Modeling of homogeneous and isotropic universe in $f(R) = R + \alpha R^2$ gravity

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Abstract: In this paper, we investigate a model of homogeneous universe powered by Hybrid Expansion Law (HEL) $a = t^m exp(\beta t)$; m and β are arbitrary constant, in modified $f(R) = R + \alpha R^2$ gravity. The physical viability of the model is examined by applying energy condition and stability condition. The derived model represents a model of transitioning universe in $f(R)$ theory of gravity. The graphical representation of energy density, ricci scalar and velocity of sound are given and also we discuss some physical properties of the universe.

Keywords: R^2 gravity; Homogeneous Universe, Hybrid expansion law.

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1 Introduction

It is believed that the equilibrium of the universe in its early times is splited spontaneously during the phase alteration and due to this, numerous customs of topological defects were produced in the form of domain walls, cosmic strings, mono-poles and textures. In last decade of twentieth century, there has been numerous study in occurrence of domain walls that revised the previous cosmological models in General Relativity [1, 2, 3]. From cosmological standpoint, we observed that the occurrence of domain walls play an important role in the formation of galaxy Hill et al. [4]. In Hill et al. [4], the authors have investigated that galaxies were formed due to domain wall which was produced during phase transitions after recombination of matter and radiation. In 2003, Fabris and Gon¸calves [5] have constructed a cosmological model where the matter content is explored by a dust fluid and a domain wall fluid - a fluid of domain walls may have an effective equation of state and also it has been observed that this equation of state is qualitatively in consistent with the supernova type-Ia observations. Some recent developments in the general theory of relativity and modified gravity are given in Refs. [6, 7, 8, 9, 10, 11, 12].

In the recent times, the cosmological models are investigating with aim to addressing the possible cause of late time acceleration of the universe. The late time acceleration of the universe may be explain either by introducing dark energy/dark matter as matter/energy content or by modifying the geometric terms in Einstein's field equation. In this paper, we confine ourself to investigate a cosmological model in modified theories of gravity in particular $f(R)$ theories of gravity. It is worthwhile to note that $f(R)$ theories not only studied at cosmological scale to explain the late time acceleration but also these was tested at the astrophysical level. Therefore, some physical characteristics of compact stars

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have been investigated within the framework of modified $f(R)$ gravity. One of the simplest modifications in Einstein's field equation is the theory popularly known as the Starobinsky model [13], given by $f(R) = R + \alpha R^2$. We can studied the structure of the compact stars by two approaches: i) perturbative methods - $f(R)$ is considered a small perturbation from general theory of relativity [14, 15, 16, 17, 18, 19] and ii) non-perturbative [20, 21, 22, 23]. According to both perturbative and non-perturbative methods, the gravitational mass decreases with term α. Although, in non-perturbative way a gravitational sphere develop outside the star, so that the astrophysical mass of compact stars increases with term α [24, 25] for the analysis on masses in R^2 gravity. In Ref. [26], it has been investigated that lowest normal-mode frequency must be real in case of compact star in R^2 gravity.

In this paper, we consider a model of isotropic and homogeneous universe in $f(R) = R + \alpha R^2$ gravity which is governed by hybrid expansion law. The structure of this paper is as follows: In Section 2, the theoretical model and its basic equations are given. In Section 3, we describes the physical properties of the transitioning universe. Finally, in Section 4, we summarize our finding.

2 The metric and field equations

The flat FRW space-time, describing homogeneous and isotropic universe is read as

$$ds^2 = dt^2 - a(t)^2 \; dx^2 + dy^2 + dz^2 \tag{1}$$

Where, $a(t)$ is a scale factor and it define the rate of expansion of the universe. The model is based on $f(R) = R + \alpha R^2$ gravity.

The action describing this modification of General Relativity is obtained from the Einstein-Hilbert action by simply trading R for a generic nonlinear function of it [28, 29, 30, 31].

$$S = \frac{c^4}{16\pi G_n} \int d^4x \sqrt{-g} f(R) + S_\Omega \tag{2}$$

where G_n and S_Ω are the Newton's constant and the action for the matter fields.

In this paper, we follow this metric conformity as described in Refs. [31, 32]

$$f_R(R) R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f - \nabla_\mu \nabla_\nu f_R + g_{\mu\nu} \Box f_R = k T^{\Omega}_{\mu\nu} \tag{3}$$

Here, $T^{\Omega}_{\mu\nu}$ is the energy momentum tensor for matter field, $f_R(R) = df(R)/dR$ is derivative of $f(R)$ with respect to R.

The Einstein's field equations are given by

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \frac{6\alpha}{a^4} \left[5\dot{a}^4 - 12a\dot{a}^2\ddot{a} + a^2\ddot{a}^2 + 4a^2\dot{a}\dddot{a} + 2a^3\ddddot{a}\right] = -8\pi p \tag{4}$$

$$3\frac{\dot{a}^2}{a^2} - \frac{18\alpha}{a^4} \left[a^2\ddot{a}^2 + 2a^2\dot{a}\dddot{a} - 5\dot{a}^4 + 2a\dot{a}^2\ddot{a}\right] = 8\pi\rho \tag{5}$$

We consider that the scale factor a = tmexp(βt); m and β are constants. This form of scale factor a describes the transitioning phenomenon of the universe from early decelerating phase to accelerating universe.

The Hubble parameter is read as

$$H = \frac{\dot{a}}{a} = \left(\frac{m}{t} + \beta\right) \tag{6}$$

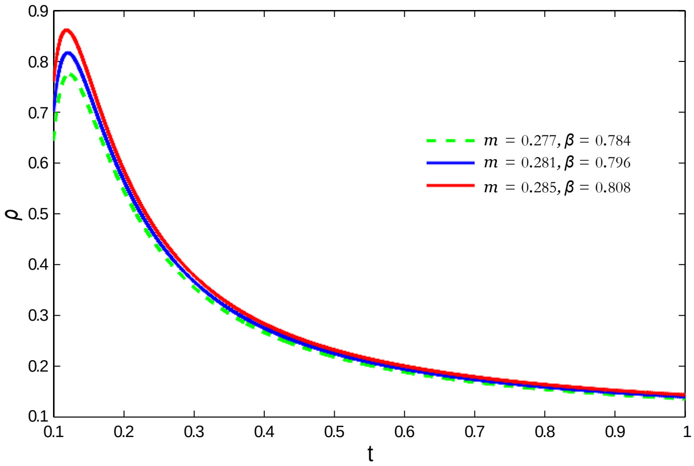

Figure 1: Variation of ρ versus time t

3 The physical properties of the universe

The expression for energy density (ρ) and pressure p of model (1) are read as

$$\rho = \frac{3}{8\pi}\left(\frac{m}{t} + \beta\right)^2 - \frac{9\alpha}{4\pi}\left[\frac{m^2}{t^4} + 4\left(\frac{m}{t} + \beta\right)\frac{m}{t^3} - 10\left(\frac{m}{t} + \beta\right)^2\frac{m}{t^2}\right] \tag{7}$$

$$p = \frac{3}{8\pi}\left[\frac{2m}{t^2} - 3\left(\frac{m}{t} + \beta\right)^2\right] + \frac{3\alpha}{4\pi}\left[\frac{(7m^2 - 12m)}{t^4} + 24\left(\frac{m}{t} + \beta\right)\frac{m}{t^3} - 14\left(\frac{m}{t} + \beta\right)^2\frac{m}{t^2}\right] \tag{8}$$

The Ricci Scalar R is computed as

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = 6\left[\frac{m}{t^2} - 2\left(\frac{m}{t} + \beta\right)^2\right] \tag{9}$$

The Energy Conditions are obtained as

$$\rho + p = \frac{1}{4\pi}\frac{m}{t^2} + \frac{3\alpha}{4\pi}\left[\frac{(4m^2 - 12m)}{t^4} + 12\left(\frac{m}{t} + \beta\right)\frac{m}{t^3} + 16\left(\frac{m}{t} + \beta\right)^2\frac{m}{t^2}\right] \tag{10}$$

$$\rho - p = \frac{1}{4\pi}\left[-\frac{m}{t^2} + \left(\frac{m}{t} + \beta\right)^2\right] - \frac{3\alpha}{4\pi}\left[\frac{(10m^2 - 12m)}{t^4} + 36\left(\frac{m}{t} + \beta\right)\frac{m}{t^3} - 44\left(\frac{m}{t} + \beta\right)^2\frac{m}{t^2}\right] \tag{11}$$

$$\rho + 3p = \frac{3}{8\pi}\left[\frac{2m}{t^2} - \left(\frac{m}{t} + \beta\right)^2\right] + \frac{9\alpha}{4\pi}\left[\frac{(6m^2 - 12m)}{t^4} + 20\left(\frac{m}{t} + \beta\right)\frac{m}{t^3} - 6\left(\frac{m}{t} + \beta\right)^2\frac{m}{t^2}\right] \tag{12}$$

In the derived model, the velocity of sound is computed as

$$v^2 = \frac{\left[-2m + 3m(m+\beta t)\right]t^2 + 3\alpha\left[(48m - 52m^2) + 28m(m+\beta t)^2 - 44m(m+\beta t)\right]}{-3m(m+\beta t)t^2 - 3\alpha\left[24m^2 + 20m(m+\beta t)^2 + 8m(m+\beta t)\right]} \tag{13}$$

4 Concludlding remarks

In this paper, we have investigated a model of transitioning universe in the modified $f(R)$ theory of gravity which describes late time acceleration of the universe without aid of dark energy/dark matter component. Fig. 1 and Fig. 2 depict the behaviour of energy density ρ and pressure p with respect

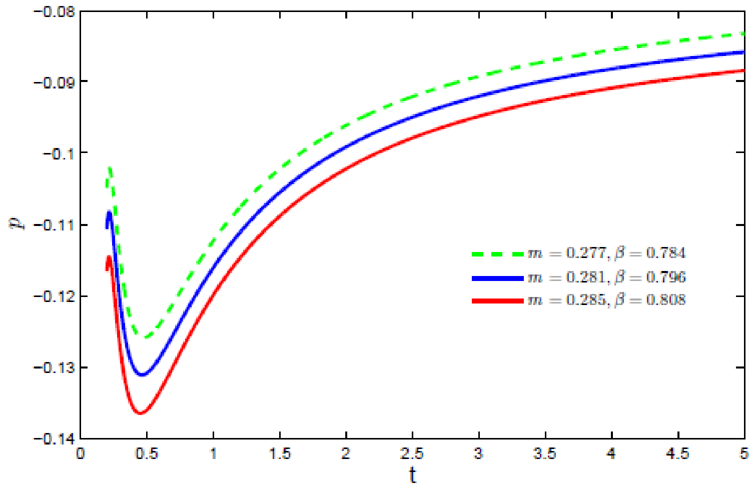

Figure 2: Variation of p versus time t

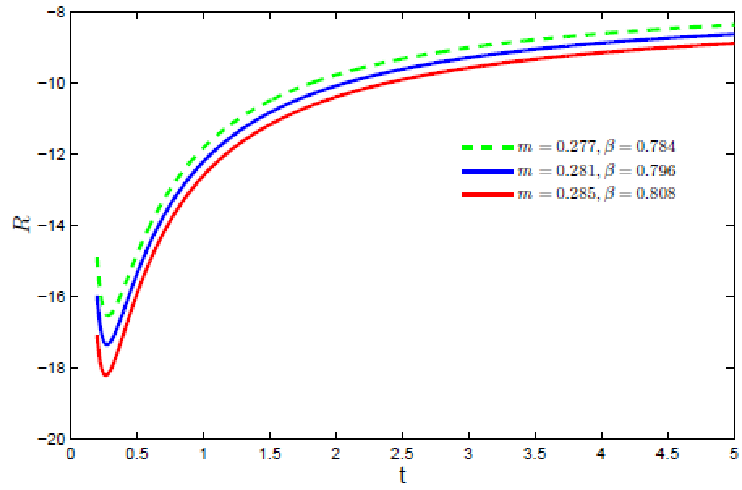

Figure 3: Variation of R versus time t

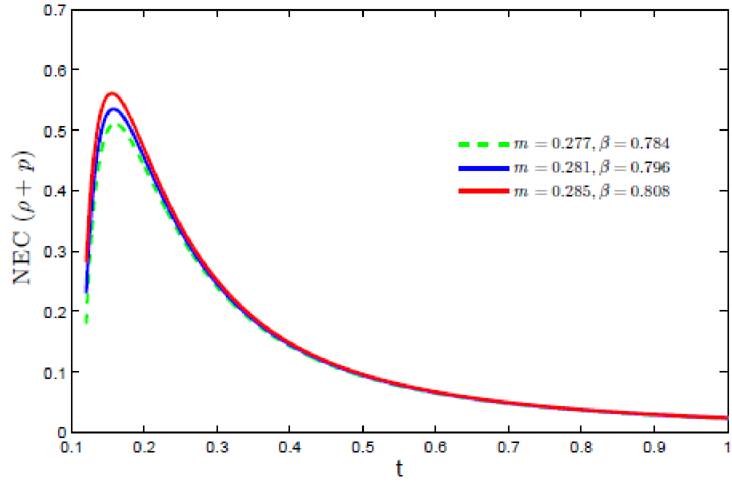

Figure 4: Plot of $\rho + p$ versus time t

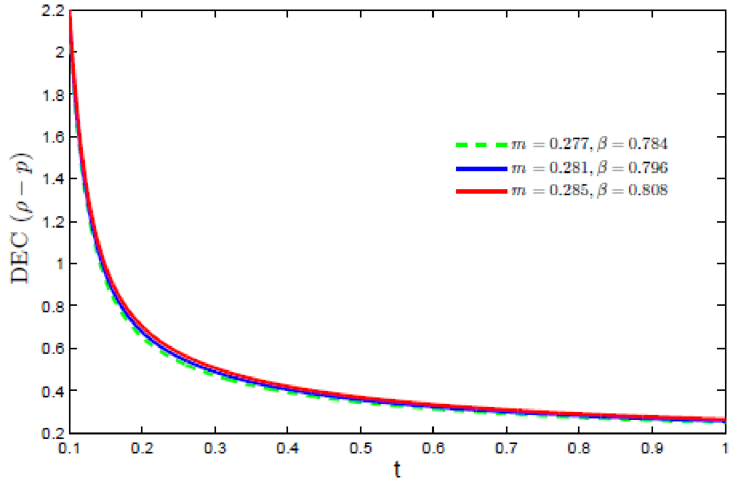

Figure 5: Plot of $\rho - p$ versus time t.

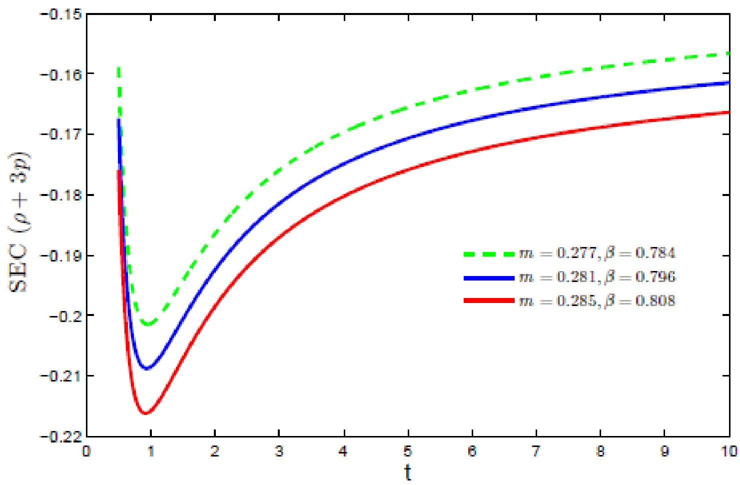

Figure 6: Plot of $\rho - 3p$ versus time t.

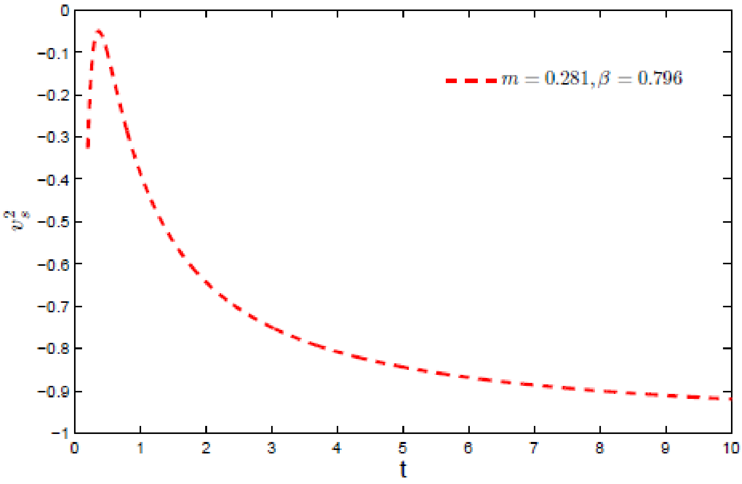

Figure 7: Plot of v^2 versus time t

to time respectively. From Fig. 1, we observe that energy density is decreasing function of time and pressure varies with its negative value at present epoch. The variation of ricci scalar R versus time is shown in Fig. 3 for some particular values of constants m and β. The null energy condition (NEC), dominant energy condition (DEC) and strong energy condition (SEC) versus time have been graphed in Figs. 4, 5 and 6 respectively. From the graphical representations of NEC, DEC and SEC, we observe that the derived model validates NEC and DEC while it violates the SEC. The graphical representation of velocity of sound is depicted in Fig. 7 for m = 0.281 and β = 0.796. As a final comment, we note that the present model of the universe may be a viable model to describe the late time acceleration of the universe and $f(R)$ theory of gravity successfully describes the dynamics of universe at present epoch without aid of dark energy/dark energy components.

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