Network-Based Cooperative Risk Analysis of a biodiesel manufacturing plant

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Abstract

The main objective of this research paper is to introduce a novel framework that applies cooperative game theory to risk analysis, taking advantage of the underlying network structure. Probabilistic risk analysis (PRA) is a widely utilized methodology for evaluating and assessing risks associated with real-world network configurations. While previous studies have focused on PRA from a physical engineering perspective, there is a growing recognition of the significant role played by human behavior in risk estimation through PRA, prompting the integration of PRA and game theory. However, the existing literature has predominantly concentrated on non-cooperative game theory, neglecting scenarios where cooperative structures, such as those prevalent in a biodiesel manufacturing plant. To address this gap, our study proposes a risk analysis approach that leverages cooperative game theory, with a specific emphasis on utilizing marginalism. By incorporating cooperative game theory into the risk analysis process, we aim to provide a more comprehensive assessment that takes into account cooperative structures and their associated risks.

Keywords: *game theory*, *cooperative game*, *Shapley value*, *risk management*, *network structure*

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1. Introduction

Probabilistic risk analysis (PRA) is a methodology used to evaluate and quantify risks associated with network structures (Bedford and Cooke, 2001). It is widely employed in various fields, including engineering, finance, environmental sciences, and project management. It involves assessing risks by considering both the likelihood (probability) and potential consequences of various events or hazards. It takes into account uncertainties, variability, and interdependencies within a system to provide a more comprehensive understanding of the overall risk profile. While extensive research has focused on PRA from a physical engineering perspective, the significance of human behavior in risk estimation using PRA has gained increasing attention. Hausken (2002) pioneered this approach by integrating PRA and game theory. In essence, game theory is a mathematical framework for decision-making in situations involving two or more relevant players, where choices depend on the actions of competitors. These scenarios are known as game situations. Originating from von Neumann and Morgenstern's "Theory of Games and Economic Behavior" (1944), game theory examines conflicts of interest and cooperation among players by formulating mathematical models of game situations (e.g., Fudenberg and Tirole, 1991). As a result, game theoretical approaches inherently consider human behavior. In this context, our study aims to leverage game theory within the realm of risk analysis by incorporating it into PRA. By recognizing the role of human behavior and the dynamics of decision-making among relevant players, we seek to enhance the understanding and assessment of risks associated with network structures. Game theory encompasses two main types of games: cooperative and non-cooperative.

- **Non-cooperative Game Theory**: Non-cooperative game theory focuses on situations where players act independently and make decisions based on their own self-interest, without any formal agreements or coordination. In this framework, each player aims to maximize their individual utility or payoff, considering the actions of other players as given. The most common solution concept in non-cooperative game theory is the Nash equilibrium, where no player has an incentive to unilaterally deviate from their chosen strategy, given the strategies of the other players.
- **Cooperative Game Theory**: Cooperative game theory, on the other hand, examines situations where players can form coalitions or groups and cooperate to achieve common goals. In this framework, players can make binding agreements, negotiate, and jointly decide on strategies to maximize the collective benefits or achieve a fair allocation of resources. Cooperative game theory seeks to analyze the potential for cooperation, study the stability of cooperative outcomes, and propose solution concepts that distribute the total payoffs among players in a desirable manner. One widely used solution concept in cooperative game theory is the Shapley value, which assigns values to players based on their marginal contributions to different coalitions.

While non-cooperative game theory often focuses on competitive scenarios, such as strategic interactions in markets, auctions, or conflicts, cooperative game theory is more applicable in situations where players can form alliances, cooperate in teams, or engage in joint decision-making. Cooperative game theory can be employed to study resource allocation, negotiation, coalition formation, and cooperative strategies in various contexts, including economics, political science, operations research, and management. It's worth noting that the distinction between cooperative and non-cooperative game theory is not always clear-cut, as there are situations that can incorporate elements of both frameworks. Hybrid models and extensions of game theory, such as repeated games, evolutionary game theory, and mechanism design, further enrich the analysis of strategic interactions by considering both cooperative and non-cooperative aspects. While non-cooperative game theory has gained popularity in disciplines such as economics, particularly in areas like industrial organization theory, cooperative game theory has not received widespread adoption. This may be due to the prevalent perception of market companies as competitors in non-cooperative games. However, it is crucial to recognize real-world situations where relevant players exhibit cooperation, especially in risk considerations.

In this paper, our focus is on highlighting scenarios where cooperation among relevant players is essential in risk analysis, taking the example of a biodiesel manufacturing plant.

Within the plant, multiple segments representing different players collaborate towards the common production goals of the company. While some economists argue that cooperative games can be transformed into non-cooperative games through the Nash program, this conversion does not hold universally. In the case of the biodiesel plant, a sustained cooperative game situation can be reasonably and unproblematically assumed. Considering the aforementioned factors, this paper aims to introduce a framework for a cooperative game theoretical approach to risk analysis, specifically addressing network structures. We propose a method for allocating risk among various segments of the biodiesel plant with a network structure, utilizing Shapley values. These values are widely employed solution concepts in cooperative game theory.

Biodiesel is a renewable fuel derived from organic sources, primarily vegetable oils and animal fats. It is considered a biofuel because it is produced from biomass, which is a sustainable and renewable resource. Biodiesel can be used as a direct replacement or blended with petroleum diesel in diesel engines without the need for engine modifications. Here are some key characteristics and benefits of biodiesel:

- **Environmental Sustainability**: Biodiesel is considered a more environmentally friendly alternative to petroleum diesel. It reduces greenhouse gas emissions, including carbon dioxide, carbon monoxide, and particulate matter, leading to improved air quality and reduced impact on climate change. Biodiesel is also biodegradable and non-toxic, minimizing environmental hazards in case of spills.
- **Renewable and Sustainable:** Biodiesel is produced from renewable resources such as vegetable oils, animal fats, and used cooking oils. These feedstocks can be grown and replenished, making biodiesel a sustainable energy option that reduces reliance on fossil fuels.
- **Energy Independence**: Biodiesel production helps reduce dependence on imported petroleum, contributing to energy independence for countries. It promotes domestic agricultural sectors by utilizing locally sourced feedstock.
- **Engine Compatibility**: Biodiesel can be used in most diesel engines without the need for major engine modifications. It can be blended with petroleum diesel at various concentrations, typically denoted as BXX (e.g., B20 indicates a blend of 20% biodiesel and 80% petroleum diesel). Biodiesel blends can be used in existing diesel infrastructure, including vehicles, trucks, buses, and industrial equipment.
- **Lubricity and Performance**: Biodiesel has excellent lubricating properties, which can enhance engine life and reduce wear on fuel system components. It also has a higher cetane number, improving ignition quality and combustion efficiency, leading to potential performance benefits such as increased power and torque.

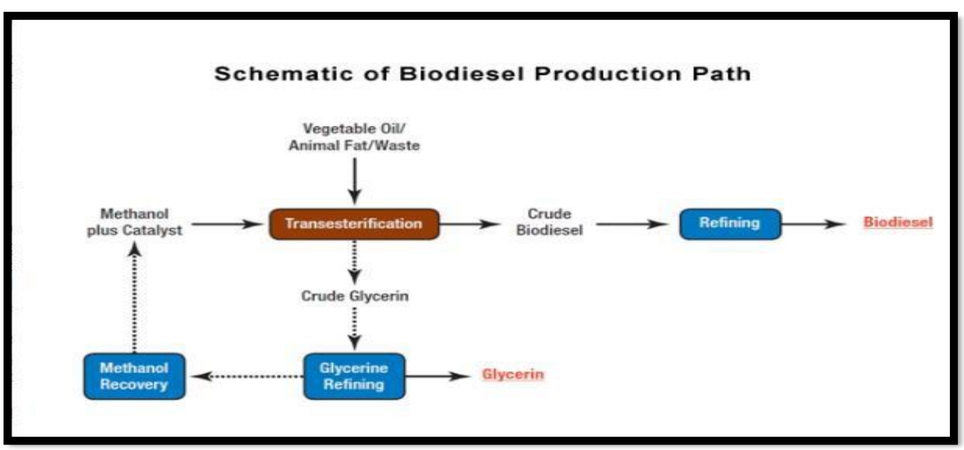
- **Biodegradable and Safe Handling**: Biodiesel is readily biodegradable,reducing environmental impacts in case of spills or leaks. It has a higher flashpoint than petroleum diesel, making it safer to handle and transport.
- **Byproduct Utilization**: The production of biodiesel also generates glycerin as a byproduct, which can be further processed and used in various industries, such as pharmaceuticals, cosmetics, and food products.

Overall, biodiesel offers a renewable and sustainable alternative to petroleum diesel, reducing environmental impacts, promoting energy independence, and contributing to a more sustainable and greener transportation sector. Hence, we consider the example of a biodiesel manufacturing plant. A biodiesel manufacturing plant typically consists of several key components that are involved in the production process. These components can vary depending on the specific design and scale of the plant, but here are some common components:

- **Feedstock Storage**: Biodiesel production starts with the storage of feedstock, which can be various types of vegetable oils or animal fats. Feedstock storage tanks are used to store the raw materials before they are processed.
- **Pre-Treatment System**: The pre-treatment system is responsible for removing impurities and contaminants from the feedstock. This may involve processes such as filtering, heating, and mixing to prepare the feedstock for further processing.
- **Reactor:** The reactor is where the chemical reaction known as transesterification takes place. Transesterification involves mixing the feedstock with an alcohol, typically methanol, and a catalyst, such as sodium hydroxide or potassium hydroxide. The reactor facilitates the conversion of the feedstock into biodiesel and glycerin.
- **Separation and Purification Units**: After the transesterification reaction, the mixture needs to be separated into biodiesel and glycerin. Separation units such as settling tanks or centrifuges are used to separate the two components. The biodiesel then goes through purification processes such as washing and drying to remove any remaining impurities.
- **Biodiesel Storage and Distribution**: Once the biodiesel is purified, it is stored in tanks for later distribution. These storage tanks are designed to ensure proper handling and storage of the biodiesel, including measures to prevent contamination or degradation. The biodiesel is then transported to customers or distributors for further use.
- **Glycerin Treatment**: The glycerin byproduct from the transesterification process undergoes further treatment to remove impurities and separate it into refined glycerin and other byproducts. The refined glycerin can be sold for various industrial applications.

Utilities and Support Systems: A biodiesel manufacturing plant requires various utilities and support systems to operate efficiently. These may include systems for heating, cooling, electricity supply, water treatment, and waste management. Additionally, control systems and automation technologies are used to monitor and control the different processes within the plant.

It's important to note that the specific configuration and equipment used in a biodiesel manufacturing plant can vary based on factors such as feedstock availability, production capacity, and regulatory requirements. Therefore, the components mentioned above provide a general overview, and actual plant designs may include additional or specialized equipment based on specific needs and circumstances.



The structure of the paper is as follows: Firstly, we introduce Shapley values through concrete examples involving simple-structure graphs. Secondly, we examine a model of a biodiesel manufacturing plant. Finally, we discuss the proposed framework and offer concluding remarks.

2. Shapley value

In this section, we will illustrate the concept of Shapley values using two simple examples. For the sake of clarity, we will use the letters A, B, and C to represent different segments or factory operations within the context of our discussion. In general, the calculation of Shapley values follows the following steps. Let N represent the set of all segments. First, we define a characteristic function c: $2N \rightarrow R$, where $2N$ and R denote the sets of all subsets of N and real numbers, respectively and the function satisfies $c(\varphi) = 0$ and $c(SUT) \leq c(S) + c(T)$. Here, S and T are subsets of N satisfying
$S \cap T = \varphi$. The characteristic function assigns a real value to each subset of segments, representing its contribution to the overall risk or any other relevant measure.
Note that $S \cap T = \varphi$ means that there no segment belonging to both sets S and T.
Then, Shapley value of segment i is defined by

$$\varphi_i(c) = \sum_{S \subseteq N \setminus i} \frac{|S|!\,(|N| - |S| - 1)!}{|N|!}(c(SUi) - c(S))$$

where $|N|$ is the total number of segments, and $|S|$ denotes the number of segments belonging to the set S. In the equation, $(c(SUi) - c(S))$ means that the marginal contribution of i to S. Hence, the Shapley value indicates a weighted average of the respective marginal contributions.

Example 1:

In our analysis, we examine a graph featuring a basic edge, as depicted in Figure 1. The characteristic functions associated with this graph are outlined below:

Fig. 1. Graph with a simple edge.

Here, we set so-called characteristic functions as follows:

$c(A) = 10$, $c(B) = 10$, $c(AB) = 15$

The values c(A) and c(B) represent the individual costs associated with each segment when they are working independently and experience a loss. These costs could include expenses related to maintenance, repair, replacement of equipment, or any other costs incurred due to accidents or production impairment specific to segment A and segment B. On the other hand, the value c(AB) represents the total cost when segments A and B are working together in concert, and an accident occurs or there is an impairment in plant production. This cost takes into account the combined effects and consequences of both segments A and B working together.

The purpose of considering these costs is to assess the financial implications and risks associated with the cooperative operation of segments A and B in the plant. By comparing the separate costs (c(A) and c(B)) with the total cost (c(AB)), it can be evaluated whether the cooperative collaboration of the segments leads to a decrease or increase in overall costs. This information is valuable for decision-making, risk analysis, and determining the appropriate allocation of resources or risk management strategies in a biodiesel manufacturing plant. The calculation of Shapley values for each segment in the context of cooperative risk analysis allows for the fair allocation of costs among the segments. In this particular case, considering segments A and B working together in a biodiesel manufacturing plant, the total cost when they collaborate is 15.

The Shapley value for segment B is calculated by subtracting its individual cost from the total cost: B = 15 - 10 = 5.

This means that if an accident occurs or there is production impairment when both segments A and B are working together, segment B would claim that it should only bear a cost of 5. However, it is important to consider the order in which the segments join the coalition. If only segment B is initially working, and then segment A joins, the situation changes. In this case, the total cost is still 15, but segment A would claim that it should only pay 5 if an accident occurs.

To account for the different coalition orders and ensure a fair allocation, the Shapley values represent the average contribution of each segment across all possible orders of joining the coalition. In this case, considering both segment orders (A, B) and (B, A), the Shapley values can be calculated as (A, B) = $(\frac{10+5}{2}, \frac{10+5}{2})$ or (A, B) = $(7.5, 7.5)$, respectively. These Shapley values reflect the average contribution of each segment based on different orderings, providing a more comprehensive and fair approach to allocating risk and cost sharing among the segments in the biodiesel manufacturing plant.

Coalition order	A's contribution	B's contribution
A→B	10	15-10=5
B→A	15-10=5	10
Shapley value	7.5	7.5

Table 1. Table for calculating the Shapley values of Example 1.

Example 2 (Tandem graph):

Here again we consider a graph with a simple edge (Figure 2), whose characteristic functions are as follows:

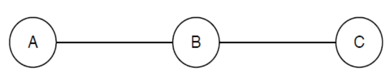


Fig. 2. Tandem graph.

$$c(A) = 10, c(B) = 10, c(C) = 10,$$
$$c(AB) = 15, c(AC) = 15,$$

$$c(BC) = 16, c(ABC) = 20$$

The values $c(AB)$ and $c(BC)$ are both equal to $c(AB)$ in Example 1, since their structure is identical. On the other hand, the relations

$$c(CA) = c(C) + c(A),$$
$$c(CA) > c(AB),$$
$$c(CA) > c(BC)$$

are satisfied, because segments *A* and *C* do not directly connect. In the context of the biodiesel manufacturing plant, cooperation between segments A and B, as well as between B and C, leads to a reduction in the individual segment costs. However, when it comes to cooperation between segments C and A, there is no decrease in the segment costs for either of them.

Coalition order	A's contribution	B's contribution	C's contribution
A→B→C	10	5	5
A→C→B	10	5	5
B→A→C	5	10	5
B→C→A	4	10	6
C→A→B	5	5	10
C→B→A	6	4	10
Shapley value	$\frac{40}{3}$	13	$\frac{41}{3}$

From Table 2, the Shapley values are $(A, B, C) = (\frac{40}{3}, 13, \frac{41}{3})$.
Note that as *B* is directly connected to *A* and *C*, the Shapley value of *B* is less than that of *A* and *C*.

3. Model of the Biodiesel manufacturing plant:

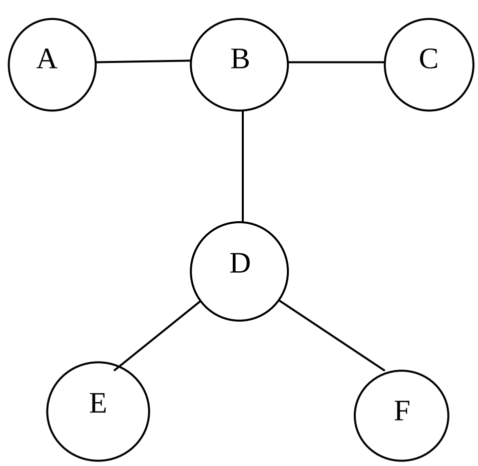
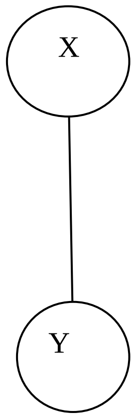


Fig.3 Biodiesel manufacturing plant

The model may at first seem somewhat complex; however, the process works in a simple manner, and by labeling the intersection points as X and Y, we can represent this model as:



Therefore, we may treat the biodiesel manufacturing plant model in the same manner as Example 1, discussed in the previous section. More precisely, by specifying the value of the characteristic functions, $c(X), c(Y)$ and $c(XY)$ we can calculate the Shapley value for this model. For example, when we specify the value of these functions in line with Example 1, the Shapley value for the model is $(7.5, 7.5)$.

Remark: In both of the aforementioned examples, we carefully constructed the characteristic functions to adhere to the rule outlined in our model. However, the specific values assigned to these functions were selected for ease of demonstration. While our primary focus in this study has been on building network models, we acknowledge the importance of determining the precise values for these characteristic functions. However, exploring this aspect in detail is a matter for future consideration and investigation.

4. Conclusions

Cooperative game theory, with its emphasis on collaboration and fairness, offers a robust framework for risk analysis and allocation. By considering the network structure and interdependencies among segments, this approach captures the intricate dynamics of risk propagation and enables a more comprehensive assessment of potential vulnerabilities.
The use of Shapley values in risk analysis allows for a nuanced understanding of each segment's contribution within a cooperative setting. By quantifying the value that each segment brings to the overall risk, decision-makers can make informed judgments about risk allocation, taking into account the specific expertise, resources, and responsibilities of each segment.
Another advantage of the cooperative game theoretical approach is its flexibility and adaptability to different risk analysis contexts. Whether applied to financial systems, supply chains, or infrastructure networks, the principles of cooperative game theory can be tailored and customized to suit the specific characteristics and dynamics of the problem at hand. This

versatility allows for the integration of domain-specific knowledge and expertise, enhancing the relevance and accuracy of risk assessment and allocation processes.
On the other hand, the nucleus concept provides an alternative perspective by focusing on the dissatisfaction of segments with their coalitions. This concept recognizes that certain segments may have inherent dissatisfaction due to their position within the network or the distribution of risks. By considering the nucleus, decision-makers can identify potential sources of discontent and address them proactively, fostering greater cooperation and consensus among segments.
It is important to note that both the Shapley value and the nucleus concept have their strengths and limitations, and their applicability may vary depending on the problem context. Further research and case studies are needed to explore the practical implications and comparative effectiveness of these solution concepts in different risk analysis scenarios.
By embracing the cooperative game theoretical approach, risk analysis practitioners and policymakers can enhance their understanding of risk dynamics, promote collaboration among segments, and ultimately improve the effectiveness and fairness of risk allocation processes. This interdisciplinary perspective bridges the gap between game theory and risk management, opening up new avenues for addressing complex challenges in a systematic and transparent manner.

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