M/M/1/N Fuzzy Queueing Models with Discouraged Arrivals in different Triangular Fuzzy Environment

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Abstract

The Performance measures for M/M/1/N Fuzzy Queueing models with Discouraged Arrivals are analysed in this paper. An important aspect of this paper is that Queueing models are studied in different environment using Triangular fuzzy numbers(TFN), Triangular Intuitionistic Fuzzy Numbers(TIFN) and Interval valued triangular fuzzy numbers(IVTFN). At last, an example and a graphical comparison have been used to successfully to show the effectiveness and accuracy of the study.

Key words: M/M/1/N Fuzzy Queueing model, Triangular fuzzy number, Interval valued triangular fuzzy number and Triangular Intuitionistic fuzzy number

1. INTRODUCTION

In today's world, queuing theory is a hot topic for solving practical problems in everyday life. It is simple as it reduces tension and ensures appropriate time management by developing a proper model. In the field of research, there are several models of queuing theory. The congestion control mechanism is used in computer and communication systems to prevent the building of long queues by adjusting packet transmission rates based on the queue length of the packets at the destination.

The fundamental preliminary steps [4] [5],[6] of the queueing model are crucial for the goals of our research. Most of the time, we use fuzzy logic and applications in real-world situations[7],[9]. Computers that process jobs have applications for queues with discouraged arrivals[1],[2],[3], where job submissions are dissuaded when the system is frequently used and arrivals are modelled as a Poisson process with state-dependent arrival rate.

The queueing system's arrival rate is impacted by discouragement. Kumar and Sharma[8] implemented the M/M/1/N queuing system while keeping track of defaulting customers. Vasanta Kumaranalyzeda M/M/1/N queue with encouraged or discouraged arrivals with a modified reneging policy under fuzzy environment [10]. Ramesh studied M/M/1/N Fuzzy Queueing Models with Discouraged Arrivals under Wingspans Fuzzy Ranking Method [11].A single-server Markovian fuzzy queuing system with encouraged arrivals studied by Ritha [12]. Ananda Prasad Panta [13] has studied a multi-server queueing model in a fuzzy environment with imposition of reneging of customers. Hamed Fazlollahtabar[14] has developed an M/M/1/N queue model using vague numbers and the corresponding economic analysis through a novel cost model.

2. PRELIMINARIES AND DEFINITIONS

2.1 TFN: A fuzzy number 'A' denoted by ($\mathfrak{m}_1$,$\mathfrak{m}_2$, $\mathfrak{m}_3$) is a triangular fuzzy number if its membership function $\mu_{\overline{A}}$(x) is given by

$$\mu_{\overline{A}}(\mathrm{x}) = \begin{cases} \dfrac{x-\mathfrak{m}_1}{\mathfrak{m}_2-\mathfrak{m}_1} & \mathfrak{m}_1 \le x \le \mathfrak{m}_2 \\ \dfrac{\mathfrak{m}_3-x}{\mathfrak{m}_3-\mathfrak{m}_2} & \mathfrak{m}_2 \le x \le \mathfrak{m}_3 \\ 0 & otherwise \end{cases}$$

2.2 TIFN: $\overline{A}^I$ is called a TIFN, if its membership and non-membership function are defined as follows:

$$u_{A^I}(x) = \begin{cases} \frac{x-a}{b-a} u_A & a \le x < b \\ u_A & x = b \\ \frac{c-x}{c-b} u_A & b < x \le c \\ 0 & otherwise \end{cases}$$

$$v_{A^I}(x) = \begin{cases} \frac{b - x + v_{\tilde{A}}(x-a')}{b-a'} & a' \le x < b \\ v_{\tilde{A}} & x = b \\ \frac{x - b + v_A(c'-x)}{c'-b} & b < x \le c' \\ 1 & otherwise \end{cases}$$

Where $0 \le u_{\bar{A}^I} \le 1; 0 \le v_{\bar{A}^I} \le 1; 0 \le u_{\bar{A}^I} + v_{\bar{A}^I} \le 1; a, b, c, a', c' \in R$

2.3 IVTFN : The interval-valued triangular fuzzy number $\tilde{A}$is represented by two fuzzy numbers $\tilde{A}^L_x = (p^L_1, p^L_2, p^L_3; \tilde{w}^L_{\tilde{A}})$ $\tilde{A}^U_x = (p^U_1, p^U_2, p^U_3; \tilde{w}^U_{\tilde{A}})$

$$\tilde{A} = [\tilde{A}^L_x, \tilde{A}^U_x]$$

$$= [(p^L_1, p^L_2, p^L_3; \tilde{w}^L_{\tilde{A}})(p^U_1, p^U_2, p^U_3; \tilde{w}^U_{\tilde{A}})]$$

satisfying $p^L_1 \le p^U_1, p^L_3 \le p^U_3$ and $\tilde{w}^L_{\tilde{A}} \le \tilde{w}^U_{\tilde{A}}$where $\tilde{w}^L_{\tilde{A}}$ and$\tilde{w}^U_{\tilde{A}}$are the corresponding heights of $\tilde{A}^L_x$, *and* $\tilde{A}^U_x$

3. M/M/1/N QUEUEING MODELS WITH DISCOURAGED ARRIVALS

Fuzzy Queueing models with Discouraged arrivals will be evaluated using the suggested ranking method applied here. The actual crisp values of the queuing models can be calculated using this simple method. The capacity of the system is assumed to be finite say ‘ N ‘. . If there are more than one customer in the system (n>1) then the arriving customer enters the system with the rate $\widetilde{\lambda_{n_I}} = \frac{\tilde{\lambda}_I}{n+1}$. Suppose that customers join the queue and wait for some time say “t” to get his service. If the service has not started then he may get reneged and leave the queue without getting served with probability -p and he may retain in the queue is with probability q = 1- p

The time ‘t’ follows exponential distribution with parameter $\tilde{\psi}$

Let $P_n$denotes the probability that there are “n” customers in the system. The steady state equations of the model are

$$\tilde{\lambda}_I P_0 = \tilde{\mu}_I P_1$$

$$\left[\left(\frac{\widetilde{\lambda_I}}{n+1}\right)+\widetilde{\mu_I}+(n-1)\tilde{\psi}p\right]P_n=\left[\widetilde{\mu_I}+n\tilde{\psi}p\right]P_{n+1}+\left(\frac{\widetilde{\lambda_I}}{n}\right)P_{n-1}$$

where $1 \le n \le N-1$

$$\left(\frac{\widetilde{\lambda_I}}{N}\right)P_{N-1}=\left[\widetilde{\mu_I}+(N-1)\tilde{\psi}p\right]P_N$$

Let us obtain the performance measures of the model under steady state conditions:

1) Expected number of customers in the system $E(N_s)=\sum_{n=1}^{N} nP_n$

$$=\sum_{n=1}^{N} n\left[\frac{1}{n!}\prod_{l=1}^{n}\frac{\widetilde{\lambda_I}}{\widetilde{\mu_I}+(l-1)\tilde{\psi}p}\right]P_0$$

2) Expected number of Customers Served $E(C_s)$ $=\sum_{n=1}^{N}\widetilde{\mu_I}P_n$

$$=\widetilde{\mu_I}\sum_{n=1}^{N}\left[\frac{1}{n!}\prod_{l=1}^{n}\frac{\widetilde{\lambda_I}}{\widetilde{\mu_I}+(l-1)\tilde{\psi}p}\right]P_0$$

3) Average reneging rate Ren_r $=\sum_{n=1}^{N}(n-1)\tilde{\psi}pP_n$

$$=\sum_{n=1}^{N}(n-1)\tilde{\psi}p\left[\frac{1}{n!}\prod_{l=1}^{n}\frac{\widetilde{\lambda_I}}{\widetilde{\mu_I}+(l-1)\tilde{\psi}p}\right]P_0$$

4) Average retention rate Ret_r $=\sum_{n=1}^{N}(n-1)\tilde{\psi}qP_n$

$$=\sum_{n=1}^{N}(n-1)\tilde{\psi}q\left[\frac{1}{n!}\prod_{l=1}^{n}\frac{\widetilde{\lambda_I}}{\widetilde{\mu_I}+(l-1)\tilde{\psi}p}\right]P_0$$

Where $P_0=\dfrac{1}{1+\sum_{n=1}^{N}\left[\frac{1}{n!}\prod_{l=1}^{n}\frac{\widetilde{\lambda_I}}{\widetilde{\mu_I}+(l-1)\tilde{\psi}p}\right]P_0}$

4. RENEGING METHODOLOGY

4.1 Ranking of Fuzzy numbers.

The fuzzy numbers are defuzzified into crisp ones by ranking measures which are given by

$$R(A)=\frac{m_1+m_2+m_3{}'}{3}$$

Where (m_1, m_2, m_3) is a TFN

$$R(A) = \frac{a' + a + b + c + c'}{5}$$

Where (a’ a, b, c, c’) is a TIFN

$$R(A) = \frac{p_1^u + p_1^l + p_2 + p_3^u + p_3^{l'}}{5}$$

$where\ (p_1^u, p_2^u, p_3^u)(p_1^l, p_2^l, p_3^l)$ is a IVTFN.

5. Numerical Example

Customers arrive to a cinema theater to buy ticket. Upon arrival, when customers notice there are many people waiting, they might decide not to join the line. The average arrival rate decreases progressively as the system's condition improves. When there are more customers waiting in line for service, it appears as though we are discouraging new arrivals (discouraged arrivals) to this queue. In this case, we only take into account a single server model queue (cost free queue(M/M/1/N)) and a finite number of devotees (N). The customers are served by a fuzzy service rate with a time distribution parameter and are served by a fuzzy arrival rate. The arrival rate ranges from 3 to 5, Service rate from 4 to 6 and the distribution parameters is from 0.1 to 0.3

TFNTable -1 Triangular Fuzzy number and rank

Arrival rate λ	3	4.2	5	$R(\lambda)$=4.066667
Service Rate μ	4	5.3	6	$R(\mu)$=5.1
Time distribution parameter ψ	0.1	0.25	0.3	$R(\psi)$=0.216667

Table- 2 Performance measure for TFN vs p

p	$L_S=E(N_S)$	$L_Q=E(C_S)$	Avg. renege Ren_r	Avg. retention Ret_r
0	0.7972901	3.5878053	0	0.0538001
0.1	0.7946234	3.5758052	0.0053351	0.0480155
0.2	0.7920132	3.5640593	0.0105823	0.042329

0.3	0.7894572	3.5525573	0.0157446	0.0367374
0.4	0.7869532	3.5412895	0.0208248	0.0312373
0.5	0.7844993	3.5302467	0.0258257	0.0258257
0.6	0.7820934	3.5194203	0.0307496	0.0204997
0.7	0.7797338	3.5088023	0.0355989	0.0152567
0.8	0.7774189	3.4983851	0.040376	0.010094
0.9	0.775147	3.4881615	0.0450829	0.0050092
1	0.7729166	3.4781247	0.0497218	0

IVTFNTable – 3 Interval valued Triangular Fuzzy numbers and rank

Arrival rate λ	3	3.5	4	4.5	5	R(λ)=4
Service Rate μ	4	4.5	5.2	5.5	6	R(μ)=5.04
Time distribution parameter ψ	0.1	0.15	0.22	0.25	0.3	R(ψ)=0.204

Table – 4 Performance measure for IVTFN vs p

p	$L_S=E(N_S)$	$L_Q=E(C_S)$	Avg. renege Ren_r	Avg. retention Ret_r
0	0.793493832	3.570722245	0	0.050151708
0.1	0.79097964	3.55940838	0.004975368	0.044778309
0.2	0.788516064	3.548322288	0.009872869	0.039491478
0.3	0.786101135	3.537455106	0.014695009	0.034288354
0.4	0.783733003	3.526798513	0.019444155	0.029166233
0.5	0.781409929	3.516344682	0.024122554	0.024122554
0.6	0.779130276	3.506086243	0.028732336	0.01915489

0.7	0.776892498	3.496016242	0.033275526	0.01426094
0.8	0.774695136	3.486128113	0.037754051	0.009438513
0.9	0.772536809	3.476415641	0.042169745	0.004685527
1	0.77041621	3.466872944	0.046524359	0

TIFNTable – 5 Triangular Intuitionistic Fuzzy Number and rank

Arrival rate λ	3	3.5	4	4.5	5	R(λ)=4
Service Rate μ	4	4.5	5.2	5.5	6	R(μ)=5.066667
Time distribution parameter ψ	0.1	0.15	0.22	0.25	0.3	R(ψ)=0.206667

Table-6 Performance measure for TIFN vs p

p	$L_S=E(N_S)$	$L_Q=E(C_S)$	Avg. renege Ren_r	Avg. retention Ret_r
0	0.789269095	3.551710928	0	0.050410513
0.1	0.786758497	3.540413235	0.005000773	0.045006958
0.2	0.784298742	3.529344341	0.009922762	0.039691047
0.3	0.781887842	3.518495291	0.014768514	0.034459866
0.4	0.779523929	3.50785768	0.01954044	0.02931066
0.5	0.777205247	3.497423611	0.024240823	0.024240823
0.6	0.77493	3.48719	0.02887	0.01925
0.7	0.772697058	3.477136762	0.033435522	0.014329509
0.8	0.770504521	3.467270345	0.037933853	0.009483463
0.9	0.768351139	3.457580127	0.042368689	0.004707632
1	0.766235595	3.448060177	0.046741807	0

Table – 7 Comparative analysis of expected number of customers in the system

p	TFN	IVTFN	TIEN

0	0.79729	0.793494	0.789269
0.1	0.794623	0.79098	0.786758
0.2	0.792013	0.788516	0.784299
0.3	0.789457	0.786101	0.781888
0.4	0.786953	0.783733	0.779524
0.5	0.784499	0.78141	0.777205
0.6	0.782093	0.77913	0.77493
0.7	0.779734	0.776892	0.772697
0.8	0.777419	0.774695	0.770505
0.9	0.775147	0.772537	0.768351
1	0.772917	0.770416	0.766236

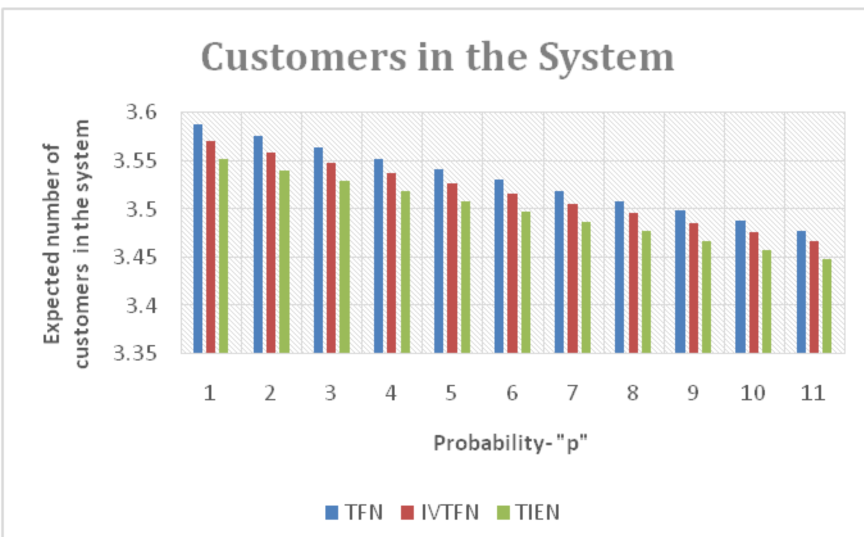

Figure – 1 Comparative analysis of expected number of customers in the system

Expected number of customers in the system remains higher for TFN than that of TIFN and IVTFN

Table – 8 Comparative analysis of expected number of Customers Served

P	TFN	IVTFN	TIFN

0	3.587805	3.570722	3.551711
0.1	3.575805	3.559408	3.540413
0.2	3.564059	3.548322	3.529344
0.3	3.552557	3.537455	3.518495
0.4	3.54129	3.526799	3.507858
0.5	3.530247	3.516345	3.497424
0.6	3.51942	3.506086	3.48719
0.7	3.508802	3.496016	3.477137
0.8	3.498385	3.486128	3.46727
0.9	3.488161	3.476416	3.45758
1	3.478125	3.466873	3.44806

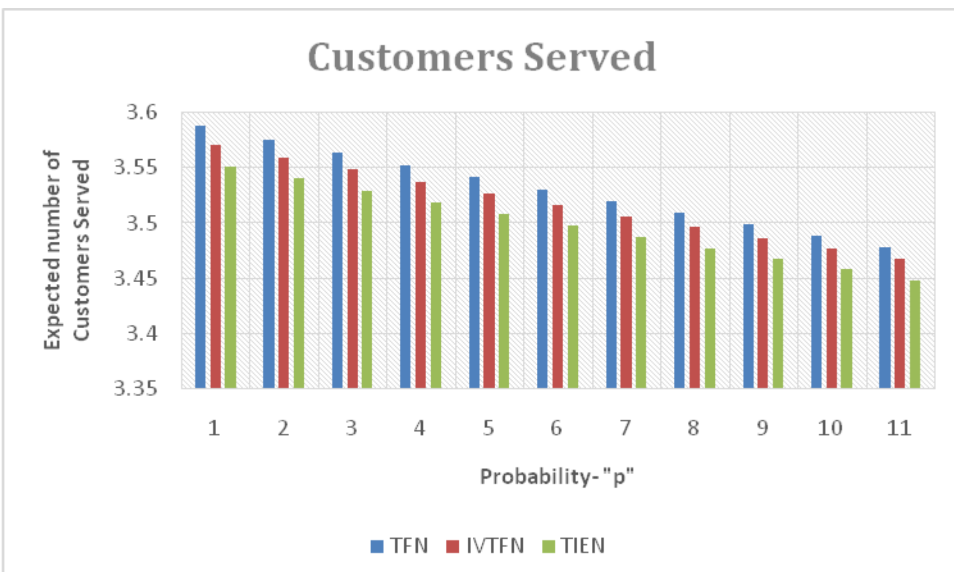

Figure – 2 Comparative analysis of expected number of Customers Served

Expected number of Customers Served is higher for TFN than that of TIFN and IVTFN

Table- 9 Comparative analysis of Average reneging rate

p	TFN	IVTFN	TIFN
0	0	0	0

0.1	0.005335	0.004975	0.005001
0.2	0.010582	0.009873	0.009923
0.3	0.015745	0.014695	0.014769
0.4	0.020825	0.019444	0.01954
0.5	0.025826	0.024123	0.024241
0.6	0.03075	0.028732	0.02887
0.7	0.035599	0.033276	0.033436
0.8	0.040376	0.037754	0.037934
0.9	0.045083	0.04217	0.042369
1	0.049722	0.046524	0.046742

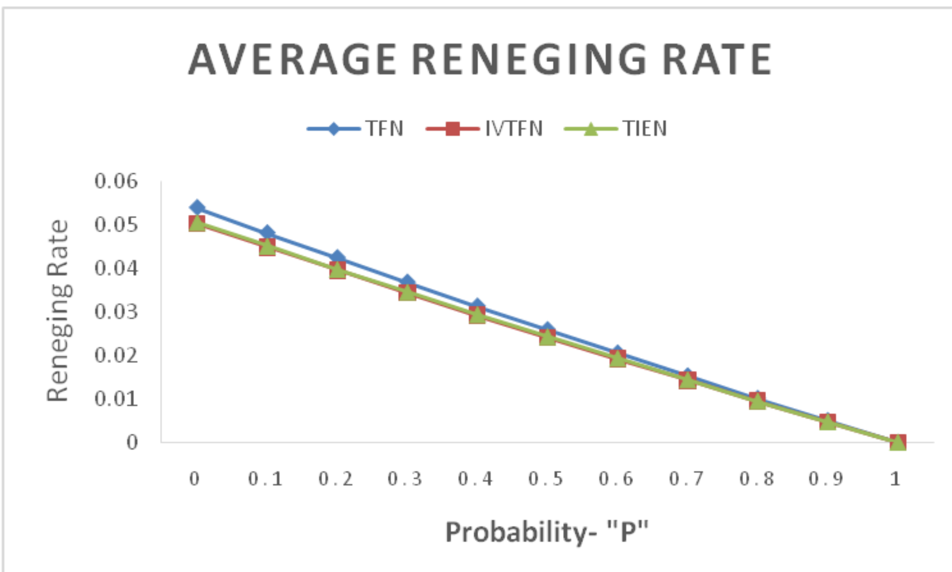

Figure- 3 Comparative analysis of Average reneging rate

There is a higher reneging rate for TFN than that of IVTFN and TIFN as “p” value increases

Table – 10 Comparative analysis of Average retention rate

p	TFN	IVTFN	TIFN

0	0.0538	0.050152	0.050411
0.1	0.048015	0.044778	0.045007
0.2	0.042329	0.039491	0.039691
0.3	0.036737	0.034288	0.03446
0.4	0.031237	0.029166	0.029311
0.5	0.025826	0.024123	0.024241
0.6	0.0205	0.019155	0.01925
0.7	0.015257	0.014261	0.01433
0.8	0.010094	0.009439	0.009483
0.9	0.005009	0.004686	0.004708
1	0	0	0

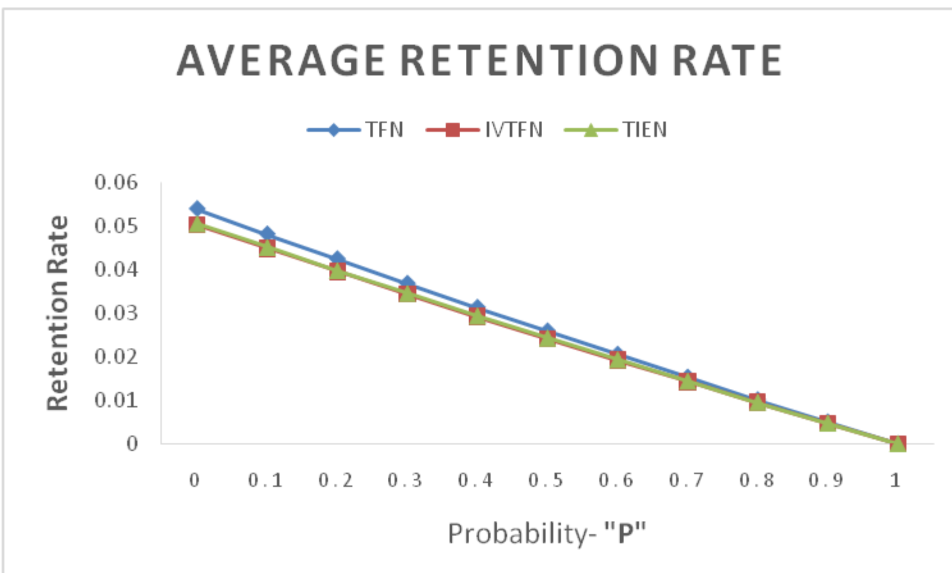

Figure- 4 Comparative analysis of Average retention rate

For smaller value of "p", there is higher retention rate for TFN than that of IVTFN and TIFN. From the aforementioned figures, we can determine the minimum scores for E(Ns) and $E(C_S)$ and the maximum scores for Ren_r and Ret_R

5. Conclusion

In this paper, an M/M/1/N fuzzy queuing model with discouraged arrivals in different fuzzy environments is considered. The performance measures of the model are found in different environments using TFN, TIFN and IVTFN. This approach would be effective in dealing with the uncertainty prevailing in the queuing models.

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