A BILLABLE TIME PRODUCTION INVENTORY MODEL FOR CRUMBLING COMPONENTS WITH NON-LINEAR ASSESS AND STOCK DEPENDENT EXIGENCY

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Abstract

The prime objective of this paper is to assert a production inventory model for crumbling components with Non-Linear assess and stock dependent exigency that co-exists with an extraneous production scope. In reality, it is regarded that the production charge may not always be stable on account of massive competition in the marketing environment and the market demand for the product. The contemporary production system is pliant to satisfy the market demand. But the holding cost of the goods may crumble when stored in the warehouse on account of various factors which may lead to the variation in the absolute state of consumption. Hence, the crumbling percentage of components will elevate the product's life span. The analogous optimization issue will be formulated, solved and validated to attain the productive cessation.

Keywords: Continual and lucubrate proffering inventory model, Non-Linear price, Linear stock dependent demand, Generalized trapezoidal intuitionistic fuzzy number, Stackelberg duopoly approach

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1. INTRODUCTION

The classical production inventory model was instigated presuming that the production rate is fixed. But in practise, the production rate may not always be fixed as a consequence of pitched strife in the marketing field and the demand in the market for the product too. In the prevailing literature of inventory control system, the life time of an article is interminable while it is in storage due to their physical characteristics. Consequently, the decay of the article cannot be left unnoticed. To regionalize, Stackelberg duopoly game approach has been applied considering the supplier as the chair person who decides the optimal wholesale price based on the retailer's optimal selling price and order cycle, in order to maximize the expected profit for the decentralised case.

2. LITERATURE REVIEW:

Ghare and Schrader [3] presumed an exponential decay percentage and investigated the effect of its on-hand products. Later, Emmons [2] espoused a two-parameter Weibull distribution decay percentage and advocated ample measures to oversee the policy when decay wavers over time. Thereafter, Misra [4] accepted another model perceiving decaying goods and integrating Weibull distributed decay prohibiting shortages. Furthermore, Khan et al., Shaikh et al., and Panda et al., [8,9,10] formulated the decaying items presuming the fixed capacity of the retailer's warehouse. Shen et al., [6] designed another model for attaining the optimal price and inventory strategies and also optimal production strategy. Dye and Ouyang [1] explained a mode for decaying articles with stock-associated market demand. Lee and Dye [7] proposed a model with Linear on-hand inventory associated market demand. Pando et al., [5] explored a model for decaying items with the Non-Linear stock amount-related consumption rate.

3. FUNDAMENTALS:

3.1 ASSUMPTIONS:

To materialize the anticipated pay off, the following assumptions are taken into consideration:

1. Sustained and lucubrate proffering system.
2. The demand rate $D\{S_p, I(t)\}$ is determined on the market price S_p of an article and the amount of commodities. It is personified by $D\{S_p, I(t)\} = a\,S_p^{-b} + c\,I(t), a, b, c \succ 0.$
3. A poor outlook for reimbursement for degenerated items.
4. The approach operates only a single phrase in addition to one stocking moment which slides for an indefinite period.
5. Every retrieval is made spontaneous. On the contrary, establishment is imperceptible.
6. Inadequacies are forbidded.

3.2 NOTATIONS:

$I(t)$ - Consignment of the vendibles at time t

β - Productivity fare

θ - Slump rate where $0 \prec \theta \prec 1$

S_c - Elevated cost of the production system

N_p - Normal production cost for every unit

O_p - Individual overtime production cost

D_p - Per capita deterioration cost

S_P - Selling price for every article

$D\{S_p, I(t)\}$ - Non-linear cost and stock dependent ultimatum

H_c - Holding price for unit time

t_1 - Time period for which the stock amount reaches the pinnacle

T - Time for which the stock amount vanishes

π - Net profit of the system

L_p - Land value

H_v - Humus cost

D_v - Amount spent for defoliant

H_p - Cost of herbicide

F_p - Fruition cost

C_p - Cost of capitulation

3.3 RANKING FOR GENERALIZED TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER USING CENTROID METHOD:

Let B be a generalized trapezoidal intuitionistic fuzzy number such that $B = \left((a, b, c, d), (a', b', c', d'), w_B, u_B\right)$, then its corresponding membership function and non-membership part are represented by the formula $S(\mu_B) = \left(\frac{2(a+d)+7(b+c)}{18}\right)\left(\frac{7w_B}{18}\right)$ and $S(v_B) = \left(\frac{2(a'+d')+7(b'+c')}{18}\right)\left(\frac{11+7u_B}{18}\right)$. The ranking formula based on the centroid method is defined as $R(B) = \frac{w_B S(\mu_B) + u_B S(v_B)}{w_B + u_B}$.

4. MATHEMATICAL FORMULATION:

The integrated inventory system can be depicted by imposing the differential equations

$$\frac{dI(t)}{dt} + \theta I(t) = S_p(t) - D\{S_p, I(t)\}, 0 \prec t \le t_1 \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D\{S_p, I(t)\}, t_1 \prec t \le T \qquad (2)$$

with supplementary conditions $I(t) = 0$ at $t = 0, T$, $I(t)$ preserves the continuity condition at $t = t_1$. Using the above condition the solution of the equation (1) and (2) can be written as $I(t) = \mu(1 - e^{\lambda t})/\lambda, 0 \prec t \le t_1$ where $\lambda = \theta + \gamma - c(\delta - 1)$ and $\mu = \beta + a{S_p}^{-b}(\delta - 1)$, then the condition becomes, $I(t) = a{S_p}^{-b}\left\{e^{(\theta+c)(T-t_1)} - 1)\right\}/(\theta + c), t_1 \prec t \le T$. Imposing the continuity conditions at $t = t_1$, we have $\mu(1 - e^{-\lambda t_1})/\lambda = a{S_p}^{-b}\left\{e^{(\theta+c)(T-t_1)} - 1\right\}/(\theta + c)$. The aggregate of ordered items is $Q = \frac{(a - bS_p)}{(\lambda + \theta)}\left[e^{(\lambda+\theta)t_1} - 1\right]$. The sum total charge to transfer all the products for the whole period is

$$HC = C_h\left\{\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right\}$$

$$= C_h\left[\frac{\mu}{\lambda}\left\{t_1 + \frac{(e^{-\lambda t_1} - 1)}{\lambda}\right\} - \frac{a{S_p}^{-b}}{(\theta + c)}\left\{\frac{1 - e^{(\theta+c)(T-t_1)}}{(\theta + c)} + (T - t_1)\right\}\right]$$

The absolute production cost to assemble all the products is

$$PC = N_p \beta t_1 + O_p \int_0^{t_1} -\gamma I(t) + \delta D\{S_p, I(t)\} dt$$

$$= N_p \beta t_1 + O_p\left\{k_1 t_1 - k_2(e^{-\lambda t_1} - 1)\right\} \text{ where } k_1 = a{S_p}^{-b}\delta + \frac{\mu}{\lambda}(c\delta - \lambda) \text{ and } k_2 = \frac{\mu}{\lambda^2}(\gamma - c\delta)$$

Since the total decayed items during the production time $[0,t_1]$ and the production off time $[t_1,T]$ are $\int_0^{t_1} I(t)dt$ and $\int_{t_1}^{T}\theta I(t)dt$ respectively, the net cost for the decayed items during the period $[0,T]$ is

$$DC = D_p\left[\int_0^{t_1}\theta I(t)dt + \int_{t_1}^{T}\theta I(t)dt\right]$$

$$= D_p\theta\left[\frac{\mu}{\lambda}\left\{t_1 + \frac{(e^{-\lambda t_1}-1)}{\lambda}\right\} - \frac{aS_p^{-b}}{(\theta+c)}\left\{\frac{1-e^{-(\theta+c)(T-t_1)}}{(\theta+c)} + (T-t_1)\right\}\right]$$

The procuring cost is $PC = \dfrac{P_w Q}{t_1} = \dfrac{P_w(a-bS_p)}{t_1(\lambda+\theta)}(e^{(\lambda+\theta)t_1}-1)$

The pre-operating cost is denoted by S_c

Redeem value is given by $\dfrac{\xi S_p c\lambda aT^{1+\beta-S_p}}{1+\beta-S_p}$

The fruition and capitulate cost is $\dfrac{F_p t_1}{Q} + \dfrac{C_p t_1}{Q} = \dfrac{(F_p + C_p)t_1}{Q}$

Cost for the humus, defoliant and herbicide is aired by $= \dfrac{(H_v + D_v + H_p)\beta t_1}{Q}$

The diminution percent based on the quantity ordered is P_d

Processing cost per cycle is u

The Salable price and the blemish cost per cycle are given by P_w and $\dfrac{ST}{Q}$ respectively

Travail cost and the land value are denoted in proportion by $\dfrac{L_p\beta(T-t_1)}{Q}$ and L_p

Shift in climate cost due to effluence of defilement from the vehicle per cycle is $\beta\dfrac{P_d}{u}$

The gross cost $\pi(t_1,T)$ is given by

$$\pi(t_1,T) = \frac{1}{T}\left[\begin{array}{l} \left\{\left[C_h\left\{\frac{\mu}{\lambda}\left\{t_1 + \frac{(e^{-\lambda t_1}-1)}{\lambda}\right\}\right\} - \frac{aS_p^{-b}}{(\theta+c)}\left\{\frac{1-e^{-(\theta+c)(T-t_1)}}{(\theta+c)} + (T-t_1)\right\}\right] + \right. \\ \left[N_p\beta t_1 + O_p\left\{k_1t_1 - k_2(e^{-\lambda t_1}-1\right\}\right] + \left[\frac{P_wQ}{t_1}\right] + \left[S_c\right] + \left[\frac{\xi S_p c\lambda aT^{1+\beta-S_p}}{1+\beta-S_p}\right] \\ \left[D_p\theta\left\{\frac{\mu}{\lambda}\left\{t_1 + \frac{(e^{-\lambda t_1}-1)}{\lambda}\right\}\right\} - \frac{aS_p^{-b}}{(\theta+c)}\left\{\frac{1-e^{-(\theta+c)(T-t_1)}}{(\theta+c)} + (T-t_1)\right\}\right] + \left[P_d\right] + \left[u\right] + \left[P_w\right] \\ \left. + \left[\frac{(F_p + C_p)t_1}{Q}\right] + \left[\frac{(H_v + D_v + H_p)\beta t_1}{Q}\right] + \left[\frac{ST}{Q}\right] + \left[\frac{L_p\beta(T-t_1)}{Q}\right] + \left[L_p\right] + \left[\beta\frac{d_p}{u}\right]\right\} \end{array}\right]$$

The eventual aspiration is to bring off the flawless out-turn of t_1 and T to thrive the net profit.

5. RANKING FOR GENERALIZED TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER USING CENTROID METHOD:

The solution of the proposed crisp model is brought out by administering geometric programming based on centroid ranking technique. The corresponding fuzzy cost for the entire phase is given by,

$$GTIF\pi(t_1,T)=\frac{1}{T}\left[\begin{array}{l}\left\{\left[GTIF(C_h)\left\{\frac{\mu}{\lambda}\left\{t_1+\frac{(e^{-\lambda t_1}-1)}{\lambda}\right\}\right\}-\frac{a\,GTIF(S_p)^{-b}}{(\theta+c)}\left\{\frac{1-e^{-(\theta+c)(T-t_1)}}{(\theta+c)}+(T-t_1)\right\}\right]+\right.\\ \left[GTIF(N_p)\beta t_1+GTIF(O_p)\left\{k_1t_1-k_2(e^{-\lambda t_1}-1\right\}\right]+\left[\frac{GTIF(P_w)Q}{t_1}\right]+\left[GTIF(S_c)\right]+\\ \left[\frac{\xi\,GTIF(S_p)\,c\,\lambda\,a\,T^{1+\beta-S_p}}{1+\beta-GTIF(S_p)}\right]+\left[P_d\right]+\left[GTIF(u)\right]+\left[GTIF(P_w)\right]+\left[\frac{GTIF(F_p+C_p)t_1}{Q}\right]+\\ \left[D_p\theta\left\{\frac{\mu}{\lambda}\left\{t_1+\frac{(e^{-\lambda t_1}-1)}{\lambda}\right\}\right\}-\frac{a\,GTIF(S_p)^{-b}}{(\theta+c)}\left\{\frac{1-e^{-(\theta+c)(T-t_1)}}{(\theta+c)}+(T-t_1)\right\}\right]+\\ \left.\left[\frac{GTIF(S)T}{Q}\right]+\left[\frac{GTIF(H_v+D_v+H_p)\beta t_1}{Q}\right]+\left[\frac{L_p\beta(T-t_1)}{Q}\right]+\left[L_p\right]+\left[\beta\ \frac{d_p}{GTIF(u)}\right]\right\}\end{array}\right]$$

On application of the geometric programming approach the above equation becomes,

$$\text{Max}\quad\prod_{r=1}^{n}\frac{1}{T}\left[\begin{array}{l}\left\{\left[\left(\left\{\frac{\mu}{\lambda}\left\{t_1+\frac{(e^{-\lambda t_1}-1)}{\lambda}\right\}\right\}-\frac{a\,S_p^{-b}}{(\theta+c)}\left\{\frac{1-e^{-(\theta+c)(T-t_1)}}{(\theta+c)}+(T-t_1)\right\}\right)\left(C_h+D_p\theta\right)\right]+\right.\\ \left[N_p\,\beta t_1+O_p\left\{k_1t_1-k_2(e^{-\lambda t_1}-1\right\}\right]\Big\}\times\frac{\gamma_{1r}}{\gamma_{1r}}+\left\{\left[\frac{P_wQ}{t_1}\right]+\left[S_c\right]+\left[\frac{\xi\,S_p\,c\,\lambda\,a\,T^{1+\beta-S_p}}{1+\beta-S_p}\right]+\right.\\ \left.\left[\frac{(F_p+C_p)t_1}{Q}\right]\right\}\times\frac{\gamma_{2r}}{\gamma_{2r}}+\left\{\left[\frac{(H_v+D_v+H_p)\beta t_1}{Q}\right]+\left[\frac{ST}{Q}\right]+\left[\frac{L_p\beta(T-t_1)}{Q}\right]\right\}\times\frac{\gamma_{3r}}{\gamma_{3r}}+\\ \left\{\left[P_d\right]+\left[u\right]+\left[P_w\right]+\left[L_p\right]+\left[\beta\ \frac{d_p}{u}\right]\right\}\times\frac{\gamma_{4r}}{\gamma_{4r}}\end{array}\right]$$

subject to the conditions

$$\gamma_{1r}+\gamma_{2r}+\gamma_{3r}+\gamma_{4r}=1$$

$$-\gamma_{1r}+\gamma_{2r}=0$$

$$\gamma_{2r}-\gamma_{3r}=0$$

$$\gamma_{2r}-\gamma_{4r}=0$$

On solving the above conditions we get $\gamma_{1r}=\gamma_{2r}=\gamma_{3r}=\gamma_{4r}={}^{1}\!/_{4}$

6. NUMERICAL EXAMPLE:

6.1 CRISP MODEL:

S_c = 50; C_h = 0.25; N_p = 10; O_p = 2.7; D_p = 17; β = 110; ξ = 0.03; δ = 0.1; c = 0.25; θ = 0.1; a = 100; b = 0.5; S_p = 15; S = 4000; F_p = 3000; u = 30; H_v = 7000; H_p = 1500; D_v = 1300; L_p = 300000; P_w = 5000; C_p = 7000; P_d = 2500

The optimal solutions are t_1 = 2.2862; T = 3.6119 and gross profit is $\pi(t_1,T)$ = Rs.1042.4384

6.2 FUZZY MODEL:

$S_c=((20,40,60,80,),(18,40,60,82),0.001,0.89),S_\mu(S_c)=0.01944;S_v(S_c)=47.8611$

Using ranking formula for centroid method, $GTIF(S_c)=47.8074$

$C_h = ((0.22, 0.24, 0.26, 0.28), (0.20, 0.24, 0.26, 0.30), 0.001, 0.868), S_\mu(C_h) = 0.00009722; S_v(C_h) = 0.2372$

Wielding the ranking formula we get, $GTIF(C_h) = 0.2369$

$N_p = ((7, 9, 11, 13,), (5, 9, 11, 15), 0.001, 0.87), S_\mu(N_p) = 0.0039; S_v(N_p) = 9.4944$

Applying the formula for centroid ranking, $GTIF(N_p) = 9.4835$

$O_p = ((2.4, 2.6, 2.8, 3), (2.2, 2.6, 2.8, 3.2), 0.001, 0.875), S_\mu(O_p) = 0.00105; S_v(O_p) = 2.5688$

Plying with the centroid formula, $GTIF(O_p) = 2.5659$

$S_p = ((12, 14, 16, 18), (10, 14, 16, 20), 0.001, 0.89), S_\mu(S_p) = 0.005833; S_v(S_p) = 14.3583$

From the ranking formula for centroid method, $GTIF(S_p) = 14.3422$

$S = ((2500, 3500, 4500, 5500), (2300, 3500, 4500, 5700), 0.001, 0.83), S_\mu(S) = 1.5556; S_v(S) = 3735.5556$

The ranking formula for centroid method gives, $GTIF(S) = 3731.0622$

$F_p = ((1500, 2500, 3500, 4500), (1300, 2500, 3500, 4700), 0.001, 0.86), S_\mu(F_p) = 1.1667; S_v(F_p) = 2836.6667$

From the centroid ranking, $GTIF(F_p) = 2833.3743$

$u = ((27, 29, 31, 33), (25, 29, 31, 35), 0.001, 0.899), S_\mu(u) = 0.01167; S_v(u) = 28.8217$

We have, $GTIF(u) = 28.7897$

$H_v = ((4000, 6000, 8000, 10000), (2000, 6000, 8000, 12000), 0.001, 0.88), S_\mu(H_v) = 2.7222; S_v(H_v) = 6673.1$

Utilizing the centroid method, $GTIF(H_v) = 6665.5286$

$H_p = ((750, 1250, 1750, 2250), (730, 1250, 1750, 2270), 0.001, 0.878), S_\mu(H_p) = 0.5833; S_v(H_v) = 1428.8333$

Exerting the centroid ranking, $GTIF(H_p) = 1427.2084$

$D_v = ((550, 1050, 1550, 2050), (530, 1050, 1550, 2070), 0.001, 0.842), S_\mu(D_v) = 0.5056; S_v(D_v) = 1220.1222$

Incorporating the ranking formula for centroid method, $GTIF(D_v) = 1205.8075$

$P_w = ((3500, 4500, 5500, 6500), (3300, 4500, 5500, 6700), 0.001, 0.85), S_\mu(P_w) = 1.9444; S_v(P_w) = 4708.3333$

Encompassing the centroid ranking formula, $GTIF(P_w) = 4702\ .8029$

$C_p = ((5500, 6500, 7500, 8500), (5300, 6500, 7500, 8700), 0.001, 0.85), S_\mu(C_p) = 2.7222; S_v(C_p) = 6591.6667$

From the ranking formula for centroid method we have, $GTIF(C_p) = 6583.9241$ and D_p = 17; β = 110; ξ = 0.03; δ = 0.1; c = 0.25; θ = 0.1; a = 100; b = 0.5; L_p = 300000; P_d = 2500

The optimal solutions are t_1 = 2.0105; T = 3.0432 and the net profit is $\pi(t_1, T)$ = Rs.871.3522

7. CONCLUSION

This paper concotes a production inventory model taking into consideration the added production system with Non-Linear price and Linear stock amount pertaining to demand. The extra production can check out scarcity during which the manufacturer should be perceptive to surpass the backorder situation. This can be well achieved by blowing away the total cost and implementing auxiliary production technique during the process of manufacture. This model is extended to be on the line of Non-Linear holding cost, advance payment, discount policy, credit-linked demand etc. by applying intuitionistic fuzzy number using geometric programming.

8. REFERENCES

1. Dye, C.Y., Ouyang, L.Y., (2005). An EOQ model for perishable items under stock-dependent selling rate and time-dependent partial backlogging, Int.J.Prod.Econ.48 (3), pp. 776-783.
2. Emmons, H., (1968). A replenishment model for radioactive nuclide generators, Manage.Sci.14, pp.263-273.
3. Ghare, P., and Schrader, G., (1963). A model for exponential decaying inventories, Eng.14, pp.238-243.

4. Misra, R.B., (1975). Optimum production lot-size model for a system with deteriorating inventory, Int.J.Prod.Res.13, pp.495-505.
5. Pando., San-Jose, L.A., Garcia-Languna, J., and Sicilia, J., (2018). Optimal lot-size policy for deteriorating items with stock-dependent demand considering profit maximization, Comput.Ind.Eng.117, pp.81-93.
6. Shen, W., Duenyas, I., and Kapuscinki, R., (2014). Optimal pricing, production and inventory for new product diffusion under supply constarints, Manuf.Service Oper. Manage.16 (1), pp.28-45.
7. Lee, Y.P., and Dye, C.Y., (2012). An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate, Comput.Ind.Eng.63 (2), pp.474-482.
8. Panda, G.C., Khan, M.A.A., and Shaikh, A.A., (2019). A credit policy approach in a two-warehouse inventory model for deteriorating items with price and stock-dependent demand under partial backlogging, J.Industr.Eng.Int.15, pp.147-170.
9. Khan, M.A.A., Shaikh, A.A., Panda, G.C., and Konstantaras, I., (2019). A two-warehouse inventory model for deteriorating items with partial backlogging and advance payment scheme, RAIRO-Oper.Res.53 (5), pp.1691-1708.
10. Shaikh, A.A., Das, S.C., Bhunia, A.K., Panda, G.C., Khan, M.A.A., (2019). A two-warehouse EOQ model with interval-valued inventory cost and advance payment for deteriorating items under particle swarm optimization, Soft.Comput.23 (4), pp.13531-13546.