between the colliding bodies (impact mass and cantilever beam) at the free end of the cantilever beam, estimated from the theoretical model, is shown in Fig. 8.

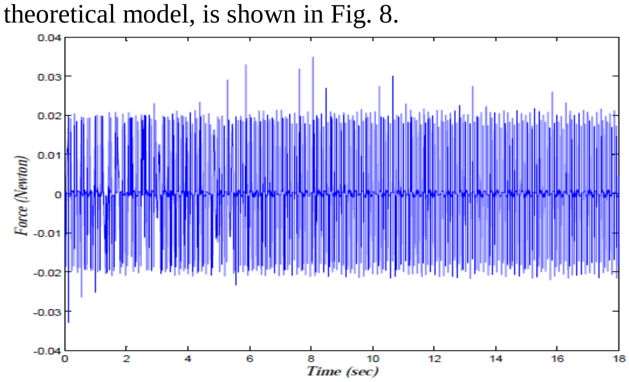

Fig.8. Contact force between colliding bodies at resonance for base excitation 0.2037 mm - Theoretical

6. CONCLUSION

The impact damper's influence on a vibrating cantilever beam is assessed by comparing the system's response with and without the impact damper.

The theoretical model developed in the study demonstrates a good match with experimental results in terms of frequencies and damping values. The results from the theoretical model indicate a significant improvement in the damping of the system with the inclusion of the impact damper. The impact system damping is shown to increase from

0.29% to 1.4%, resulting in a fivefold improvement. Similarly, the experimental results also exhibit similar damping enhancements.

Additionally, the theoretical model predicts the impact force between the colliding bodies. It is observed that the impact force stabilizes at 0.02N, indicating a consistent and predictable behavior during the collision process.

These findings emphasize the positive impact of the impact damper on the vibrating cantilever beam system. The theoretical model's predictions align well with the experimental results, highlighting the effectiveness of the impact damper in improving damping performance.

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