CONSTRUCTING MINIMIZATION AND MAXIMIZATION OF MULTIPLICATIVE LABELLING GRAPHS

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ABSTRACT

In this study, we offer the novel idea of maximizing multiplicative labeling and develop a method for doing so. A function f is called a maximization of multiplicative labeling of a graph G with q edges, if f is a bijective function from the vertices of G to the set $\{0,1,2,\ldots p-1\}$ such that when each edge uv is assigned the label $f(uv) = f(u) * f(v) + \max\{f(u), f(v)\}$, then the resulting edge labels are distinct numbers. A function is said to be minimization of multiplicative labeling of a graph G with m −edges if f is bijective function from the vertices of G to the set $\{1,2,3,\ldots\ldots n\}$ to such an extent that when each edge pq is attach with the label $f(e = pq) = f(p).f(q) - \min\{f(p), f(q)\}$ then the resulting edge labels are different numbers. Moreover, here we are investigating some examples like Jahangir graph, Gear graph G_n, C_n^d graph, Triangular Book graph, $C_n(C_n)$ and $C_n \odot K_1$ graph for the minimization of multiplicative labeling. In this paper, we have investigated some families of graphs which admit maximization of multiplicative labeling.

KEYWORDS:

Labeling, Multiplicative Labeling, Strongly Multiplicative Labeling, Minimization Of Multiplicative Labeling.

INTRODUCTION

The graphs considered here are small, acyclic, and elementary. V(G) and E(G) stand for the sets of vertices and edges, respectively, of a graph G. The majority of modern approaches to graph labeling may be traced back to variations on Rosa's original 1967 research [1]. Labeling a graph entail applying rules to assign integer values to nodes, edges, or both. Database administration, and so on. Shalini.P et al. proposed the idea of minimizing multiplicative labeling. In this work, we show how to build multiplicative graphs that minimize a given quantity, and we also prove that the route union of 'n' copies of a cycle minimizes a multiplicative quantity associated with the solution of an equation system. [2]

One of the most useful tools in discrete mathematics is graph labeling. GLeonhard Euler, a Swiss mathematician working in the eighteenth century, is often credited with developing the idea of graph theory [3]. Network flow, fuzzy graph theory, coding theory, robotics, channel assignment, and many more are just few of the numerous areas where graph labeling has been put to use. Rosa proposed the concept of graph labeling in 1967. Gallian presents a thorough analysis of the labeling of several graphs.

Beineke and Hedge introduced the idea of highly multiplicative labeling. For highly multiplicative graphs of order n, Adiga et al. calculated an improved upper limit on the maximum number of sharp edges, which is then used to prove that the constraint obtained by is tighter [4]. Vaidya and Kanani made a broad statement on the multiplicative nature of graphs resulting from arbitrary super-subdivision of cycle, route, star, and tadpole graphs in 2010. The highly multiplicative labeling of circulant graphs was studied by Punitha et al. We point readers in the direction of for more fascinating findings on labeling.In this study, we explore the phenomenon of significantly multiplicative labeling of several graph classes [5]. The vertices of a diamond graph, umbrella graph, generalized Petersen graph, double comb graph, and sunflower planar graph are labeled under the condition of strongly multiplicative labeling after their graphs have been plotted.

LITERATURE REVIEW

Rekavandi, Aref (2022) [6] Due to the complexity of the task, even the most advanced generic object identification systems are unable to precisely position and identify small objects (SOD) in optical pictures and videos. Due to the vast camera-object distance, often little things appear in real life. The information collected from such a tiny region is not necessarily rich enough to help decision making since small objects occupy a little area in the input picture. Researchers at the intersection of deep learning and computer vision are creating cross-disciplinary approaches to improve the efficacy of SOD deep learning-based technologies. In this work, we explore this expanding field by critically examining more than 160 scholarly articles published between 2017 and 202. This article provides a brief overview of the relevant literature and organizes it into a taxonomy that depicts the big picture of current study. In this study, we look at methods for enhancing the efficiency of tiny item detection in marine settings, where such improvements are particularly important. Future avenues have been suggested by the linking of general and marine SOD studies. We also cover the most widely-used datasets for SOD in both general and maritime settings, and present some commonly-used assessment criteria for state-of-the-art algorithms on these datasets.

Marpaung, Faridawaty (2022) [7] Getting from one place to another is crucial for personal, governmental, and societal reasons. The purpose of this research was to use the Ford-Fulkerson Algorithm Network Model to determine Medan's road network's maximum capacity for relieving congestion. Several steps were required to get the result, the first of which was gathering the data to be examined in the form of the total number of transportation routes in Medan city and the region through which these routes traverse. Medan city's transportation road capacity was analyzed with the help of the Ford-Fulkerson method, and a model was built with the help of MATLAB. Some transit routes were found to have surpassed road capacity, while other routes were found to have inadequate road capacity.

MA, Rajan (2019) [8] An $((ER,G_{pq}))$ is a labelled graph where the vertex set V has p vertices labeled [1, 2, ...P] and q edges such that there exists an edge connecting two unique vertices labeled I and [j] if I and [j] are coprime to each other. This work investigates many aspects of the (($ER-,G_{pq}$)). In addition, we provide a technique for using an (($ER-,G_q$)) graph to get the GCD and LCM of any pair of integers between 1 and p.

PRELIMINARIES

Definition 1 Let $G=(V(G),E(G))$ behave like a graph. If there is a bijective function from graph G to itself, then we say that G maximizes the multiplicative labeling $f:V(G)\to\{1,2,3,......p-1\}$ such that, when each edge uv be tagged as $f(uv)=f(u)*f(v)+\max\{f(u),f(v)\}$, When this happens, the resultant edge labels are isolated[9]-[10] integers.

Definition 2 Perfect minimization of a multiplicative graph is a kind of minimization where the weights are all perfect squares.

MINIMIZATION OF MULTIPLICATIVE GRAPHS

Theorem-1:

The graph $C_n^d, n\geq 5$ minimization of multiplicative graphs with no crossing edges.

Proof: -Let $V=\{v_1,v_2,.\ .v_n\}$ be the vertex set and $E=E_1\cup E_2\cup E_3$ be the edge set of the graph C_n^d with non-intersecting chords.

Where $E_1=\{v_j v_{j+1}/1\leq j<n\}$ $E_2=\left\{v_{j+1}v_{n-j+1}/1\leq j<\frac{n}{2}-1\right\}$ and $E_3=\{v_n v_1\}$ and

Define a bijection $f:V\to\{1,2,3,......n\}$ such that

Example

When $n\equiv 0(\bmod 2)$

$$f(v_j)=\begin{cases}2j-1;1\leq j\leq\frac{n}{2}\\ 2(n-j)+2;\ \frac{n}{2}+1\leq j\leq n\end{cases}$$

Then we get the induced edge function $f^*:E(G)\to N$ such that $f(e=pq)=f(p).f(q)-\min\{f(p),f(q)\}$ as

$$f(v_j v_{j+1}) = f(v_j)f(v_{j+1}) - \min\{f(v_j), f(v_{j+1})\};\ where\ 1 \le j < n$$

$$f(v_{j+1} v_{n-j+1}) = f(v_{j+1})f(v_{n-j+1}) - \min\{f(v_{j+1}), f(v_{n-j+1})\};\ where\ 1 \le j < \frac{n}{2} - 1$$

$$f(v_n v_1) = f(v_n).f(v_1) - \min\{f(v_n), f(v_1)\}$$

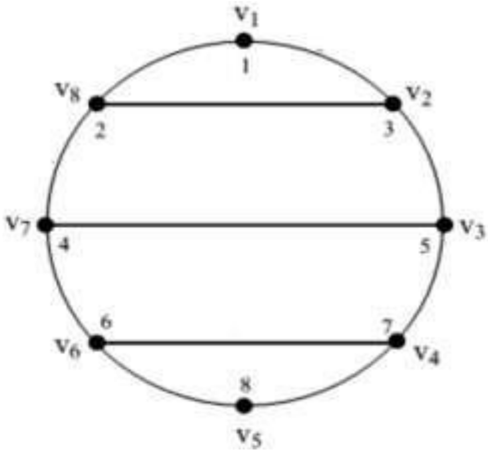


Fig.1 C_8^3 with minimization multiplicative graph

Theorem-2:

The Jahangir [11] graphs $J_{m,n}$ $for\ m \ge 2, n \ge 3$ has the form of a multiplicative graph who's minimum $mn \equiv 0 (\mathrm{mod}\ 2)$

Proof: -Let $G = J_{m,n}$ be a Jahangir Graph with $V(G) = \{v_1, v_2, .. .v_{mn+1}\}$ and

$$E(G) = \{v_i v_{i+1} / 1 \le i \le mn - 1\} \cup v_{mn} v_1 \cup \{v_{1+jm} v_{mn+1} / 0 \le j \le n - 1\}$$

Note that, $p = |V(J_{m,n})| = mn + 1\ and\ q = |E(J_{m,n})| = (m+1)n$

Here the set $\{v_j v_{j+1} / 1 \le i \le mn - 1\} \cup v_{mn} v_1$ represent as edges of the cycle and the set $\{v_{1+jm} v_{mn+1} / 0 \le j \le n - 1\}$ denotes the set of all edges adjacent to the vertex v_{mn+1}

The vertex labeling of $J_{m,n}$ is $f : V(G) \to \{1,2,3,. ....p\}$ such that $f(v_i) = i + 1$

For $1 \le i \le mn$ and $f(v_{mn+1}) = 1$. Thus, with $mn+1$ vertices and $mn+1$ labeling edges and f is bijective function. Then we get the induced edge function $f^* : E(G) \to N$ such that

$$f(e = pq) = f(p).f(q) - \min\{f(p), f(q)\}$$

$$f(v_i v_{i+1}) = f(v_i) f(v_{i+1}) - \min\{f(v_i), f(v_{i+1})\}; \; where 1 \le i < mn - 1$$

$$f(v_{mn} v_1) = f(v_{mn}).f(v_1) - \min\{f(v_{mn}), f(v_1)\}$$

$$(f v_{1+jm} v_{mn+1}) = (f v_{1+jm}).f(v_{mn+1}) - \min\{f(v_{1+jm}).f(v_{mn+1})\}; \; where 0 \le j \le n-1$$

It is evident from the aforementioned edge function that each edge is being given a unique edge label [11]. This means that the Jahangir graph $J_{m,n}$ minimizes a multiplicative graph, where $m \ge 2, n \ge 3$

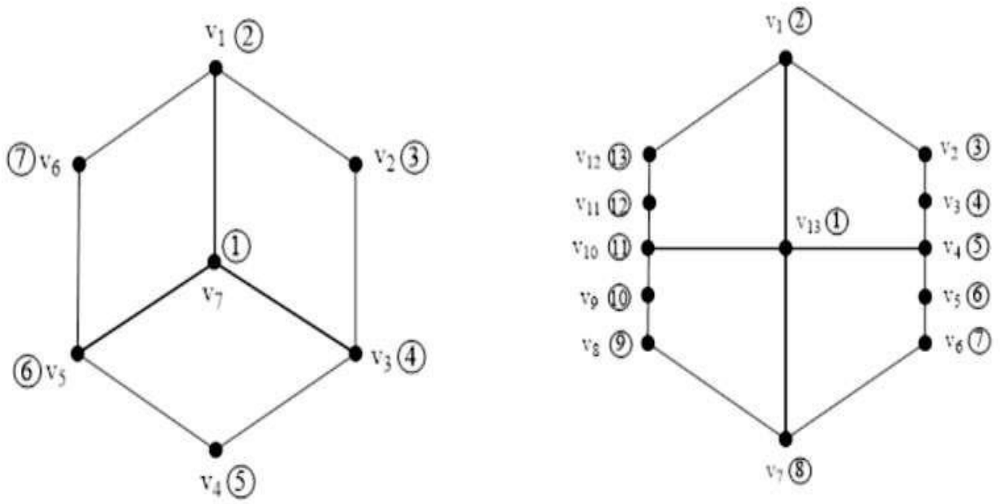


Fig.2 Jahangir graph $J_{2,3}$ and $J_{3,4}$ with minimization multiplicative graph

Theorem-3: As a special case of minimal multiplicative graphs, the triangular book graph B_n^3 may be defined, if $n \equiv 0(mod\ 2)$.[12].

Proof: -

-Let $G = B_n^3$ make use of the triangle plot in a novel. Let $V(G) = \{v_1, v_2, .\ ., v_n, v_{n+1} v_{n+2}\}$ and

$$E(G) = \{v_1 v_{j+1} \,/\, j = 1,2,.....n\} \cup \{v_{j+1} v_{n+2} \,/\, j = 1,2,.....n\} \cup \{v_1 v_{n+2}\}$$

Note that $p = |V(G)| = n+2$ and $q = |E(G)| = 2n+1$ Hence

$p+q = 3n+3$

Define a labeling $f : V(G) \cup E(G) \to \{1,2,3,.....(p+q)\}$ as follows:

Example

$$f(v_j) = \begin{cases} 1, & 1 \le j \le m-1 \\ j+1, & m \le j \le n+1 \\ m, & j = n+2 \end{cases}$$

$$f(e_j) = 2+n+j; \ where \ 1 \le j \le 2n+1$$

Where m is the largest prime number such that $m \le n+2$

Then we get the induced edge function $f^* : E(G) \to N$ such that

$$f(e = pq) = f(p).f(q) - \min\{f(p), f(q)\}$$

$$f(v_j v_{j+1}) = f(v_j) f(v_{j+1}) - \min\{f(v_j), f(v_{j+1})\}; \ where 1 \le j \le n$$

$$f(v_{j+1} v_{n+2}) = f(v_{j+1}) f(v_{n+2}) - \min\{f(v_{j+1}), f(v_{n+2})\}; where 1 \le j \le n$$

$$f(v_{n+2} v_1) = f(v_{n+2}).f(v_1) - \min\{f(v_{n+2}), f(v_1)\}$$

It is evident from the aforementioned edge function that each edge is being given a unique edge label [12]. Minimizing multiplicative graphs leads to the triangular book graph B_n^3. Where $n \equiv 0 (\mod 2)$.

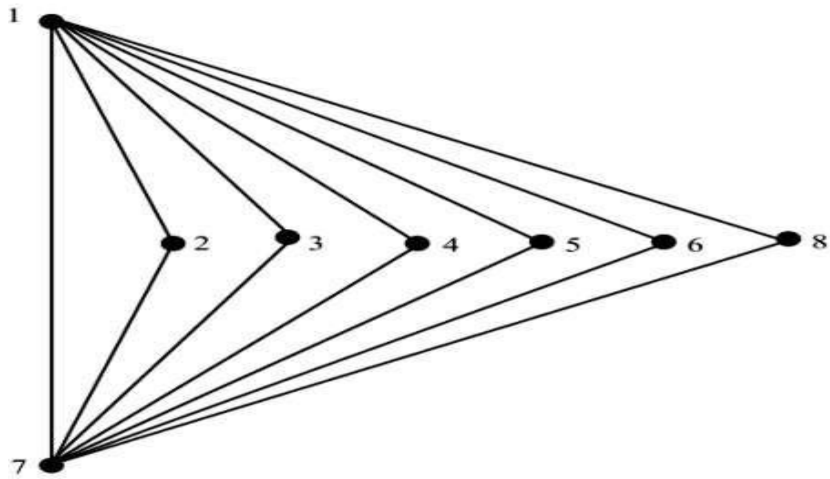


Fig.3 Book graph B_3^6 with minimization multiplicative graph

Theorem-4: Union of helm graph H_n and gear graph G_n is minimization of multiplicative graphs.[14]

Proof: -Let G be the union of helm graph H_n and gear graph G_n. Suppose the vertex set of the helm graph H_n and gear graph G_n are

$\{v_0, v_1, ...v_n, v'_1, v'_2, ...v'_n\}$ and $\{u_0, u_1, ... u_{2n}\}$ respectively. [13]

Example

Define $f : V(G) \to \{1,2,3,......4n+2\}$ by

$f(v_0) = 1$ $f(u_0) = 2$

$f(v_1) = 4$ $f(v'_1) = 3$

$f(v_j) = 2j+1$ $f(v'_j) = 2(j+1)$

$f(u_i) = 2n+i+2$; $where\ i \in \{1,2,...2n\}$

Then we get the induced edge function $f* :(G) \to N$ such that

$f(e = pq) = f(p).f(q) - \min\{f(p), f(q)\}$

Then for $j \in \{1,2,...n\}$

$f(v_0 v_j) = f(v_0) f(v_j) - \min\{f(v_0), f(v_j)\}$

$$f(u_0 u_{2j+1}) = f(u_0) f(u_{2j+1}) - \min\{f(u_0), f(u_{2j+1})\}$$

$$f(v_n v_1) = f(v_n) f(v_1) - \min\{f(v_n), f(v_1)\}$$

$$f(u_1 u_{2n}) = f(u_1) f(u_{2n}) - \min\{f(u_1), f(u_{2n})\}$$

It is evident from the aforementioned edge function that each edge is being given a unique edge label. As a result, combining the helm graph H_n with the gear graph Minimization of multiplicative graphs, or G_n. [14]

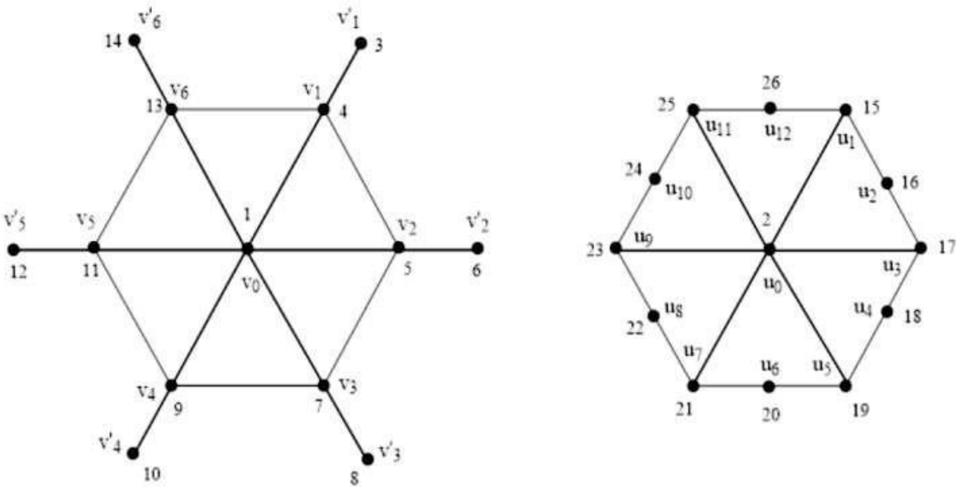


Fig. 4 Union of $H_6 \cup G_6$ with minimization multiplicative graph

Maximization of Multiplicative Labeling

Theorem 5: The path P_n is a multiplicative graph with perfect maxima for $n \geq 2$.

Proof:

Let G be a graph of path P_n. Let $\{v_1, v_2, \ldots, v_n\}$ be the vertices of P_n and $\{e_1, e_2, e_3, \ldots, e_{n-1}\}$ be the edges of P_n which are denoted as in the Fig.5. [15]

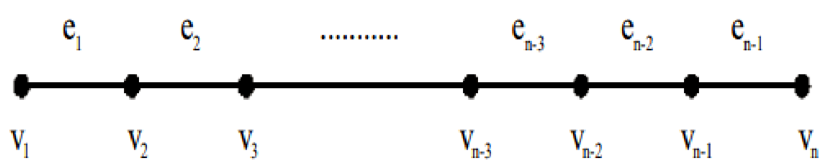


Fig. 5: Path P_n with ordinary labeling

The path P_n consists of n vertices and n – 1 edge. The vertices of P_n are labelled as given below.

Define $f: V(G) \to \{0,1,2,3,\ldots n-1\}$ by $f(v_i) = i-1; 1 \le i \le n$

Then the edge labels are:

$f(e_i) = i^2;\ 1 \le i \le n-1$

With perfect square, unique numbers are assigned to each edge of the route graph.

Hence, the path $P_n\ (n \ge 2)$ has multiplicative graphs with perfect maxima.

Example

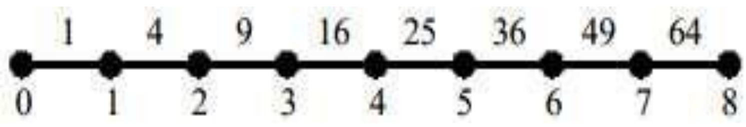


Fig. 6 Path P$_9$

Example

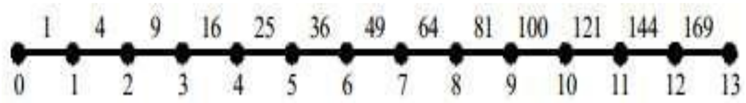


Fig. 7 Path P$_{14}$

Theorem 6

The star $K_{1,n}$ is the optimal multiplication of graphs, $n \ge 2$ [17]

Proof: Let G be a graph of star $K_{1,n}$.

Let v_0 be the center vertex and $\{v_1, v_2, \ldots, v_n\}$ be the pendant vertices of

$K_{1,n}$ and $\{e_1, e_2, e_3, \ldots, e_n\}$ be the edges of $K_{1,n}$ which are denoted as in the Fig.8

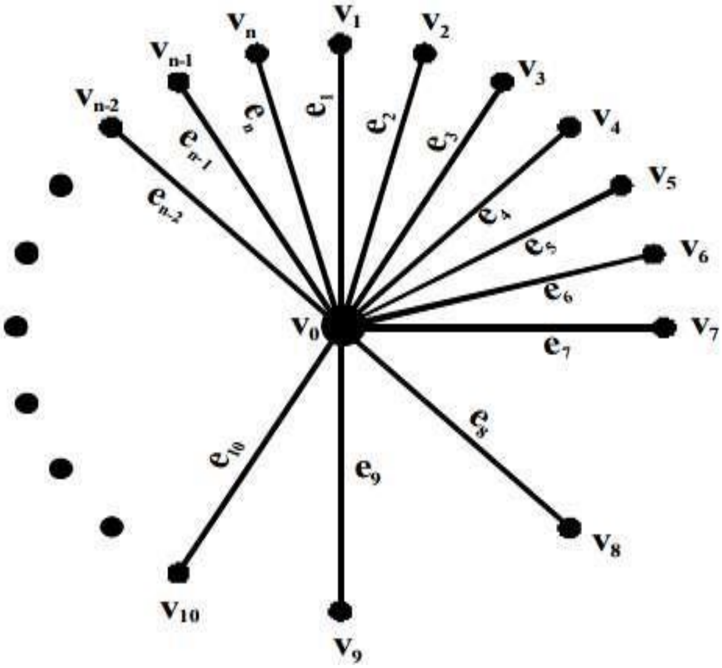


Fig. 8: Star $K_{1,n}$ with ordinary labeling

The star $K_{1,n}$ consists of n + 1 vertices and n edges. The vertices of $K_{1,n}$ is labelled as given below. Define $f: V(G) \rightarrow \{0,1,2,3,....n\}$ by

$f(v_0) = 0$

$f(v_i) = i; 1 \leq i \leq n$

Then the edge labels are:

$f(e_i) = i;\ 1 \leq i \leq n$

The edges of the star graph receive distinct numbers. [17]

Hence, the star $K_{1,n}\,(n \geq 2)$ is a maximization of multiplicative graphs.

Example

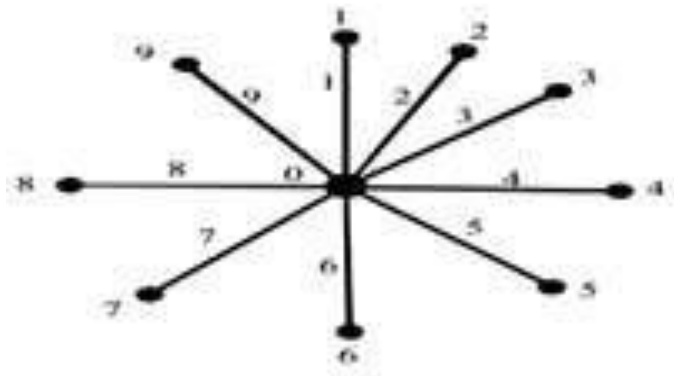


Fig. 9: Star $K_{1,9}$

Example

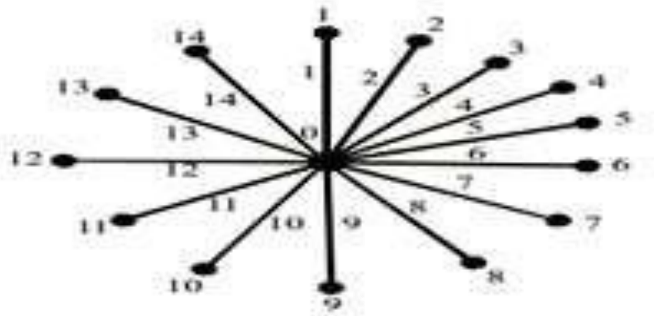


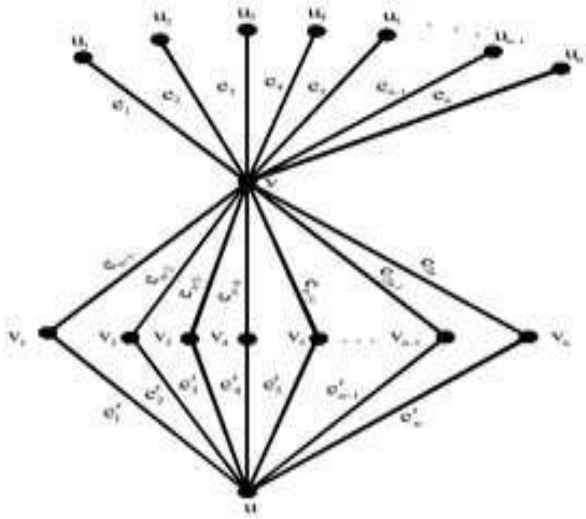
Fig. 10: Star $K_{1,14}$

Theorem 7:

The split $Spl(K_{1,n})$ is a maximization of multiplicative graphs for $n \geq 3$ [19]

Proof: Let G be a graph of split $Spl(K_{1,n})$

Let $\{u, v, u_1, u_2, u_3, \ldots\ldots u_n, v_1, v_2, v_3, \ldots\ldots, v_n\}$ be the vertices of $Spl(K_{1,n})$ and $\{e_1, e_2, e_3, \ldots, e_{n-1}, e_n, e_{n+1}, e_{n+2}, \ldots\ldots e_{2n-1}, e_{2n}, e'_1, e'_2, e'_3, \ldots\ldots e'_n\}$ be the edges of $Spl(K_{1,n})$



which are denoted as in the Fig.11. [18]

Fig. 11: Split $Spl(K_{1,n})$ with ordinary labeling

The split $Spl(K_{1,n})$ consists of $2n+2$ vertices and $3n$ edges.

The vertices of $Spl(K_{1,n})$ are labelled as given below.

Define $f : V(G) \rightarrow \{0,1,2,3,....2n+1\}$ by

$f(v) = 0$

$f(u_i) = i; 1 \leq i \leq n$

$f(v_i) = n + i; \ 1 \leq i \leq n$

$f(u) = 2n + 1$

Then the edge labels are:

$f(e_i) = i; \ 1 \leq i \leq 2n$

$f(e_1') = (2n+1)(n+i+1); \ 1 \leq i \leq n$

Separate digits are assigned to each edge of the partitioned graph. [19]

Hence, the split $Spl(K_{1,n})(n \geq 3)$ optimizes the multiplicative graphs.

Example

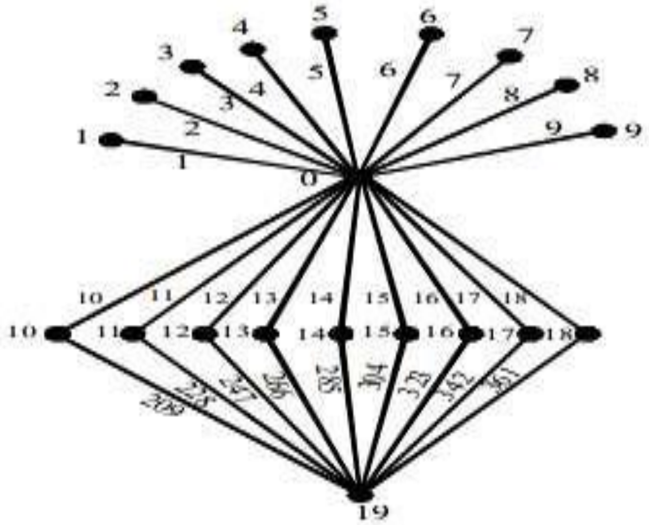


Fig. 12: Split $Spl(K_{1,9})$

Example

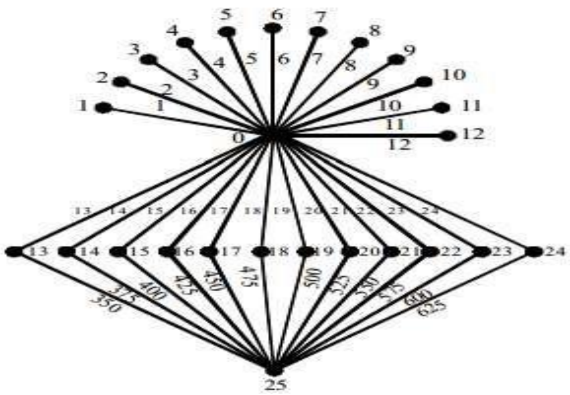


Fig. 13: Split $Spl(K_{1,12})$

Theorem: 8

The graph Specs $K_2 \Theta C_n$ optimizes the multiplicative graphs.[20]

Proof: Let G be a graph of Specs $K_2 \Theta C_n$

Let $\{v_1, v_2, v_3 ..... v_n, v_1', v_2', v_3' ..... v_n'\}$ be the vertices of Specs $K_2 \Theta C_n$ and $\{e_1, e_2, ... e_n, e_1', e_2', ... e_n'\}$ be the edges of Specs $K_2 \Theta C_n$ which are denoted as in the Figure. 14

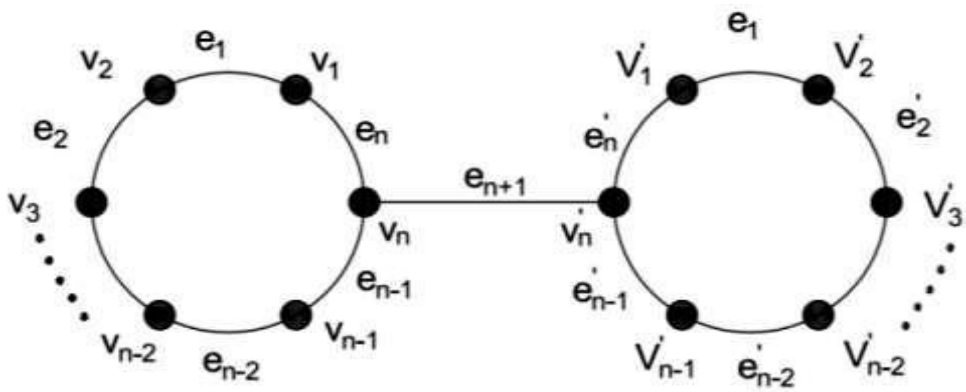


Fig.14. The Specs $K_2 \Theta C_n$ with ordinary labeling

The Specs $K_2 \Theta C_n$ consists of $2n$ vertices and $2n+1$ edges.

The vertices of Specs $K_2 \Theta C_n$ are labeled as given below.

Define $f: V(G) \rightarrow \{0,1,2,3,. ...2n-1\}$ by

$f(v_i) = i - 1;\ 1 \le i \le n$

$f(v_i') = n - 1 + i\ ;\ 1 \le i \le n$

If so, the resulting edge labels are induced:

$f(e_i) = i^2\ ; 1 \le i \le n-1$

$f(e_1') = n + i; 1 \le i \le n-1$

$f(e_n') = (2n^2 + n - 1)$

$f(e_{n+1}) = n$

The edges of $K_2 \Theta C_n$ get a set of even numbers that are all different. Hence, Specs $K_2 \Theta C_n$ optimizes the multiplicative graphs. [20]

Example

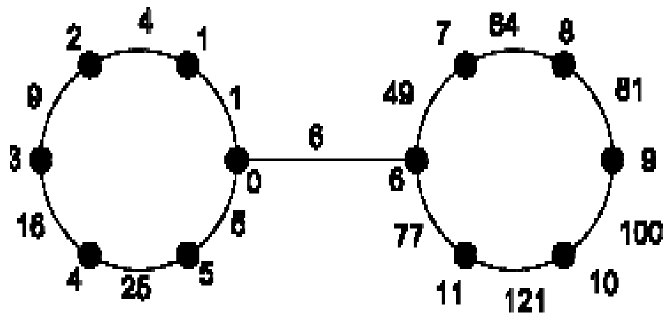


Fig.15 $K_2 \Theta C_6$

Theorem 9

The graph Stair $Str_n$is a bound on multiplicative graphs,

Proof:

Let G be a graph of stair Str_n. Let $\{v_1, v_2, v_3 ....v_n, v_{n+1}, v'_1, v'_2, v'_3 \cdots v'_n\}$ be the vertices of Str_n and $\{e_1, e_2, e_3 ....e_n, e'_1, e'_2, e'_3 \cdots e'_n\}$ be the edges of Str_n which are denoted as in the Figure.17

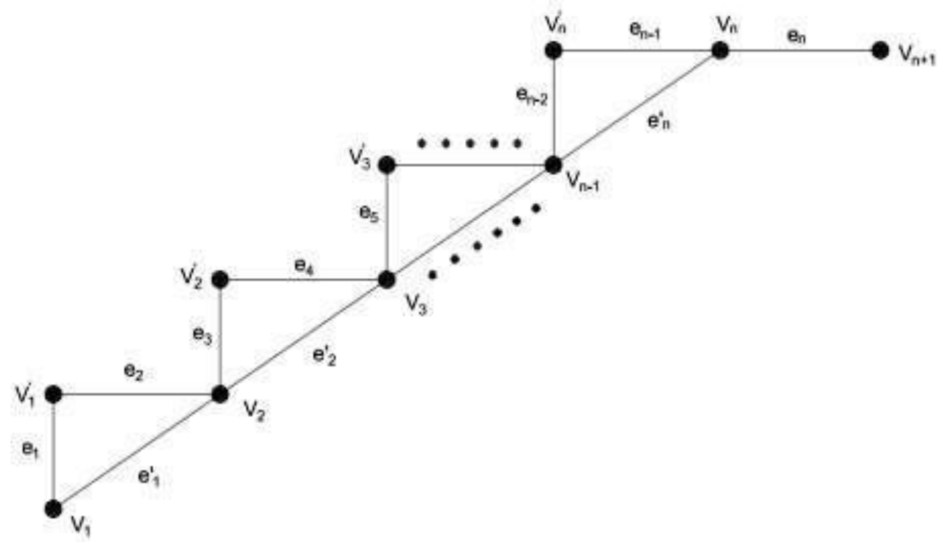


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Fig.17 The Stairs Str_n with ordinary labeling

Example

The graph Stairs Str_n consists of $2n$ vertices and $3n-2$ edges. The vertices of Str_n are labeled as given below. Define $f: V(G) \to \{0,1,2,3,....2n-1\}$by

$f(v_i) = 2i-2;\ 1 \le i \le n$

$f(v_i') = 2i - 1\ ;\ 1 \le i \le n-1$

$f(v_{n+1}) = 2n - 1$

And last, we have the edge labels, which are:

$f(e_i) = i^2\ ;\ 1 \le i \le n-1$

$f(e_1') = 4i^2 - 2i\ ;\ 1 \le i \le n-1$

The edges of Str_n unique digits are sent to the graph.

Hence, Str_n is a bound on multiplicative graphs,

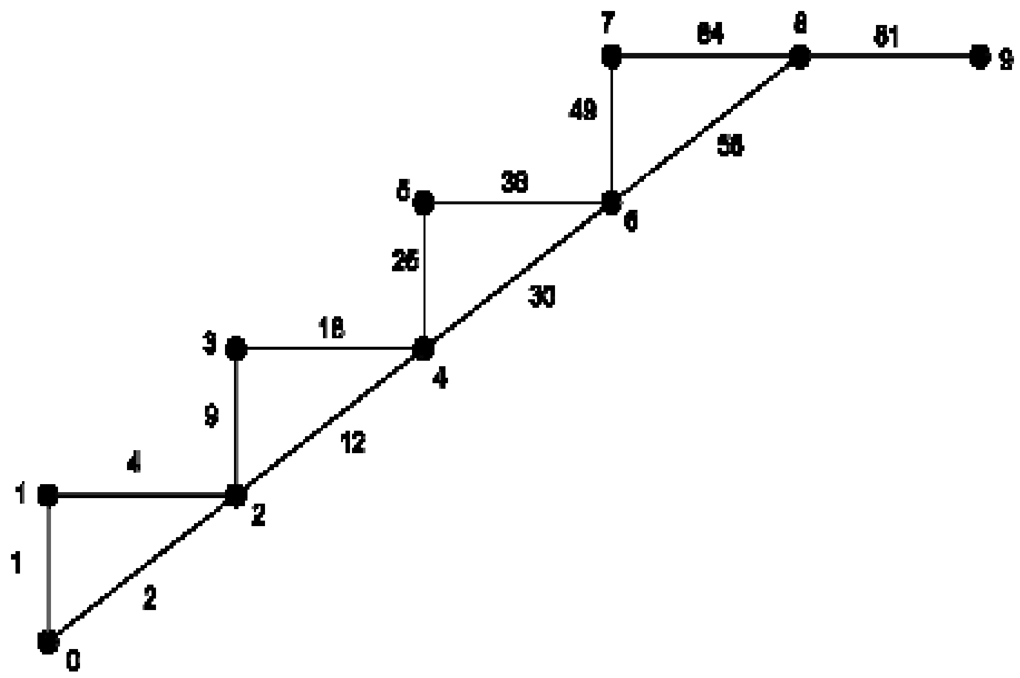

Fig.18 stair Str_5

CONCLUSION

In this study, we looked at how to optimize the multiplicative labeling of certain families of graphs. We found a way to maximize & minimize the use of multiplicative labels. In conclusion, we state that the aforementioned graphs are the multiplicatively maximal graphs. We looked at examples like the slingshot, the stair, the stethoscope, and the pair of glasses and explained the maximize & minimize of the multiplicative labeling.

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