INTUITIONISTIC FELICITOUS FUZZY GRAPHS

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Article History: Received: 08.05.2023 **Revised:** 20.06.2023 **Accepted:** 15.07.2023

Abstract

Intuitionistic Fuzzy Graphs is a highly growing research area dealing with real life applications. We introduced the new concept of IFFGs and defined properties of IFFGs. We have newly analyzed irregular IFFGs, highly irregular IFFGs and their complements.

Keywords: Felicitous Fuzzy Graphs (FFGs), Bipolar Felicitous Fuzzy Graphs (BFFGs), Intuitionistic Felicitous Fuzzy Graphs (IFFGs).

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DOI: 10.31838/ecb/2023.12.5.489

1. Introduction

There are many uses of graph theory to solve problems in engineering, computer science, operation research, system analysis, networking routing, economics and transpotation. In 1975, A. Rosenfeld [1] launched the concept of fuzzy graphs. T. Krassimir and K. Atanassov [2] introduced intuitionistic fuzzy sets and intuitionistic fuzzy graphs in 1986 and 1999 respectively. R. Parvathi [3] investigated intuitionistic fuzzy graphs and its properties in 2002. M. Akram and W. Dudek [4] added the notion of regular intuitionistic fuzzy graphs. A.
Nagoorgani and S. R. Latha [5] established regular intuitionistic fuzzy graphs and its properties. V. Vignesh and Muthuraj [6] proposed and discussed on complement of intuitionistic fuzzy graphs. K. R. Bhutani [7] dealt with weak isomorhism and isomorphism between fuzzy graphs . T. Bharathi and S. Felixia [8] newly introduced felicitous fuzzy graphs and bipolar felicitous fuzzy graph in 2020 and 2021 respectively. In this article, we define the concept of intuitionistic felicitous fuzzy graphs with its properties.

Preliminaries

Definition 1.

Let (P,Q)-graph, G be a proper labeling $f : V(G) \to \{Q+1, Q+2, ..., P+Q\}$, where P and Q represent the number of arcs and the number of nodes respectively. The arc label $\beta(uv)$ of each arc $uv \in E(G)$ is defined as $\beta(uv) = (\eta(u) + \eta(v))(\text{mod}(P * h))$, where $h = 0.01$ if $3 \leq n \leq 49$ and $h = 0.001$ if $50 \leq n \leq 99$ and so on. G is said to be felicitous fuzzy graph, such that $\{\beta(uv) : uv \in E(G)\} = [1,Q]$,

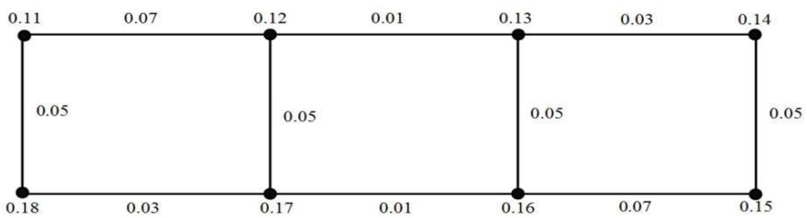

Figure 1: Felicitous fuzzy graph

Definition 2.

An IFFG is of the form, $G = (P,Q)$ where

(i) $P = V_1, V_2, ..., V_n$ such that $\eta_P : V \to [0,1]$ and $\delta_P : V \to [0,1]$ denote the membership and the non-membership values of any $v \in V$ respectively and $0 \leq \eta_P(v_i) + \delta_P(v_i) \leq 1$, for $i = 1,2,...,n$.

(ii) $E \subseteq V \times V$ where $\eta_Q : V \times V \to [0,1]$ and $\delta_Q : V \times V \to [0,1]$,
$\eta_Q(v_i,v_j) \leq min\{\eta_P(v_i), \eta_P(v_j)\}$
$\delta_Q(v_i,v_j) \leq max\{\delta_P(v_i), \eta_P(v_j)\}$
and $0 \leq \eta_Q(v_i,v_j) + \delta_Q(v_i,v_j) \leq 1$ for every $(v_i,v_j) \in E$
where $1 \leq i,j \leq n$ and $i \neq j$.

(iii) G has a proper labeling $f : V(G) \to \{Q+1, Q+2, ..., P+Q\}$ where P and Q represent the number of nodes and the number of arcs. $E(G)$ is defined as
$\eta_Q(u,v) = (\eta_Q(u) + \eta_Q(v))(\text{mod } P)$
$\delta_Q(u,v) = (\delta_Q(u) + \eta_Q(v))(\text{mod } P)$
where $h = 0.01$ if $3 \leq n \leq 49$ and h=0.001 if $50 \leq n \leq 99$ and so on. G is said to be
intuitionistic felicitous fuzzy graph such that $\{\eta_Q(u,v), \delta_Q(u,v) : uv \in E(G)\} = [1,P]$.

2. Main Results

3.1 Properties of Intuitionistic Felicitous Fuzzy Graphs

Theorem 1. If $G_1 = (P_1,Q_1)$ and $G_2 = (P_2,Q_2)$ are two complete intuitionistic felicitous fuzzy graphs, then $G_1 + G_2$ *is* also a complete intuitionistic felicitous fuzzy graph.

Proof. Let $G_1 = (P_1,Q_1)$ and $G_2 = (P_2,Q_2)$ are two complete intuitionistic felicitous fuzzy graphs.

So $\eta_Q(v_i,v_j) = min(\eta_P(v_i),\eta_P(v_j))$ *and*

$\delta_Q(v_i,v_j) = min(\delta_P(v_i),\delta_P(v_j))$, *if* $v_i,v_j \in P_1$

$\eta Q'(vi,vj) = min(\eta_P'(v_i),\eta_P'(v_j))$ *and*

$\delta'_Q(v_i,v_j) = min(\delta'_P(v_i),\delta'_P(v_j))$, *if* $v_i,v_j \in P2$

But $G_1 + G_2 = (P_1 \cup P_2, Q_1 \cup Q_2 \cup Q')$ is defined by

$(\eta_P+\eta'_P)(v) = (\eta_P \cup \eta'_P)(v)$ *if* $v \in P_1 \cup P_2$

$(\delta_P+\delta'_P)(v) = (\delta_P \cup \delta'_P)(v)$ *if* $v \in P_1 \cup P_2$

$(\eta Q + \eta Q')(v_iv_j) = (\eta Q \cup \eta Q')(v_iv_j)$ *if* $v_iv_j \in Q_1 \cup Q_2$

$= min\ (\eta_P(v_i),\delta,\eta'_P(v_j))$, if $v_iv_j \in Q'$

$(\delta Q + \delta Q')(v_iv_j) = (\delta Q \cup \delta Q')(v_iv_j)$ *if* $v_iv_j \in Q_1 \cup Q_2$

$= max\ (\delta_P(v_i),\delta,\eta'_P(v_j))$, if $v_iv_j \in Q'$

Therefore $G_1 + G_2$ is a complete intuitionistic felicitous fuzzy graph.

Theorem 2. When G is a complete intuitionistic felicitous fuzzy graph, the arc set is empty in $\bar{G}$.

Proof. Let $G = (P,Q)$ be a complete intuitionistic felicitous fuzzy graph.

So $\eta_Q(v_i,v_j) = min\{\eta_P(v_i),\eta_P(v_j)\}$ and

$\delta_Q(v_i,v_j) = max\{\delta_P(v_i),\delta_P(v_j)\}$ if $v_i,v_j \in p$

Hence in $\bar{G}$, $\bar{\eta}_Q(v_i,v_j) = min\{\eta_P(v_i),\eta_P(v_j)\} - \eta_Q(v_i,v_j)$

$= min\{\eta_P(v_i),\eta_P(v_j)\} - min\{\eta_P(v_i),\eta_P(v_j)\}$, if $i,j = 1,2,...,n$

$= 0$, if $i,j = 1,2,...,n$.

and $\bar{\delta}_Q(v_i,v_j) = min\{\delta_P(v_i),\delta_P(v_j)\} - \delta_Q(v_i,v_j)$

$= min\{\delta_P(v_i),\delta_P(v_j)\} - min\{\delta_P(v_i),\delta_P(v_j)\}$

$= 0$, if $i,j = 1,2,...,n$.

Thus $(\eta_Q(v_i,v_j),\eta_Q(v_i,v_j)) = (0,0)$

Hence the arc set is empty in $\bar{G}$ if G is complete intuitionistic felicitous fuzzy graph.

Theorem 3. If $G = (P_1,Q_1)$ and $G_2 = (P_2,Q_2)$ are two complete intuitionistic felicitous fuzzy graphs, then $G_1 + G_2 = \bar{G}_1 + \bar{G}_2$.

Proof. Let $G_1 = (P_1,Q_1)$ and $G_2 = (P_2,Q_2)$ are two complete intuitionistic felicitous fuzzy graphs. WKT, $G_1 + G_2$ is also a complete intuitionistic felicitous fuzzy graph.

Also by the above theorem, $\bar{G}_1$ and $\bar{G}_2$ are two intuitionistic felicitous fuzzy graphs with $\bar{Q}_1 = \Phi$ and $\bar{Q}_2 = \Phi$. Also $\bar{P}_1 = P_1$ and $\bar{P}_2 = P_2$ without any changes in membership and non-membership values. In $\bar{G}_1 + \bar{G}_2$, the membership and the non-membership values of arcs are defined by,

$(\eta_{Q+}\eta'_Q)(v_iv_j) = min\{\eta(v_i),\eta'_P(v_{ij})\}$ and

$(\delta_{Q+}\delta'_Q)(v_iv_j) = min\{\delta(v_i),\delta'_P(v_{ij})\}$

So $\bar{G}_1 + \bar{G}_2$ is complete with,

$P = P_1 \cup P_2$ and $(\eta_{p+}\eta'_p)(v) = \begin{cases} \eta_p(v)\ if\ v \in P_1 - P_2 \\ \eta'_p(v)\ if\ v \in P_1 - P_2 \end{cases}$

$(\delta_{p+}\delta'_p)(v) = \begin{cases} \delta_p(v)\ if\ v \in P_1 - P_2 \\ \delta'_p(v)\ if\ v \in P_1 - P_2 \end{cases}$

Hence $G_1 + G_2 = G_1 + G_2$

Proposition 1. It is not necessary that the complement of regular intuitionistic felicitous fuzzy graph is regular.

Let $G = (P,Q)$ be a regular intuitionistic felicitous fuzzy graph.

The membership and the non-membership values of G are as follows:

Example 1.

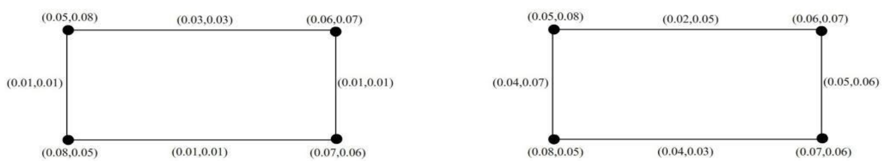

Figure 2: Intuitionistic felicitous fuzzy graph G and its complement G'

$(\eta_P(v_1),\delta_P(v_1))$ = (0.05,0.08),$(\eta_P(v_2),\delta_P(v_2))$ = (0.06,0.07)
$(\eta_P(v_3),\delta_P(v_3))$ = (0.07,0.06),$(\eta_P(v_4),\delta_P(v_4))$ = (0.08,0.05)
$(\eta_Q(v_1,v_2),\delta_Q(v_1,v_2))$ = (0.03,0.03),$(\eta_Q(v_2,v_3),\delta_Q(v_2,v_3))$ = (0.01,0.01)
$(\eta_Q(v_3,v_4),\delta_Q(v_3,v_4))$ = (0.03,0.03),$(\eta_Q(v_4,v_1),\delta_Q(v_4,v_1))$ = (0.01,0.01)
Here, $d(v_1) = d(v_2) = d(v_3) = d(v_4)$ = (0.04,0.04)
Thus G is a regular intuitionistic felicitous fuzzy graph.

The complement of G is the graph *G* with membership and non-membership values as follows:
$(\eta_P(u_1),\delta_P(u_1))$ = (0.05,0.08),$(\eta_P(u_2),\delta_P(u_2))$ = (0.06,0.07)
$(\eta_P(u_3),\delta_P(u_3))$ = (0.07,0.06),$(\eta_P(u_4),\delta_P(u_4))$ = (0.08,0.05)
$(\eta_Q(u_1,u_2),\delta_Q(u_1,u_2))$ = (0.02,0.05),$(\eta_Q(u_2,u_3),\delta_Q(u_2,u_3))$ = (0.05,0.06)
$(\eta_Q(u_3,u_4),\delta_Q(u_3,u_4))$ = (0.04,0.03),$(\eta_Q(u_4,u_1),\delta_Q(u_4,u_1))$ = (0.04,0.07)
Here, $d(v_1)$ = (0.06,0.12),$d(v_2)$ = (0.07,0.11),$d(v_3)$ = (0.09,0.09),$d(v_4)$ = (0.08,0.10)

Therefore G is not a regular intuitionistic felicitous fuzzy graph.

Proposition 2. A degree of a intuitionistic felicitous fuzzy graph need not be the same in its complement.

Example 2.
Let G=(P,Q) be an intuitionistic felicitous fuzzy graph and *G* be its complement.
The membership and non-membership values of G are as follows:
$(\eta_P(v_1),\delta_P(v_1))$ = (0.04,0.07),$(\eta_P(v_2),\delta_P(v_2))$ = (0.05,0.06)
$(\eta_P(v_3),\delta_P(v_3))$ = (0.06,0.05),$(\eta_P(v_4),\delta_P(v_4))$ = (0.07,0.04)
$(\eta_Q(v_1,v_2),\delta_Q(v_1,v_2))$ = (0.01,0.01),$(\eta_Q(v_2,v_3),\delta_Q(v_2,v_3))$ = (0.03,0.03)
$(\eta_Q(v_3,v_4),\delta_Q(v_3,v_4))$ = (0.01,0.01)
The membership and non-membership values of *G* are as follows:
$(\eta_P(v_1),\delta_P(v_1))$ = (0.04,0.07),$(\eta_P(v_2),\delta_P(v_2))$ = (0.05,0.06)
$(\eta_P(v_3),\delta_P(v_3))$ = (0.06,0.05),$(\eta_P(v_4),\delta_P(v_4))$ = (0.07,0.04)
$(\eta_Q(v_1,v_2),\delta_Q(v_1,v_2))$ = (0.03,0.06),$(\eta_Q(v_2,v_3),\delta_Q(v_2,v_3))$ = (0.02,0.03)
$(\eta_Q(v_3,v_4),\delta_Q(v_3,v_4))$ = (0.05,0.06)

Degree	in G	in *G*
$d(v_1)$	(0.01,0.01)	(0.03,0.06)
$d(v_2)$	(0.04,0.04)	(0.05,0.09)
$d(v_3)$	(0.04,0.04)	(0.07,0.09)
$d(v_4)$	(0.01,0.01)	(0.05,0.06)

Proposition 3. The minimum degree of an intuitionistic felicitous fuzzy graph, G and that
of its complement $\bar{G}$ need not be the same. Also, the maximum degree of an intuitionistic felicitous fuzzy graph, G and that of its complement $\bar{G}$ need not be the same.

Example 3.
Let G=(P,Q) be an intuitionistic felicitous fuzzy graph.
Them

embership and non-membership values of G are as follows:
$(\eta_P(v_1),\delta_P(v_1))$ = (0.04,0.07),$(\eta_P(v_2),\delta_P(v_2))$ = (0.05,0.06)
$(\eta_P(v_3),\delta_P(v_3))$ = (0.06,0.05),$(\eta_P(v_4),\delta_P(v_4))$ = (0.07,0.04)
$(\eta_Q(v_1,v_2),\delta_Q(v_1,v_2))$ = (0.01,0.01),$(\eta_Q(v_2,v_3),\delta_Q(v_2,v_3))$ = (0.03,0.03)
$(\eta_Q(v_3,v_4),\delta_Q(v_3,v_4))$ = (0.01,0.01)

The membership and non-membership values of $\bar{G}$ are as follows:
$(\eta_P(v_1),\delta_P(v_1))$ = (0.04,0.07),$(\eta_P(v_2),\delta_P(v_2))$ = (0.05,0.06)
$(\eta_P(v_3),\delta_P(v_3))$ = (0.06,0.05),$(\eta_P(v_4),\delta_P(v_4))$ = (0.07,0.04)
$(\eta_Q(v_1,v_2),\delta_Q(v_1,v_2))$ = (0.03,0.06),$(\eta_Q(v_2,v_3),\delta_Q(v_2,v_3))$ = (0.02,0.03)
$(\eta_Q(v_3,v_4),\delta_Q(v_3,v_4))$ = (0.05,0.06)

Degree	in G	in *G*
$d(v_1)$	(0.01,0.01)	(0.03,0.06)
$d(v_2)$	(0.04,0.04)	(0.05,0.09)
$d(v_3)$	(0.04,0.04)	(0.07,0.09)
$d(v_4)$	(0.01,0.01)	(0.05,0.06)

$\lambda(G)$ = (0.01,0.01) and $\bar{\lambda}(G)$ = (0.03,0.06)
$\Delta(G)$ = (0.01,0.01) and $\bar{\Delta}(G)$ = (0.03,0.06)

Algorithm 1.

Step 1: Fix any (s,t)− fuzzy gragh G,(n ≤ 3), where n is odd.

Step 2: Fix the membership function for the nodes, $\eta_P(v_i) = t + 1$,
$t \leq n$ where i = 1,2,...,n.

Step 3: Fix the non-membership function for the nodes,
$\delta_P(v_i) = (S + t) - i, n \geq t$, where i = 0,1,2,...,S − 1.

Step 4: Fix the membership function for the arcs,
$\eta_Q(v_i,v_{i+1}) = (\eta_P(v_i)+\eta_P(v_{i+1}))(\mathrm{mod}(s * h))$, where i = 1,2,...,t and
h=0.01 if $3 \leq n \leq 49$
h=0.001 if $50 \leq n \leq 99$ and so on.

Step 5: Fix the non-membership function for the arcs,
$\delta_Q(v_i,v_{i+1}) = (\delta_P(v_i) + \delta_P(v_{i+1}))(\mathrm{mod}(s * h))$, where i = 1,2,...,t and
h=0.01 if $3 \leq n \leq 49$
h=0.001 if $50 \leq n \leq 99$ and so on.

Step 6 : Every membership function of the nodes and the arcs must satisfy the fuzzy graph condition, $\eta_Q(u,v) \leq \eta_P(u) \wedge \eta_P(v)$ $\delta_Q(u,v) \leq \delta_P(u) \wedge \delta_P(v)$, then it fails.

Algorithm 2.

Step 1: Fix any cycle related (s,t)− graph G with n ≤ 3.

Step 2: Fix the membership function for the nodes,
$\eta_P(v_i) = t + 1, n \geq t$, where i = 1,2,...,n.

Step 3: Fix the non-membership function for the nodes,
$\delta_P(v_i) = (S + t) - i, n \geq t$, where i = 0,1,2,...,S − 1.

Step 4: Fix the membership function for the arc,
$\eta_Q(v_i,v_{i+1}) = (\eta_P(v_i) + \eta_P(v_{i+1}))(\mathrm{mod}(s * h))$, where i = 1,2,...,t and
h=0.01 if $3 \leq n \leq 49$
h=0.001 if $50 \leq n \leq 99$ and so on.

Step 5: Fix the non-membership function for the arcs,

$\delta_Q(v_i,v_{i+1}) = (\delta_P(v_i) + \delta_P(v_{i+1}))(mod(s * h))$, where $i = 1,2,...,t$ and
$h=0.01$ if $3 \leq n \leq 49$
$h=0.001$ if $50 \leq n \leq 99$ and so on.

Step 6 : Every membership function of the nodes and the arcs must satisfy the fuzzy graph condition, $\eta_Q(u,v) \leq \eta_P(u) \wedge \eta_P(v)$ $\delta_Q(u,v) \leq \delta_P(u) \wedge \delta_P(v)$, then it fails.

Corollary 1. The complement of the theorem 4.24 need not be true.

Proof. Consider intuitionistic felicitous fuzzy graphs G and G' with the underlying sets S and S' resepectively. The membership and non-membership values of graph G,
$(\eta_P(u_1),\delta_P(u_1)) = (0.04,0.07),(\eta_P(u_2),\delta_P(u_2)) = (0.05,0.06)$

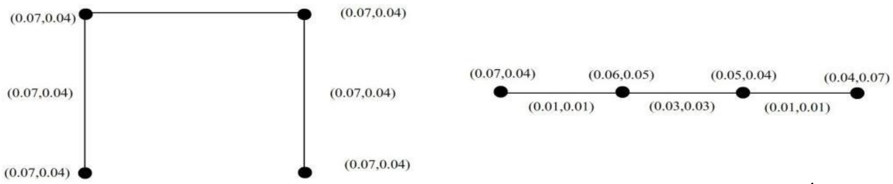

Figure 3: Intuitionistic felicitous fuzzy graph of G and G'

$(\eta_P(u_3),\delta_P(u_3)) = (0.06,0.05),(\eta_P(u_4),\delta_P(u_4)) = (0.07,0.04)$
$(\eta_Q(u_1,u_2),\delta_Q(u_1,u_2)) = (0.01,0.01)$
$(\eta_Q(u_2,u_3),\delta_Q(u_2,u_3)) = (0.03,0.03)$
$(\eta_Q(u_3,u_4),\delta_Q(u_3,u_4)) = (0.01,0.01)$
The membership and non-membership values of graph G',
$(\eta'_P(u_1),\delta'_P(u_1)) = (0.07,0.04),(\eta'_P(u_2),\delta'_P(u_2)) = (0.06,0.05)$
$(\eta'_P(u_3),\delta_P(u_3)) = (0.05,0.06),(\eta'_P(u_4),\delta'_P(u_4)) = (0.04,0.07)$
$(\eta_{Q'}(u_1,u_2),\delta_{Q'}(u_1,u_2)) = (0.01,0.01)$
$(\eta_{Q'}(u_2,u_3),\delta_{Q'}(u_2,u_3)) = (0.03,0.03)$
$(\eta_{Q'}(u_3,u_4),\delta_{Q'}(u_3,u_4)) = (0.01,0.01)$
From the above two intuitionistic felicitous fuzzy graphs,
Size of G and G' = (0.05,0.05)
Size of G and G' = (0.22,0.22)
but $G \neq G'$ Hence the proof.

3. Conclusion

Graph theory is used in a variety of ways to tackle issues in the fields of transportation, system analysis, economics, and operations research. However, certain solutions to a graph theoretical problem may leave some parts in doubt or ambiguous. The fuzzy set method is a natural solution for ambiguity and vagueness. The new idea of IFG has shown advantages in tackling vagueness and ambiguity when compared to fuzzy set. In this article, we went into great detail on the novel idea of IFFGs.

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