SORET EFFECT ON AN UNSTEADY MHD HEAT AND MASS TRANSFER FLOW WITH CONSUMPTION OF CHEMICAL SPECIES

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ABSTRACT: The influence of thermal diffusion with a special reference to chemical reaction parameter and magnetic parameter in a non-linear MHD heat and mass transfer flow are studied parametrically. The MHD boundary layer equations are solved by adopting Homotopy Perturbation method (HPM) which is restricted by a system of non-linear partial differential equations. The impacts of thermal diffusion with other relevant physical parameters are graphically illustrated and discussed.

KEYWORDS: Soret effect, Chemical reaction, Schmdit number, Homotopy Perturbation method.

2010 AMS subject classification: 76W05

INTRODUCTION:

Magnetohydrodynamics *(MHD)* concerns with the study of fluids under electro-magnetic effects. Now-a-days applications of MHD principles obtain great importance because of its wide-ranging utilities in various fields such as geophysics, astronomical science, space science etc. Because of importance of MHD principle in different field, many researchers give their attentions to do work in the field of MHD. Significant contribution in the MHD field was given by Alfven (1942). After works of Alfven several researchers were doing many good works in MHD field. The name of some of them are Sarada and Shankar (2013), Ahmed and Sinha (2014), Abbas et al. (2020), Bera (2020), Manzoor et al. (2021) etc.

Effects of chemical reactions are getting an exceedingly sensible value especially in practical applications in different fields of science and technology. Under characteristics of the fluid, the chemical reaction effect without or with MHD flow was studied by several authors which are specified in Raju *et al.* (2020), Sarada, K. and Shanker, B. (2013), and Reddy *et al.* (2013).

In fluid mechanics, the discussion of flow related problems to stagnation point is of great scientific importance due to its numerous applications in technology and engineering. Studies on *MHD* induced fluid moving on non-linear stretching sheet recently assumes great significance and importance because of their wide-ranging applications in the process relating to technology and industry. Importance of these phenomena is so increasing that quite a good number researchers are contributing their research works in this field. Attia (2006) studied numerically the flow of Stagnation Point on a stretching sheet with heat generation in a porous media, Kazem *et al.* (2011) has given an improved analytical solution of this problem. Some notable researchers are Jhankal, A. K. (2014), Makinde, O. D. (2010), Sinha S., Sarma M. K. (2020), Kai *et al.* (2019), Ekang *et al. (*2021) etc.

In this paper, the imposition of thermal diffusion or Soret effect is utilized with a classical perturbation technique namely Homotopy Perturbation Method (HPM) to take a broad view to the work of Boboi (2023). In the procedure of generalization, almost accurate results are drawn which is revealed by virtue of comparison graph with the work of Boboi (2023).

MATHEMATICAL FORMULATION OF THE PROBLEM:

The physical model of the present flow problem contains a steady two-dimensional stagnation point flow of an electrically conducting viscous incompressible fluid in the neighborhood of a stagnation point O, at the surface which is well thought-out along the plane y = 0 (x-axis). The significant conditions describing the model with initial velocity u_w, T_w be the temperature whereas $u_w(x)$, $T_\infty(x)$ are the velocity and temperature of the flow external to the boundary layer is given in Figure 1.

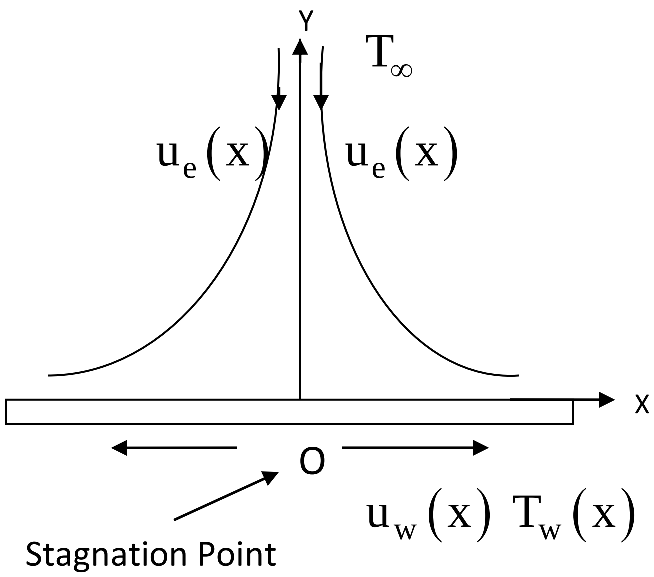

Figure 1: Physical model of the problem

The governing equations, equation of continuity and equation of momentum, for two dimensional steady flows are given as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\upsilon}{K}\left[U(x)-u\right]-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\frac{k\frac{\partial^2 T}{\partial y^2}+Q\left(T-T_\infty\right)}{\rho C_p}\right) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_M\frac{\partial^2 C}{\partial y^2}-Kr'\left(C-C_\infty\right) \tag{4}$$

Where the symbols used above have their typical meanings.

Also,

The boundary conditions of the above flow are given by:

$$\left.\begin{array}{ll} u = u_w(x) = cx, v = 0; \; T = T_w, C = C_w & \text{for } y = 0 \\ u = u_c(x) \to ax; \quad T \to T_\infty, C = C_\infty & \text{for } y \to \infty \end{array}\right\} \quad (5)$$

The stream function $\psi(x, y)$ is given by: $u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x}$

Using, $\upsilon = \mu / \rho$ as kinematic viscosity, applying similarity transformations:

$$\psi(x,y) = \sqrt{c\upsilon}\, x f(\eta); \; \eta = \sqrt{\frac{c}{\nu}}\, y, T = T_\infty + (T_w - T_\infty)\theta(\eta)$$

$$C = C_\infty + (C_w - C_\infty)\phi(\eta) \quad (6)$$

To normalize the flow model the following non dimensional terms heat transfer is introduced:

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \; \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

$$Sr = \frac{D_T}{c\upsilon}\frac{T_w - T_\infty}{C_w - C_\infty}, \; Kr = \frac{Kr'}{c}, Pr = \frac{\mu C_p}{K},$$

$$M = \frac{\nu}{cK}, C = \frac{a}{c}, B = \frac{Q}{c\rho C_p}, Sc = \frac{\upsilon}{D_M} \quad (7)$$

The following nonlinear coupled differential equations are produced by applying (6), (7) in (2), (3) and (4):

$$f'''(\eta) + f(\eta).f''(\eta) - f'^2(\eta) + (\lambda + M) f'(\eta) + C(C + \lambda) = 0 \quad (8)$$

$$\theta''(\eta) + Pr.f(\eta).\theta'(\eta) + Pr\, B\, \theta(\eta) = 0 \quad (9)$$

$$\phi''(\eta) + f(\eta)\, Sc\, \phi'(\eta) - Kr\, Sc\, \phi(\eta) = -Sc\, Sr\, \theta''(\eta) \quad (10)$$

Subject to the boundary conditions

$$\left.\begin{array}{ll} f(\eta) = 0, f'(\eta) = 1, \theta(\eta) = 1, \phi(\eta) = 1 & \text{as } \eta = 0 \\ f'(\eta) \to C, \theta(\eta) \to 0, \phi(\eta) \to 0 & \text{as } \eta \to \infty \end{array}\right\} \quad (11)$$

SOLUTION OF THE PROBLEM:

The *nonlinear coupled differential equations* (8), (9) and (10) can be rewritten as:

$$f''' + f.f'' - f'^2 - M_1 f' + M_2 = 0 \quad (12)$$

$$\theta'' + Pr\, f\theta' - M_3\theta = 0 \quad (13)$$

$$\phi'' + f\, Sc\, \phi' - Kr\, Sc\, \phi = -Sc\, Sr\, \theta'' \quad (14)$$

Applying HPM, the equations (12), (13) and (14) can take the following form:

$$(1-p)(f''' - M_1 f') + p\left(f''' + ff'' - f'^2 - M_1 f'\right) = -M_2 \quad (15)$$

$$(1-p)(\theta'' - M_3\theta) + p\left(\theta'' + Pr\, f\, \theta' - M_3\theta\right) = 0 \quad (16)$$

$$(1-p)\phi'' + p\left(\phi'' + Sc\, f\phi' - Kr\, Sc\, \phi - Sc\, Sr\, \theta''\right) = 0 \quad (17)$$

Let us consider f, θ and ϕ as

$$\left.\begin{array}{l} f = f_0 + pf_1 + p^2 f_2 + \ldots\ldots\ldots\ldots \\ \theta = \theta_0 + p\theta_1 + p^2\theta_2 + \ldots\ldots\ldots\ldots \\ \phi = \phi_0 + p\phi_1 + p^2\phi_2 + \ldots\ldots\ldots\ldots \end{array}\right\} \quad (18)$$

Using (18) in (15), (16) and (17) and then by simplifying, we obtain:

$$f_0(\eta) = C_1 + C_2 e^{\sqrt{M_1}\eta} + C_3 e^{-\sqrt{M_1}\eta} + \frac{M_2}{M_1}\eta \tag{19}$$

$$\theta_0(\eta) = C_4 e^{\sqrt{M_3}\eta} + C_5 e^{-\sqrt{M_3}\eta} \tag{20}$$

$$\phi_0(\eta) = 1 - \frac{1}{6}\eta \tag{21}$$

$$f_1(\eta) = C_6 + C_7 e^{\sqrt{M_1}\eta} + C_8 e^{-\sqrt{M_1}\eta} + A_{10}\eta^2 + A_{11}\eta^3 + A_{12}\eta - \left[A_{15} e^{\sqrt{M_1}\eta} - A_{16} e^{-\sqrt{M_1}\eta}\right]$$
$$-\eta\left[E_1 e^{\sqrt{M_1}\eta} + E_2 e^{-\sqrt{M_1}\eta}\right] + \eta^2\left[A_{17} e^{\sqrt{M_1}\eta} + A_{18} e^{-\sqrt{M_1}\eta}\right] \tag{22}$$

$$\theta_1(\eta) = C_9 e^{\sqrt{M_3}\eta} + C_{10} e^{-\sqrt{M_3}\eta} + \eta\left[A_{39} e^{\sqrt{M_3}\eta} + A_{40} e^{-\sqrt{M_3}\eta}\right]$$
$$-\eta^2\left[A_{35} e^{\sqrt{M_3}\eta} - A_{36} e^{-\sqrt{M_3}\eta}\right] - \left[A_{31} e^{M_4\eta} - A_{32} e^{-M_4\eta}\right] + \left[A_{33} e^{M_5\eta} - A_{34} e^{-M_5\eta}\right] \tag{23}$$

$$\phi_1(\eta) = \left(A_9\eta - A_3\right) + \frac{Sc}{6}\left(\frac{C_1\eta^2}{2} + \frac{C_2}{M_1} e^{\sqrt{M_1}\eta} + \frac{C_3}{M_1} e^{-\sqrt{M_1}\eta} + \frac{M_2}{6M_1}\eta^3\right)$$
$$+ Kr\,Sc\left(\frac{\eta^2}{2} - \frac{\eta^3}{36}\right) - Sc\,Sr\left(C_4 e^{\sqrt{M_3}\eta} + C_5 e^{-\sqrt{M_3}\eta}\right) \tag{24}$$

The above zeroth and first order expressions of velocity, temperature and concentration are found out by using the following restrictions:

$$\left.\begin{array}{l} f_0(0)=0,\ f_0'(0)=1,\ f_0'(6)=C;\ \theta_0(0)=1, \theta_0(6)=0;\ \phi_0(0)=1,\ \phi_0(6)=0 \text{ at } \eta=0 \\ f_1'(0)=0,\ f_1'(6)=1;\ \theta_1(0)=0, \theta_1(6)=0;\ \phi_1(0)=0,\ \phi_1(6)=0 \text{ as } \eta\to\infty \end{array}\right\} \tag{25}$$

(in the boundary layer theory, $\eta\to\infty$ is replaced by $\eta = 6$ those at in concurrent practice)

Neglecting higher order perturbed terms we finally obtain:

$$f(\eta) = f_0 + pf_1$$

$$\theta(\eta) = \theta_0 + p\theta_1$$

$$\phi(\eta) = \phi_0 + p\phi_1$$

The terminologies for *viscous drage* in terms of *skin friction* (τ), the coefficient of rate of heat transfer (Nu) and the coefficient of rate of mass transfer (Sh) are articulated as:

$$\tau = \left(\frac{\partial f}{\partial \eta}\right)_{\eta=0} = \sqrt{M_1}(C_2 - C_3) + \frac{M_2}{M_1}$$
$$+ p\left[\sqrt{M_1}(C_7 - C_8) + A_{12} - (A_{13} + A_{14}) - 2\sqrt{M_1}(A_{15} + A_{16}) + (A_{19} - A_{20})\right]$$

$$Nu = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} = -\sqrt{M_3}(C_4 - C_5) + p\left[\sqrt{M_3}(C_9 - C_{10}) + A_{39} + A_{40} - M_3(A_{31} - A_{32})\right]$$

$$Sh = -\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=0} = \frac{1}{6} - p\,A_{12}$$

FINDINGS:

In this study, the problem of boundary layer for MHD flow placed vertically in presence of heat and mass transfer is considered by HPM. The present analysis also reveals the expressions of boundary layer equations with *viscous drag* and the *co-efficient of rate of heat* *and mass transfer* are piled up. A variety of graphs with their significant interprtation are drawn against the expressions of species concentration and sherwood number by opting some standard values of different parameters occupied in the dilemma. The obtained outcomes are revealed graphically and are compared with the accurate solutions. The result shows that the estimated solution obtained in this paper has an exceptional concurrence with the work done by Boboi (2023).The mathematical results are obtained for various values of physical parameters with the fixed value of Homotopy Perturbation Parameter (p= 0.1) implanted in the flow system.

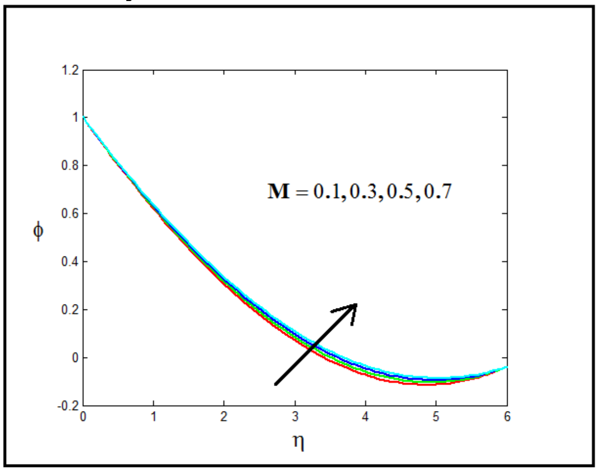

Figure 2: Concentration versus η under λ=3, C=1.5, Sc=0.62, Kr=1, Pr=0.71, B=0.1, Sr=0.1, p=0.1

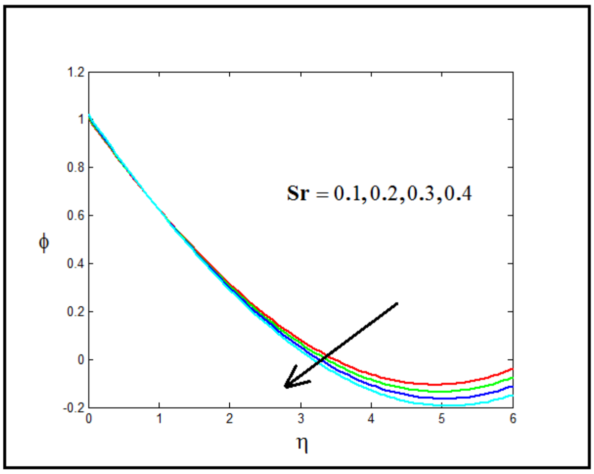

Figure 3: Concentration versus η under λ=3, C=1.5, Sc=0.62, M=0.25, Kr=1, Pr=0.71, B=0.1, p=0.1

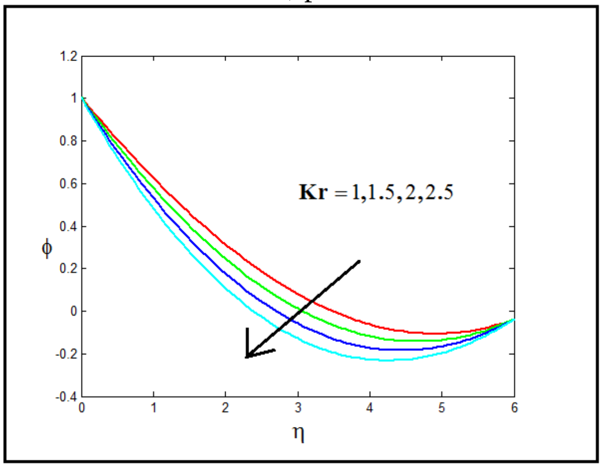

Figure 4: Concentration versus η under λ=3, M=0.25, C=1.5, Sc=0.62, Pr=0.71, B=0.1, Sr=0.1, p=0.1

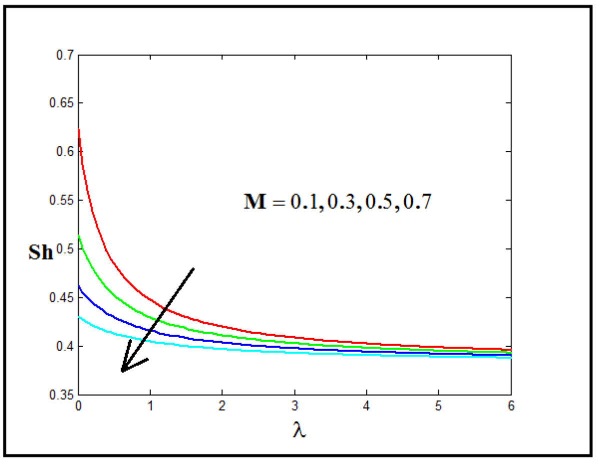

Figure 5: Sherwood number versus λ under C=1.5, Sc=0.62, Kr=1, Pr=0.71, B=0.1, Sr=0.1, p=0.1

Figures 2-4 exhibit the consequence of Hartman number, Soret number and Chemical raction parameter . It is evidently seen from the figures that the concentration intensity of the fluid accelerates due to the obligation of magnetic field while cosumption of thermal diffusion and chemical species, the incidence of the boundary layer close to a stretching sheet and the transport of mass brings the species to fall down. i.e the fluid concetration is enhanced and restricted on account of the substantial parameters occupied in the problem. It is also found that the high temperature gradient made the fluid concentration to fall down. i.e the diffusion due to temperature difference made the species to minimize.

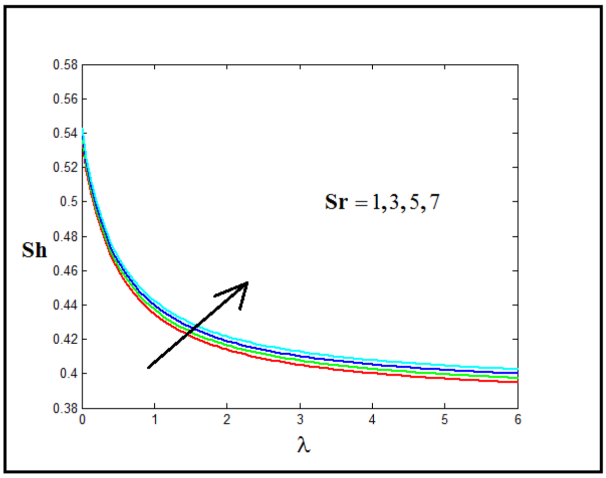

Figure 6: Sherwood number versus λ under M=0.25, C=1.5, Sc=0.62, Pr=0.71, B=0.1, Kr=1, p=0.1

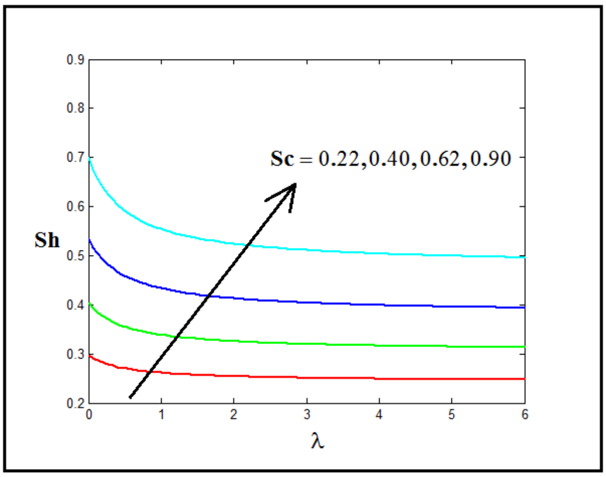

Figure 7: Sherwood number versus λ under C=1.5, Sc=0.62, M=0.25, Kr=1, Pr=0.71, B=0.1, Kr=1, p=0.1

The induction of the strength of the applied magnetic field, thermal diffusion and Schmdit number versus the co-efficient of rate of mass transfer are depicted in figures 5-7. Fom figure 5, it is established that the magnetic intensity of the flow model decelerates the mass transfer rate.

The observable fact of the augmentation of Sherwood number on account of Soret effect and mass diffusion are placed in figures 6-7. It is obserbed that the mass transfer rate is directly relative with the the mentioned parameters. It also suggests that the Sherwood number accelerates for any increasing value of Sc, i.e. mass flux from the plate to the fluid gets accelerated under the influence of mass diffusivity.

COMPARISON OF RESULTS

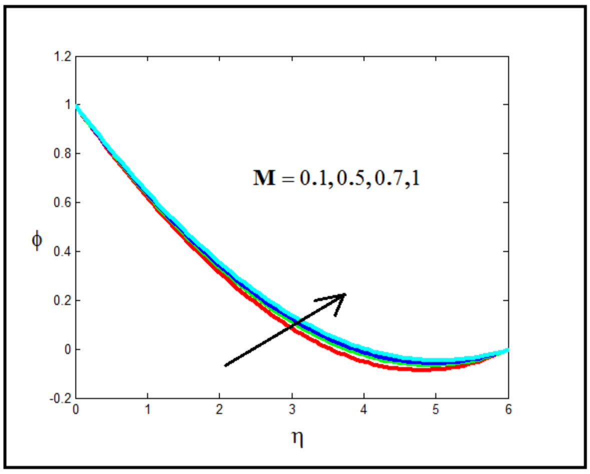

Figure 8: Concentration versus η under λ=3, C=1.5, Sc=0.62, Kr=1, Pr=0.71, B=0.1, Sr=0, p=0.1

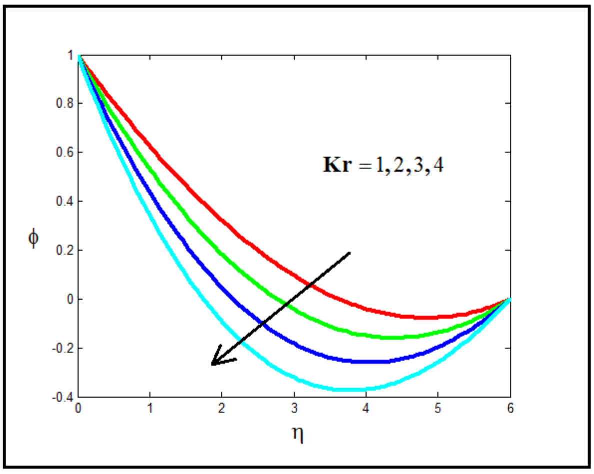

Figure 9: Concentration versus η under λ=3, M=0.25, C=1.5, Sc=0.62, Pr=0.71, B=0.1, Sr=0, p=0.1

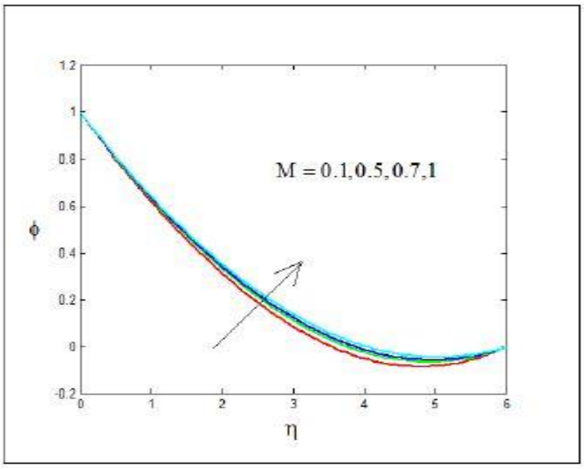

Figure 2: Concentration versus η under λ=3, C=1.5, Sc=0.62, Kr=1, p=0.1

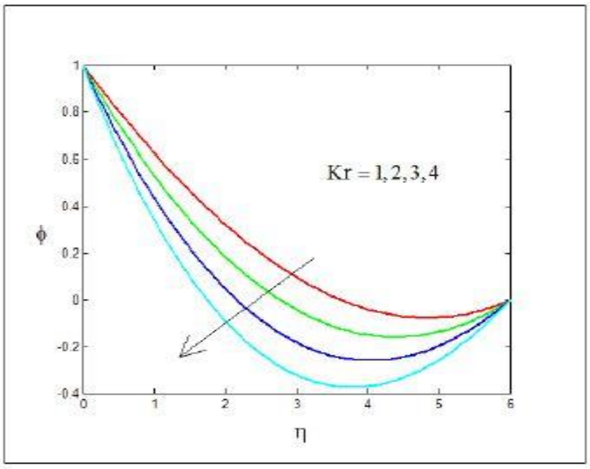

Figure 3: Concentration versus η under λ=3, C=1.5, Sc=0.62, M=0.25, p=0.1

To compare the results of the present paper, the work of Boboi (2023) is considered.

Comparing figures 8 and 9 with figures 2 and 3 of the work done by Boboi (2023), it is pragmatic that due to the execution of thermal diffusion, similar sort of characteristics is found. That is there is an admirable conformity between the results obtained by Boboi (2023) and the present authors.

CONCLUDING REMARKS:

1. The concentration intensity of the fluid accelerates due to the obligation of magnetic field.
2. The cosumption of thermal diffusion and chemical species, the incidence of the boundary layer close to a stretching sheet and the transport of mass brings the species to fall down.
3. The magnetic intensity of the flow model decelerates the mass transfer rate.
4. The mass flux from the plate to the fluid gets accelerated under the influence of mass diffusivity.

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