AN ANALYSIS OF FRACTAL IN VARIOUS TYPESOF GRAPHS

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Abstract—*This paper introduces fractal graphs, which are based on the study of fractal geometry, and deduces some of its properties. For this derivation, fractal methods are used, and they are explained via graphs. Fractal graph consequences were created because of the focus of graph theory on the interaction between edges and vertices. Additionally, the differences between fractal graphs and other well-known graphs are illustrated using a few relevant graphs.*

Keywords—*Cycle, Fractal, Graph, Strongly Regular Graph, Tree.*

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Introduction

Fractals[2] are typically mathematical shapes that are uneven or fractured and may be divided into pieces, each of which is roughly a smaller version of the total. In 1975, Benoit Mandelbrot coined the word "Fractal."[4] The Latin word "Fractus," which meaning "broken," is the source of this term. The Qualities of Fractals It mirrors itself. It has a superb structure, meaning it has incredibly minute features. By definition, it is uncomplicated. It is obtained by an ongoing process. It is difficult to adequately define the geometry of fractals in classical words. There are gaps of varied lengths inside the Fractal Geometry. The size is not measured using common measurements like lengths.

The study of graphs[1,7], or the connection between points and lines as vertices and edges, is known as graph theory. A graph is a graphic illustration of a collection of objects where two objects are connected by links.[6]

This paper's main goal is to present fractal graphs and to talk about some of its characteristics. The example of Few Graphs is used to illustrate how this fractal graph differs significantly from other graphs.

A graph X[3] is made up of two sets: its vertex set V(X) and its edge set E(X), where an edge is an unordered pair of X's distinctive vertices. The greatest distance between a vertex v and any other vertex u in Graph X is known as the vertex's eccentricity[5].

1. Fractal Graph

A $k-regular\ graph\ X$ with $n \geq 3$ vertices is said to be **Fractal Graph[4],** only if there exists at-least n spanning trees. Fractal Graph is denoted by F_X.

Example

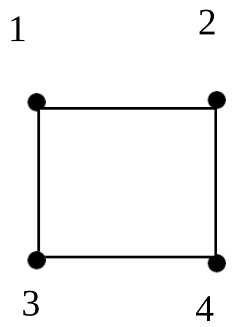

Figure 1 – Fractal Graph

Theorem 1

Every connected Fractal graph contains Hamiltonian path.

Proof:

Let F_X be the fractal graph.

We know that F_X is a $k-regular\ graph$ with $n \geq 3$ vertices.

Given F_X is the connected fractal graph.

A path P_n in the graph F_X that passes through every vertex exactly once.

Therefore F_X contains Hamiltonian path.

Example

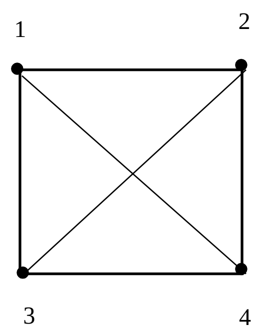

Figure 2

The above Fractal graph contains Hamiltonian Path $1-2-3-4$.

Corollary 1

Every Fractal Graph need not to be connected.

Example

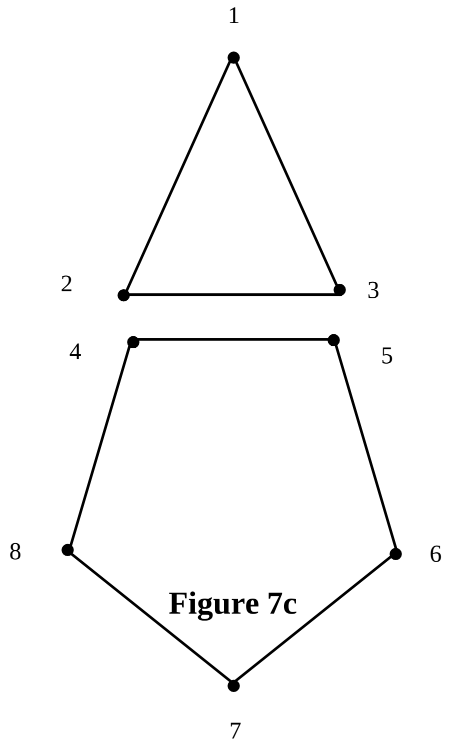

Figure 7c

Figure 3

The above graph is a 2-regular graph.

It has 8 ($8 \geq 3$) vertices. And it has more than 8 spanning trees.

It is not a connected graph.

Therefore, Every Fractal Graph need not to be a connected graph.

Theorem 2

For every complete bipartite graph with $n = 2s, s \neq 1$ and equal number of vertices in the partitions is a fractal graph.

Proof:

Let V_1 and V_2 be the two partitions in a graph X.

Let $K_{a,b}$ be a complete bipartite graph with $|V_1| = a, |V_2| = b$

Given $n = 4, 6, 8, \ldots, 2s$ and $a = b$.

Since it is a complete bipartite graph, any two vertices in V_1 have no edges, similarly any two vertices in V_2 have no edges but each element of V_1 have edges with elements of V_2.

Hence $K_{a,b}$ is a $k - regular\ graph$.

Then $K_{a,b}$ has at-least n spanning subgraphs as tree for n vertices.

Therefore, $K_{a,b}$ is a fractal graph.

Example

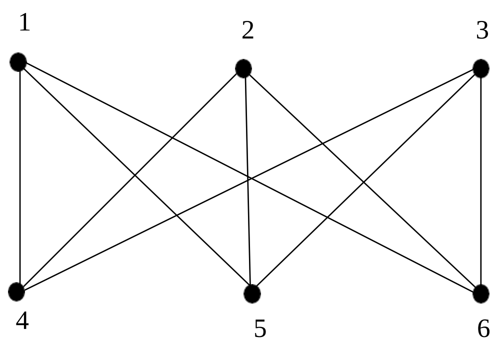

Figure 4 – Complete bipartite graph $K_{3,3}$

It is 3-regular graph.

$K_{3,3}$ has 6 spanning trees.

Corollary 2

Every Complete bipartite graph need not to be Fractal Graph.

Example

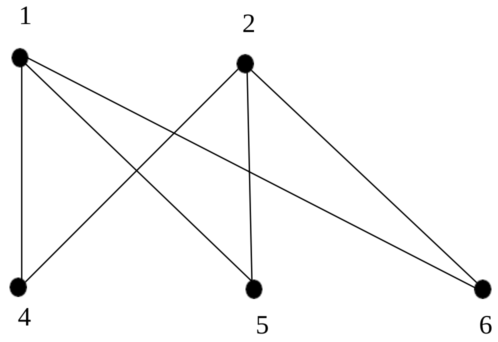

Figure 5 – Complete bipartite graph $K_{2,3}$

The Figure 5 is not a k-regular graph. So, it is not a Fractal Graph.

Theorem 3

For any fractal graph with $n \geq 4$ and non-complete graph, the diameter is two.

{This is exceptional for C_n $where\ n \geq 6$}

Proof:

Let F_X be the fractal graph.

We shall prove this theorem by induction method.

Let $n = 4$,

Given F_X is a non-complete graph.

We know that F_X is a $k - regular\ graph$

For $n = 4$, Since it is non-complete graph, obviously it a $2 - regular\ graph$.

Let X be the graph with 4 vertices and $V(X)\ = \{a, b, c, d\}$

Since it is $2 - regular\ graph$, every vertex has 2 neighbours.

Let the neighbours of a be $b\ and\ c$.

There is no edge between $a\ and\ d$.

But the vertices $b\ or\ c$ must have a edge between d.

Hence eccentricity between $a\ and\ d$ is 2.

Similarly, all non-adjacent vertices have eccentricity 2.

Therefore, diameter is 2 for $n = 4$.

Let us assume for m vertices and $m + 1$ vertices.

There must be a common neighbour between adjacent and non-adjacent vertices.

Therefore, diameter is two.

Example

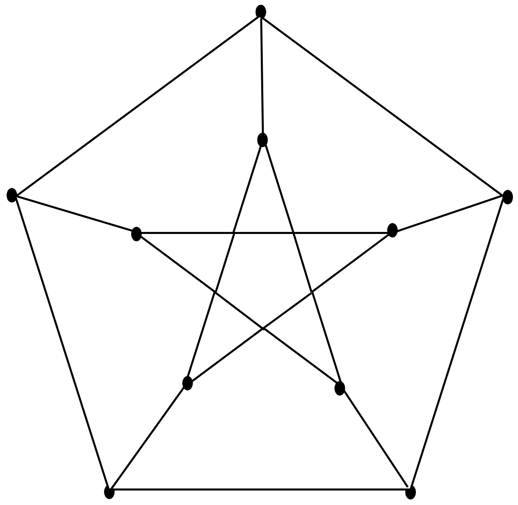

Figure 6 – Petersen Graph with 10 vertices

The diameter of Petersen Graph is 2.

Corollary 3

The diameter of fractal graph is one for the complete graphs.

Example

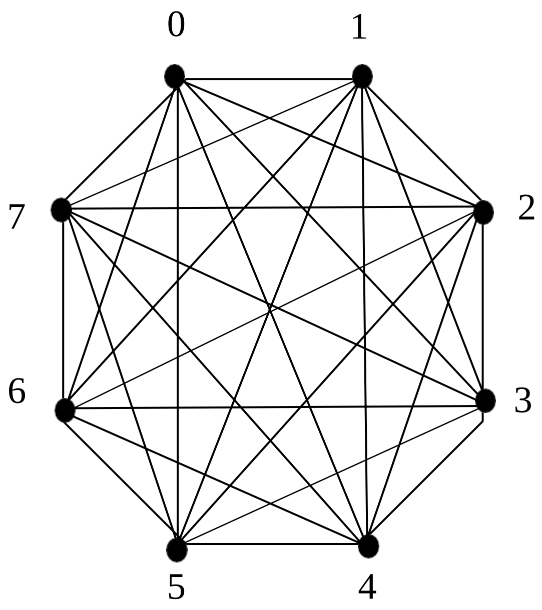

Figure 7 – Complete Graph K_8

The diameter of Complete Graph K_8 is 1.

Corollary 4

The diameter of Cycle Graph Cn with $n \geq 6$ vertices is greater than 2.

Example

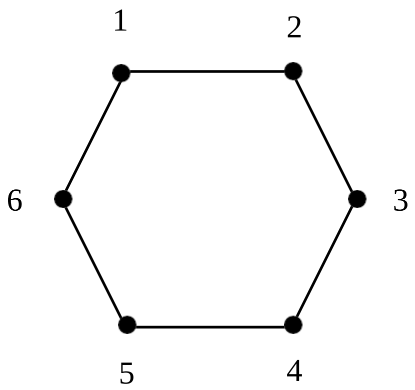

Figure 8 – Cycle Graph C_6

The diameter of Cycle Graph C_6 is 3.

Theorem 4

Every Fractal graph with 5 vertices is Eulerian.

Proof:

Let F_X be the fractal graph with 5 vertices.

We know that F_X is a $k-regular\ graph$ with 5 vertices

F_X can be $2-regular$ or $4-regular$.

Take a closed walk H starting with an vertex v and passing through the edges of the graph with no repetition in edges.

Case (i)

F_X is a Euler graph if H becomes a Euler line if it completely encloses all of F_X's edges.

Case (ii)

Remove all of H's edges from F_X if H does not cover all F_X's edges to get the remaining graph F_X.

All the vertices on F_X and F_X ' are of even degree. Additionally, every vertex in F_X ' has an even degree.

F_X is connected, hence H will traverse at least one vertex (v) of F_X'.

We can again create a new walk H' in F_X' starting from v.

The path H, which when joined with H creates a closed walk, now contains more edges than H and begins and finishes at vertex v.

We keep doing this until we have a closed walk that encompasses all of G's edges.

F_X is a Euler graph as a result.

Hence Fractal graph with 5 vertices is Eulerian.

Theorem 5

All strongly regular graphs are fractal graphs.

Proof:

Let $srg(n, k, \lambda, \mu)$ be the strongly regular graph with n vertices, k neighbours, λ common neighbours for adjacent vertices and μ common neighbours for non-adjacent vertices.
We know that $srg(n, k, \lambda, \mu)$ is $k - regular$
And it has spanning trees.
Hence $srg(n, k, \lambda, \mu)$is a fractal graph.

Theorem 6

All cycle graphs are fractal graphs.

Proof:

Let $C_n$be the cycle graph with n vertices.

Cycle is a non-empty trail in which only the first and last vertices are equal.

Since it is a cycle, obviously the graph is $2 - regular$.

And also, it has atleast n spanning trees.

Hence C_n is a fractal graph for all n.

2. Conclusion

This paper introduces a graph known as a fractal graph. Additionally, a few fractal graphs are used as examples and some of their features are discussed.

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