ρ-SETS WHERE $\rho \in \{ r\omega\delta, r^*\omega\delta \}$

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Abstract: In this paper, the mixed and ordinary operators are characterized in the classes of rωδ-sets and r*ωδ-sets. Certain topological sets that are inherited from ω-open set [2], open set and δ-open set [13] are characterized using rωδ -sets and r*ωδ-sets. More over the behavior of rωδ-sets and r*ωδ-sets in spaces are investigated. The Inclusion chains among the mixed and ordinary operators are refined in the domains of rωδ -sets and r*ωδ-sets.

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1 INTRODUCTION

In the year 1982, Hdeib [2] introduced the notion of a ω-closed set. A subset B of a topological space is called ω-closed if it contains all its condensation points. Recently general topologists introduced and studied new types of topological sets by mixing interior, closure operators with δ-interior, δ-closure operators. In this paper, our investigations on ω-open sets and ω-closed sets in the sense of Hdeib, lead to the development in the domains of topology.

2 PRELIMINARIES

Throughout this paper (X, τ) (or X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. The concept of δ-closure was introduced and studied by Velicko [13] in the year 1968. A point x is in the δ-closure of A if every regular open nbd of x intersects A. $Cl_\delta A$ denotes the δ-closure of A.

Definition 2.1 A subset A of X is δ-closed [13] if A = $Cl_\delta A$. The complement of a δ-closed set is δ-open. The collection of all δ-open sets is a topology denoted by τ^δ. This τ^δ is called the semi - regularization of τ.

Let $Int_\delta A$ and $Cl_\delta A$ denote the δ-interior and δ-closure of A respectively. Velicko established that the operators $Cl(.)$ and $Cl_\delta(.)$ have the same effect on the class of open sets and the operators $Int(.)$ and $Int_\delta(.)$ coincide on the class of closed sets.

Lemma 2.2 [13]

(i) For any open set A, $Cl_\delta A = ClA$,

(ii) For any closd set B, $Int_\delta B = IntB$.

Definition 2.3

A subset M of a space X is called:

(i) semi-open [4] if $M \subseteq Cl(Int(M))$;

(ii) regular open [9] if $M = Int(Cl(M))$;

(iii) preopen [5] if $M \subseteq Int(Cl(M))$.

The complements of the above-mentioned open sets are called their respective closed sets.

Definition 2.4

A subset A of a space X is called:

(i) δ-semi-open [7,11] if $A \subseteq Cl(Int_\delta(A))$;

(ii) δ-pre-open [11] if $A \subseteq Int(Cl_\delta A)$.

The complements of the above-mentioned open sets are called their respective closed sets.

Definition 2.5 [**2**] Let H be a subset of a space (X, τ), a point p in X is called a condensation point of H if for each open set U containing p, U ∩ H is uncountable.

Definition 2.6 [**2**] A subset H of a space (X, τ) is called ω-closed if it contains all its condensation points.

The complement of an ω-closed set is called ω-open. The family of all ω-closed sets is denoted by ωC(X, τ). The family of all ω-open sets is denoted by ωO(X). It is well known that a subset W of a space (X, τ) is ω-open [2] if and only if for each x ∈ W, there exists U ∈ τ such that x ∈ U and U − W is countable. The family of all ω-open sets, denoted by τ_ω, is a topology on X, which is finer than τ. The interior and closure operator in (X, τ_ω) are denoted by Int_ω and Cl_ω respectively.

Definition 2.7 The set A of a space X is called

(i) regular ω-open [6] if A= $Int_\omega Cl$A

(ii) semi-ω-open [10] if A$\subseteq$$Cl\ Int_\omega$A

(iii) pre-ω-open [10,12] if A$\subseteq$ $Int_\omega Cl$ A

The complements of the above-mentioned open sets are called their respective closed.

Definition 2.8 [3] A space (X,τ) is said to be Anti Locally Countable (briefly ALC) if every non empty open set is uncountable.

Proposition 2.9 [12] Let A be a subset of an ALC space. Then the following chains hold.

(i) $Cl_\omega Int_\delta A = Cl Int_\delta A = Cl_\delta Int_\delta A \subseteq Cl_\delta Int A = Cl Int A = Cl_\omega Int A \subseteq Cl_\omega Int_\omega A = Cl Int_\omega A \subseteq Cl_\delta Int_\omega A$.

(ii) $Int_\omega Cl_\delta A = Int Cl_\delta A = Int_\delta Cl_\delta A \supseteq Int_\delta Cl A = Int Cl A = Int_\omega Cl A \supseteq Int_\omega Cl_\omega A = Int Cl_\omega A \supseteq Int_\delta Cl_\omega A$.

3 rωδ-SETS AND r*ωδ-SETS

we study r-sets and r*-sets that are defined using interior and closure operators in topology and its associated delta topology. In this paper, the notions of rωδ-set and r*ωδ -set are introduced using mixed and ordinary two level operators in topology. The applications of the above two types sets to Anti Locally Countable Spaces are investigated.

Definition 3.1 A subset A of a space (X,τ) is

(i) an rω-set if $IntCl_\omega A = IntClA$,

(ii) an r*ω-set if $ClInt_\omega A = ClIntA$,

(iii) an rr*ω-set if it is both an rω-set and an r*ω-set,

It is noted that

(i) The set A is an rω-set $\Leftrightarrow IntCl_\omega A = IntClA = Int_\delta ClA$.

(ii) The set A is an r*ω-set $\Leftrightarrow ClInt_\omega A = ClIntA = Cl_\delta IntA$.

Proposition 3.2 A subset A of a space (X,τ) is

(i) an rωδ-set $\Leftrightarrow$ X\A is an r*ωδ-set,

(ii) an rr*ωδ-set $\Leftrightarrow$ X\A is an rr*ωδ-set.

Proof. The set A is an rωδ-set $\Leftrightarrow IntCl_\omega A = IntCl_\delta A$,

$\Leftrightarrow X \setminus IntCl_\omega A = X \setminus IntCl_\delta A$

$\Leftrightarrow ClInt_\omega(X\setminus A) = ClInt_\delta(X\setminus A)$

$\Leftrightarrow$A is an r*ωδ-set. This proves (i).

The set A is an rr*ωδ-set $\Leftrightarrow$ A is an rωδ-set and an r*ωδ-set.

$\Leftrightarrow$X\A is an r*ωδ-set and an rωδ-set.

$\Leftrightarrow$ X\A is an rωδ-set and an r*ωδ-set.

$\Leftrightarrow$ X\A is an rr*ωδ-set.

This proves (ii).

Proposition 3.3 Let A and B be an rω-set and rωδ-set respectively in an ALC space. Then the following chains hold.

(i) $Int_\delta Cl_\omega A \subseteq IntCl_\omega A = Int_\omega Cl_\omega A = Int_\omega ClA = IntClA = Int_\delta ClA \subseteq Int_\delta Cl_\delta A = IntCl_\delta A = Int_\omega Cl_\delta A$.

(ii) $Int_\delta Cl_\omega B \subseteq IntCl_\omega B = Int_\omega Cl_\omega B = Int_\omega ClB = IntClB = Int_\delta ClB = Int_\delta Cl_\delta B = IntCl_\delta B = Int_\omega Cl_\delta B$.

Proof. Let A be an rω-set in an ALC space. Then using Proposition 2.9(ii), we have

$Int_\delta Cl_\omega A \subseteq IntCl_\omega A = Int_\omega Cl_\omega A \subseteq Int_\omega ClA = IntClA = Int_\delta ClA \subseteq Int_\delta Cl_\delta A = IntCl_\delta A = Int_\omega Cl_\delta A$.

Since A is an rω-set, using the result in Definition 3.1 (i) in the above expression we have

$Int_\delta Cl_\omega A \subseteq IntCl_\omega A = Int_\omega Cl_\omega A = Int_\omega ClA = IntClA = Int_\delta ClA \subseteq Int_\delta Cl_\delta A = IntCl_\delta A = Int_\omega Cl_\delta A$.

This proves (i).

Now let B be an rωδ-set in an ALC space. Therefore replacing A by B in Proposition 2.9(ii) we have

$Int_\delta Cl_\omega B \subseteq IntCl_\omega B = Int_\omega Cl_\omega B \subseteq Int_\omega ClB = IntClB = Int_\delta ClB \subseteq Int_\delta Cl_\delta B = IntCl_\delta A = Int_\omega Cl_\delta B$. Since B is an rωδ-set, using the result in Definition (i) in the above expression we have

$Int_\delta Cl_\omega B \subseteq IntCl_\omega B = Int_\omega Cl_\omega B = Int_\omega ClB = IntClB = Int_\delta ClB = Int_\delta Cl_\delta B = IntCl_\delta B = Int_\omega Cl_\delta B$. This proves (ii).

Proposition 3.4 Let A be an rω-set in an ALC space. The followings are equivalent.

(i) A is regular open

(ii) A is regular ω-open

(iii) A = $IntCl_\omega A$

(iv) A is regular open in (X, τ_ω)

(v) A = $Int_\delta ClA$

Proof. Let A be an rω-set in an ALC space. Then we have

$IntCl_\omega A = Int_\omega Cl_\omega A = Int_\omega ClA = IntClA = Int_\delta ClA$……..(1)

Therefore A is regular open $\Leftrightarrow$ A= $IntClA$ $\Leftrightarrow$ A= $Int_\delta ClA$

$\Leftrightarrow$ A= $Int_{\omega}Cl$A$\Leftrightarrow$ A is regular ω-open

$\Leftrightarrow$ A= $Int_{\omega}Cl_{\omega}$A$\Leftrightarrow$ A is regular open in (X, τ_{ω})

$\Leftrightarrow$A= $IntCl_{\omega}$A . This proves the proposition.

Proposition 3.5 Let A be an rω-set in an ALC space. The followings are equivalent.

(i) A is pre-open

(ii) A is pre-ω-open

(iii) A$\subseteq IntCl_{\omega}$A

(iv) A is pre-open in (X, τ_{ω})

(v) A $\subseteq Int_{\delta}Cl$A

Proof. Let A be an rω-set in an ALC space. Then using (1) we have

A is pre-open $\Leftrightarrow$ A$\subseteq IntCl$A $\Leftrightarrow$ A$\subseteq Int_{\delta}Cl$A

$\Leftrightarrow$ A$\subseteq Int_{\omega}Cl$A$\Leftrightarrow$ A is pre- ω-open

$\Leftrightarrow$ A$\subseteq Int_{\omega}Cl_{\omega}A\Leftrightarrow$ A is pre-open in (X, τ_{ω})

$\Leftrightarrow$A$\subseteq IntCl_{\omega}$A. This proves the proposition.

Proposition 3.6 Let A be an rω-set in an ALC space. The followings are equivalent.

(i) A is semi-closed

(ii) A is semi-ω-closed

(iii) $IntCl_{\omega}$A$\subseteq$ A

(iv) A is semi-closed in (X, τ_{ω})

(v) $Int_{\delta}Cl$A $\subseteq$ A

Proof. Let A be an rω-set in an ALC space. Then we have

A is semi-closed $\Leftrightarrow IntCl$A $\subseteq$ A $\Leftrightarrow Int_{\delta}ClA\subseteq$ A

$\Leftrightarrow Int_{\omega}ClA\subseteq$ A$\Leftrightarrow$ A is semi-ω-closed

$\Leftrightarrow Int_{\omega}Cl_{\omega}$A $\subseteq$ A$\Leftrightarrow$ A is semi-closed in (X, τ_{ω})

$\Leftrightarrow IntCl_{\omega}A\subseteq$ A. This proves the proposition.

Proposition 3.7 Let A be an r*ω-set in an ALC space. The followings are equivalent.

(i) A is regular closed

(ii) A is regular ω-closed

(iii) A = $ClInt_{\omega}$A

(iv) A is regular closed in (X, τ_{ω})

(v) A = $Cl_{\delta}Int$A

Proof. Let A be an r*ω-set in an ALC space. Then we have

$Cl_{\delta}Int$A=$Cl_{\omega}Int$A=$ClInt$A=$ClInt_{\omega}$A=$Cl_{\omega}Int_{\omega}$A.

Therefore A is regular closed ⇔ A = $ClInt$A ⇔ A= $Cl_{\delta}Int$A.

⇔ A = $Cl_{\omega}Int$A ⇔ A is regular ω-closed

⇔ A = $ClInt_{\omega}$A

⇔ A = $Cl_{\omega}Int_{\omega}$A⇔ A is regular closed in (X,τ_{ω})

Proposition 3.8 Let A be an r*ω-set in an ALC space. The followings are equivalent.

(i) A is pre-closed

(ii) A is pre-ω-closed

(iii) A$\supseteq ClInt_{\omega}$A

(iv) A is pre-closed in (X, τ_{ω})

(v) A $\supseteq Cl_{\delta}Int$A

Proof. Let A be an r*ω-set in an ALC space. Then we have

A is pre-closed ⇔ A $\supseteq ClInt$A ⇔ A$\supseteq Cl_{\delta}Int$A.

⇔ A $\supseteq Cl_{\omega}Int$A ⇔ A is pre-ω-closed.

⇔ A$\supseteq ClInt_{\omega}$A

⇔ A $\supseteq Cl_{\omega}Int_{\omega}$A⇔ A is pre-closed in (X,τ_{ω})

Proposition 3.9 Let A be an r*ω-set in an ALC space. The followings are equivalent.

(i) A is semi-open

(ii) A is semi-ω-open

(iii) A $\subseteq Cl_\omega Int$A

(iv) A is semi-open in (X, τ_ω)

(v) A$\subseteq Cl_\delta Int$A

Proof. Let A be an r*ω-set in an ALC space. Then we have

A is semi-open $\Leftrightarrow$ A$\subseteq ClInt$A $\Leftrightarrow$ A$\subseteq Cl_\delta Int$A.

$\Leftrightarrow$ A$\subseteq ClInt_\omega$A $\Leftrightarrow$ A is semi-ω-open.

$\Leftrightarrow$ A $\subseteq Cl_\omega Int$A

$\Leftrightarrow$ A$\subseteq Cl_\omega Int_\omega$A$\Leftrightarrow$ A is semi-open in (X,τ_ω) .

This proves the proposition.

Proposition 3.10 Let B be an rωδ-set in an ALC space. The followings are equivalent.

(i) B is regular open

(ii) B is regular ω-open

(iii) B = $IntCl_\omega$B

(iv) B is regular open in (X, τ_ω)

(v) B is regular open in (X, τ^δ)

(vi) B = $IntCl_\delta$A

(vii) B = $Int_\delta Cl$A

(viii) B = $Int_\omega Cl_\delta$B

Proof. Let B be an rωδ-set in an ALC space. Then we have

$IntCl_\omega$B=$Int_\omega Cl_\omega$B=$Int_\omega Cl$B=$IntCl$B=$Int_\delta Cl$B=$Int_\delta Cl_\delta$B=$IntCl_\delta$B=$Int_\omega Cl_\delta$B.

Therefore B is regular open $\Leftrightarrow$ B = $IntCl$B$\Leftrightarrow$ B = $IntCl_\omega$B

$\Leftrightarrow$ B = $Int_\omega Cl_\omega$B$\Leftrightarrow$ B is regular open in (X,τ_ω)

$\Leftrightarrow$ B = $Int_\omega Cl$B$\Leftrightarrow$ B is regular ω-open

$\Leftrightarrow$ B = $Int_\delta Cl$B$\Leftrightarrow$ B = $IntCl_\delta$B $\Leftrightarrow$ B = $Int_\omega Cl_\delta$B

$\Leftrightarrow$ B = $Int_\delta Cl_\delta$B$\Leftrightarrow$B is regular open in (X,τ^δ)

This proves the proposition.

Proposition 3.11 Let B be an rωδ-set in an ALC space. The followings are equivalent.

(i) B is pre-open

(ii) B is pre-ω-open

(iii) $B\subseteq IntCl_\omega B$

(iv) B is pre-open in (X, τ_ω)

(v) B is pre-open in (X, τ^δ)

(vi) B is δ-pre-open

(vii) $B \subseteq Int_\delta Cl B$

(viii) $B \subseteq Int_\omega Cl_\delta B$

Proof. Let B be an r$\omega\delta$-set in an ALC space. Then we have

B is pre-open $\Leftrightarrow B\subseteq IntClB \Leftrightarrow B\subseteq IntCl_\omega B$

$\Leftrightarrow B \subseteq Int_\omega Cl_\omega B \Leftrightarrow$ B is pre-open in (X,τ_ω)

$\Leftrightarrow B\subseteq Int_\omega ClB \Leftrightarrow$ B is pre-ω-open

$\Leftrightarrow B \subseteq IntCl_\delta B \Leftrightarrow$ B is δ-pre-open

$\Leftrightarrow B \subseteq Int_\delta ClB \Leftrightarrow B \subseteq Int_\omega Cl_\delta B$

$\Leftrightarrow B \subseteq Int_\delta Cl_\delta B \Leftrightarrow$ B is pre-open in (X,τ^δ)

This proves the proposition.

Proposition 3.12 Let B be an r$\omega\delta$-set in an ALC space. The followings are equivalent.

(i) B is semi-closed

(ii) B is semi-ω-closed

(iii) $B \supseteq Int_\omega ClB$

(iv) B is semi-closed in (X, τ_ω)

(v) B is semi-closed in (X, τ^δ)

(vi) B is δ- semi-closed

(vii) $B \supseteq Int_\delta ClB$

(viii) $B \supseteq Int_\omega Cl_\delta B$

Proof. Let B be an r$\omega\delta$-set in an ALC space. Then we have

B is semi-closed $\Leftrightarrow$ B $\supseteq IntCl$B$\Leftrightarrow$ B $\supseteq Int_\omega Cl$B

$\Leftrightarrow$ B $\supseteq Int_\omega Cl_\omega$B$\Leftrightarrow$ B is semi-closed in (X,τ_ω)

$\Leftrightarrow$ B $\supseteq IntCl_\omega$B$\Leftrightarrow$ B is semi-ω-closed

$\Leftrightarrow$ B $\supseteq IntCl_\delta$B$\Leftrightarrow$ B is δ- semi-closed

$\Leftrightarrow$ B $\supseteq Int_\delta Cl$B $\Leftrightarrow$ B $\supseteq Int_\omega Cl_\delta$B

$\Leftrightarrow$ B $\supseteq Int_\delta Cl_\delta$B$\Leftrightarrow$B is semi-closed in (X,τ^δ)

This proves the proposition.

Proposition 3.13 Let B be an r*ωδ-set in an ALC space. The followings are equivalent.

(i) B is regular closed

(ii) B is regular ω-closed

(iii) B is regular closed in (X, τ_ω)

(iv) B is regular closed in (X, τ^δ)

(v) B = $ClInt_\delta$A

(vi) B = $Cl_\delta Int$A

(vii) B = $Cl_\omega Int_\delta$B

Proof. Let B be an r*ωδ-set in an ALC space. Then we have

$Cl_\omega Int_\delta B = ClInt_\delta B = Cl_\delta Int_\delta B = Cl_\delta Int B = ClInt B = Cl_\omega Int B = Cl_\omega Int_\omega B = ClInt_\omega B$.

Therefore B is regular closed $\Leftrightarrow$ B = $ClInt$B $\Leftrightarrow$ B = $ClInt_\omega$B

$\Leftrightarrow$ B = $Cl_\omega Int$B $\Leftrightarrow$B is regular ω-closed

$\Leftrightarrow$B = $Cl_\omega Int_\omega$B$\Leftrightarrow$ B is regular closed in (X,τ_ω)

$\Leftrightarrow$ B = $Cl_\delta Int_\delta$B $\Leftrightarrow$B is regular closed in (X,τ^δ)

$\Leftrightarrow$ B = $ClInt_\delta$B $\Leftrightarrow$ B = $Cl_\delta Int$B $\Leftrightarrow$ B = $Cl_\omega Int_\delta$B

Proposition 3.14 Let B be an r*ωδ-set in an ALC space. The followings are equivalent.

(i) B is pre-closed

(ii) B is pre-ω- closed

(iii) B $\supseteq ClInt_\omega$B

(iv) B is pre- closed in (X, τ_ω)

(v) B is pre- closed in (X, τ^δ)

(vi) B is δ-pre- closed

(vii) $B \supseteq Cl_\delta Int B$

(viii) $B \supseteq Cl_\omega Int_\delta B$

Proof. Let B be an r*ωδ-set in an ALC space. Then we have

B is pre-closed $\Leftrightarrow B \supseteq ClIntB \Leftrightarrow B \supseteq ClInt_\omega B$

$\Leftrightarrow B \supseteq Cl_\omega IntB \Leftrightarrow$ B is pre-ω-closed

$\Leftrightarrow B \supseteq Cl_\omega Int_\omega B \Leftrightarrow$ B is pre-closed in (X,τ_ω)

$\Leftrightarrow B \supseteq Cl_\delta Int_\delta B \Leftrightarrow$ B is pre-closed in (X,τ^δ)

$\Leftrightarrow B \supseteq ClInt_\delta B \Leftrightarrow$ B is δ-pre-closed

$\Leftrightarrow B \supseteq Cl_\delta IntB \Leftrightarrow B \supseteq Cl_\omega Int_\delta B$

Proposition 3.15 Let B be an r*ωδ-set in an ALC space. The followings are equivalent.

(i) B is semi-open

(ii) B is semi-ω-open

(iii) $B \subseteq Cl_\omega IntB$

(iv) B is semi-open in (X, τ_ω)

(v) B is semi-open in (X, τ^δ)

(vi) B is δ- semi-open

(vii) $B \subseteq Cl_\delta IntB$

(viii) $B \subseteq Cl_\omega Int_\delta B$

Proof. Let B be an r*ωδ-set in an ALC space. Then we have

B is semi-open $\Leftrightarrow B \subseteq ClIntB \Leftrightarrow B \subseteq Cl_\omega IntB$

$\Leftrightarrow B \subseteq ClInt_\omega B \Leftrightarrow$ B is semi-ω-open

$\Leftrightarrow B \subseteq Cl_\omega Int_\omega B \Leftrightarrow$ B is semi-open in (X,τ_ω)

$\Leftrightarrow B \subseteq Cl_\delta Int_\delta B \Leftrightarrow$ B is semi-open in (X,τ^δ)

$\Leftrightarrow B \subseteq ClInt_\delta B \Leftrightarrow$ B is δ-semi-open

$\Leftrightarrow B \subseteq Cl_\delta Int B \Leftrightarrow B \subseteq Cl_\omega Int_\delta B$

REFERENCES

[1] Ahmad Al-Omari and Mohd Salmi Md Noorani 2007, 'Regular generalized ω-closed sets', *Int. J. Math. and Math. Sci.*, 11 pages, doi:10.1155/2007/16292.

[2] Hdeib HZ 1982, 'ω-closed mappings', *Rev.Colomb.Mat.*, vol.16, no.3-4, pp.65-78.

[3] Khalid Y. Al-Zoubi 2005, 'On generalized ω-closed sets', *Int. J. Math. Math. Sci.*, vol.13, pp. 2011-2021.

[4] Levine, N 1963, 'Semi-open sets and semi-continuity in topological spaces', *Amer. Math. Monthly*, vol.70, pp. 36-41.

[5] Mashhour, AS, Abd El-Monsef, ME and El-Deeb, SN 1982, On pre-continuous and weak pre-continuous mappings', *Proc. Math. Phys. Soc. Egypt*, vol.53, pp. 47-53.

[6] Murugesan, S 2014, 'On Rω-open sets', *Journal of Advanced Studies in Topology*. vol.5, no 3, pp. 24-27.

[7] Park, JH, Lee, BY and Son, MJ 1997, On δ-semi-open sets in topological spaces, *J. Indian. Acad. Math.*, vol.19, no 1, pp. 59-67.

[8] Stephen Willard 1970, 'General Topology' Addison Weseley,

[9] Stone, M 1937, 'Applications of the theory of boolean rings to the general topology', *Trans. Amer. Math. Soc.*, vol. 41, pp.375-481.

[10] Takashi Noiri, Ahmad Al-Omari and Mohd Salmi Md Noorani 2009, 'Weak forms of ω-open sets and decomposition of continuity' *European Journal of Pure and Applied Mathematics*, vol.2 , no.1, pp.73-74.

[11] Takashi Noiri, 2003, 'Remarks on δ-semi-open sets and δ-preopen sets' *Demonstratio Mathematica*, vol.36 , no.4, pp.1007-1020.

[12] Thangavelu, P, Rayshima, N, Rajan, C and Ravi, O (2017), The mixed structures of ω-closure and δ-closure in topological spaces, out of print.

[13] Velicko, NV 1968, 'H-closed topological spaces', *Amer. Math. Soc. Transl.*, vol.78, no.2, pp.103-118.