FOURIER SERIES FOR FUZZY VALUED FUNCTION USING HEPTADECAGONAL AND REVERSE ORDER HEPTADECAGONAL FUZZY NUMBER

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Abstract
Fourier analysis is the way of study to represent and approximate the general functions by sums of simpler trigonometric functions. The area of fourier analysis was founded on the idea of the Fourier Series. In terms of sines and cosines, it is an infinite expansion of a function. The Fourier Series is particularly important in the domains of electronics, quantum physics, and electrodynamics. Generally, it is not always necessary that the data obtained from the experimental results from these fields is precise. In order to handle the uncertainty with seventeen distinct points, we constructed the fourier series periodic function for heptadecagonal fuzzy number and defined the membership function for the closed interval in the current study. For the reverse order heptadecagonal fuzzy number, a fourier series periodic function is also introduced, and it is discovered that both are symmetrical in nature.

Keywords and phrases. : Fuzzy number, Heptadecagonal fuzzy number, Reverse order heptadecagonal fuzzy number, Fourier Series, Fourier coefficients.

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1 Introduction

Zadeh[7] developed fuzzy set theory to cope with ambiguous, vague, or partially accurate information. According to classical set theory, an element's membership in a set is believed to be binary, meaning that either the element falls under to that set or it does not. However, fuzzy set theory permits a set element to have a membership value that falls inside the range [0, 1]. Fuzzy numbers are a helpful tool to handle vagueness of the problem when there is uncertainty in decision-making problems when the membership function is expressed in a continuous form. A fuzzy number represents an approximation of some value and usually provides a better working state than the corresponding crisp value. Depending on the parameters, various fuzzy numbers are presented such as triangular, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, nonagonal, decagonal, dodecagonal, triskaidecagonal, pentadecagonal and heptadecagonal.
Joseph Fourier first introduced the Fourier series in 1807 in order to solve the heat equation in a metal plate. Since then, it has been used to discover the key techniques for studying digital signal processing, spectrum analysis, controllers, as well as some physics and engineering difficulties. The periodic function can also be applied in wave motion, time frequency analysis in which sine and cosine terms combined linearly. Kadak & Basar [3] gave the new mythodology on fuzzy fourier series of fuzzy valued function. Perlieva [6] also gave some implications on fuzzy and fourier transform. Firstly, Pathinathan et. al. [5] introduced the periodic fourier series with symmetricity with the help of pentagonal fuzzy number and further Naveena et. al.[4] developed the symmetric periodic fourier series using pentadecagonal and reverse order pentadecagonal fuzzy number. In this research paper, we have tried to solve procedure for controlling a structure using heptadecagonal fuzzy number [1] and reverse order heptadecagonal fuzzy number with its membership function to operate the vague experimental problems. Here, we have used level sets to find the fuzzy valued function which is periodic as well as symmetric in nature.
The rest of this paper is organised in the manner that follows to do this. Heptadecagonal and reverse order heptadecagonal fuzzy numbers are being used in any fuzzy number's advancement to introduce the fourier series for fuzzy valued function using some of its basic definitions which is mentioned in section 2. Following that, section 3 introduces the fourier series for fuzzy valued functions with the help of heptadecagonal fuzzy numbers along with examples and pictorial representation. Similarly, in section 4 fourier series for fuzzy valued function with the help of reverse order heptadecagonal fuzzy number is introduced along with examples and then in section 5, an easily understood conclusion is drawn.

2 Preliminaries

Here, few fundamental ideas of fuzzy set theory and fourier analysis are covered such as fuzzy set, fuzzy number, heptadecagonal fuzzy number, reverse order heptadecagonal fuzzy number, period of a fuzzy valued function and fourier series of a fuzzy valued function.

2.1 Fuzzy set [7]

A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0, 1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs:
$A = \{(x,\mu_A(x)|x \in X)\}$
where $\mu_A(x) : X \to [0,1]$ is a mapping called the degree of membership function in the fuzzy set A.

2.2 Fuzzy number [2]

A fuzzy set A on R must possess at least the following three properties to satisfy as a fuzzy number:
(i) A must be a normal fuzzy set.
(ii) α_A must be closed interval for every $\alpha \in (0,1]$.
(iii) The support and strong α-cut of A must be bounded.

2.3 Heptadecagonal fuzzy number [1]

A Heptadecagonal fuzzy number $\tilde{A}_{HD}$ as shown in fig. 1 is denoted as $\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a$ $a13,a14,a15,a16,a17\}$ where $\{a1,a2,a3,a4,a5,a6,a7,a8,a9,a10,a11,a12,a13,a14,a15,a16,a17\}$ are real values and its degree of membership $\mu_{\tilde{A}HD}(x)$ is shown below:

$$\mu_{\tilde{A}_{HD}(x)} = \begin{cases} \frac{1(x-a_1)}{8(a_2-a_1)} & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{8} + \frac{1(x-a_2)}{8(a_3-a_2)} & \text{for } a_2 \leq x \leq a_3 \\ \frac{2}{8} + \frac{1(x-a_3)}{8(a_4-a_3)} & \text{for } a_3 \leq x \leq a_4 \\ \frac{3}{8} + \frac{1(x-a_4)}{8(a_5-a_4)} & \text{for } a_4 \leq x \leq a_5 \\ \frac{4}{8} + \frac{1(x-a_5)}{8(a_6-a_5)} & \text{for } a_5 \leq x \leq a_6 \\ \frac{5}{8} + \frac{1(x-a_6)}{8(a_7-a_6)} & \text{for } a_6 \leq x \leq a_7 \\ \frac{6}{8} + \frac{1(x-a_7)}{8(a_8-a_7)} & \text{for } a_7 \leq x \leq a_8 \\ \frac{7}{8} + \frac{1(x-a_8)}{8(a_9-a_8)} & \text{for } a_8 \leq x \leq a_9 \\ 1 - \frac{1(x-a_9}{8(a_{10}-a_9))} & \text{for } a_9 \leq x \leq a_{10} \\ \frac{7}{8} - \frac{1(x-a_{10})}{8(a_{11}-a_{10})} & \text{for } a_{10} \leq x \leq a_{11} \\ \frac{6}{8} - \frac{1(x-a_{11})}{8(a_{12}-a_{11})} & \text{for } a_{11} \leq x \leq a_{12} \\ \frac{5}{8} - \frac{1(x-a_{12})}{8(a_{13}-a_{12})} & \text{for } a_{12} \leq x \leq a_{13} \\ \frac{4}{8} - \frac{1(x-a_{13})}{8(a_{14}-a_{13})} & \text{for } a_{13} \leq x \leq a_{14} \\ \frac{3}{8} - \frac{1(x-a_{14})}{8(a_{15}-a_{14})} & \text{for } a_{14} \leq x \leq a_{15} \\ \frac{2}{8} - \frac{1(x-a_{15})}{8(a_{16}-a_{15})} & \text{for } a_{15} \leq x \leq a_{16} \\ \frac{1(a_{17-x})}{8(a_{17}-a_{16})} & \text{for } a_{16} \leq x \leq a_{17} \\ 0 & \text{otherwise} \end{cases}$$

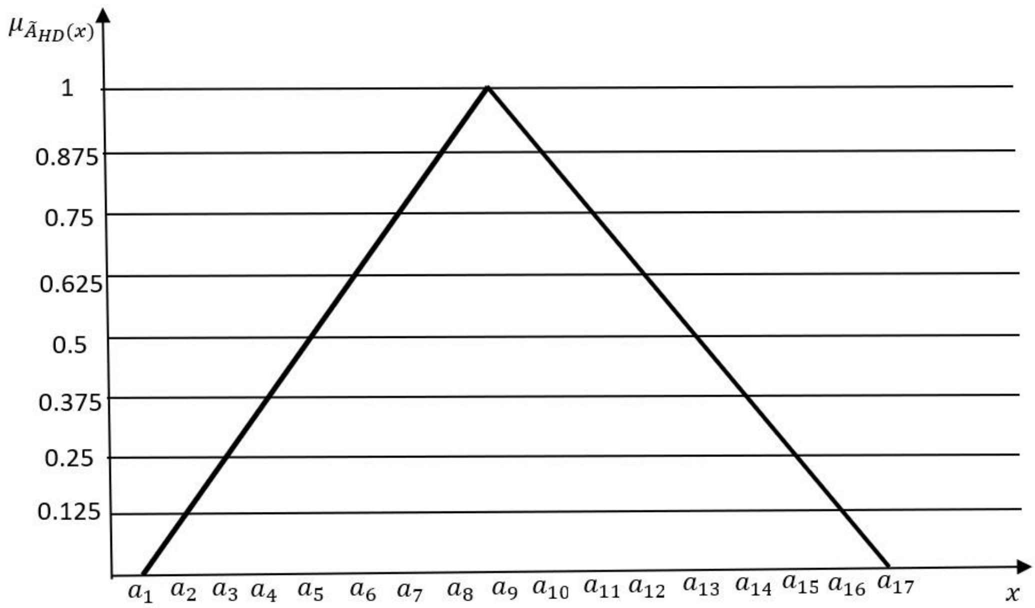

Figure 1: Graphical representation of heptadecagonal fuzzy number

2.4 Reverse order heptadecagonal fuzzy number

The reverse order heptadecagonal fuzzy number $\tilde{R}_{HD}$ as shown in fig. 2 is defined by $\tilde{R}_{HD} = \{-a_8,-a_7,-a_6,-a_5,-a_4,-a_3,-a_2,-a_1,0,a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8\}$ where $\{-a_8,-a_7,-a_6,-a_5,-a_4,-a_3,-a_2,-a_1,0, a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8\}$ are real values and its degree of membership $\mu_{\tilde{R}HD}(x)$ is shown below:

$$\mu_{\tilde{R}_{HD}(x)} = \begin{cases} 1 & \text{for } x \le -a_8 \\ -\frac{x}{a_8} & \text{for } -a_8 \le x \le -a_7 \\ -\frac{x}{a_7} & \text{for } -a_7 \le x \le -a_6 \\ -\frac{x}{a_6} & \text{for } -a_6 \le x \le -a_5 \\ -\frac{x}{a_5} & \text{for } -a_5 \le x \le -a_4 \\ -\frac{x}{a_4} & \text{for } -a_4 \le x \le -a_3 \\ -\frac{x}{a_3} & \text{for } -a_3 \le x \le -a_2 \\ -\frac{x}{a_2} & \text{for } -a_2 \le x \le -a_1 \\ -\frac{x}{a_1} & \text{for } -a_1 \le x \le 0 \\ 0 & \text{for } x = 0 \\ \frac{x}{a_1} & \text{for } 0 \le x \le a_1 \\ \frac{x}{a_2} & \text{for } a_1 \le x \le a_2 \\ \frac{x}{a_3} & \text{for } a_2 \le x \le a_3 \\ \frac{x}{a_4} & \text{for } a_3 \le x \le a_4 \\ \frac{x}{a_5} & \text{for } a_4 \le x \le a_5 \\ \frac{x}{a_6} & \text{for } a_5 \le x \le a_6 \\ \frac{x}{a_7} & \text{for } a_6 \le x \le a_7 \\ \frac{x}{a_8} & \text{for } a_7 \le x \le a_8 \\ 1 & \text{for } x \ge a_8 \end{cases}$$

2.5 Periodic fuzzy membership function [4]

The majority of membership functions for fuzzy sets are defined on a fixed range that is typically represented by [0, 1]. For some of the fuzzier approximation reasoning, time, season, and direction, the periodic function can be different. The fuzzy grades provide the same value at regular intervals since the membership functions of such fuzzy sets are periodic functions. The degree of membership is referred to be periodic with period $\omega > 0$, if $\mu(v) = \mu(v + \omega)$ for all v of the degree of membership.

In the trigonometric series, let f^{t} be any fuzzy valued function stated on $[-\pi, \pi]$. If the fuzzy valued fourier series converges to f^{t}, then the function is a periodic function and the degree sum of this yields the necessary periodic extension of f^{t}.

2.6 Fourier series for fuzzy valued functions of the period 2π

A fuzzy valued function f^{ω} of period 2π on the set A, then the fourier series for fuzzy valued function of f^{ω} in the time period 2π is referred below:

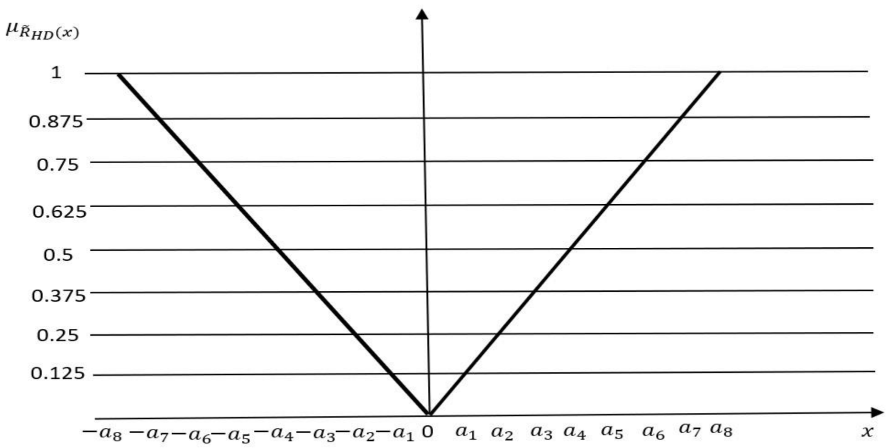

Figure 2: Graphical representation of reverse order heptadecagonal fuzzy number

$$f^{\omega}(t) \approx \frac{a_0}{2} \oplus \sum_{n=1}^{\infty}(a_n \cos(n\omega) \oplus b_n \sin(n\omega))$$

w.r.t the fuzzy fourier constants p_0, p_n and q_n, which converges uniformly for $\lambda \in [0,1]$ for all $n \in N$ and $x,t \in A$. The fourier constants with respect to the degree sets $[f_\lambda^\omega]=$
$[f_\lambda^-(\omega), f_\lambda^+(\omega)]$

$$p_0 = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f_\lambda^-(\omega)dw, \int_{-\pi}^{\pi} f_\lambda^+(\omega)dw\right]$$

$$p_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f_\lambda^-(\omega)\cos(n\omega)dw, \int_{-\pi}^{\pi} f_\lambda^+(\omega)\cos(n\omega)dw\right](n \geq 0)$$

$$q_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f_\lambda^-(\omega)\sin(n\omega)dw, \int_{-\pi}^{\pi} f_\lambda^+(\omega)\sin(n\omega)dw\right]$$

3 Fuzzy fourier series for heptadecagonal fuzzy number

Let f_ω be 2π periodic fuzzy valued function, then the degree of membership of the heptadecagonal fuzzy number under the interval $[-\pi,\pi]$ is defined by:

$$f_\lambda^\omega(x) = \begin{cases} 0 & \text{for } x < -\pi \\ \frac{(x+\pi)}{\frac{\pi}{8}-t)} & \text{for } -\pi \leq x \leq -\frac{7\pi}{8}-t \\ \frac{(x+\frac{7\pi}{8}+t)}{\frac{\pi}{8}} & \text{for } -\frac{7\pi}{8}-t \leq x \leq -\frac{6\pi}{8}-t \\ \frac{(x+\frac{6\pi}{8}+t)}{\frac{\pi}{8}} & \text{for } -\frac{6\pi}{8}-t \leq x \leq -\frac{5\pi}{8}-t \\ \frac{(x+\frac{5\pi}{8}+t)}{\frac{\pi}{8}} & \text{for } -\frac{5\pi}{8}-t \leq x \leq -\frac{4\pi}{8}-t \\ \frac{(x+\frac{4\pi}{8}+t)}{\frac{\pi}{8}} & \text{for } -\frac{4\pi}{8}-t \leq x \leq -\frac{3\pi}{8}-t \\ \frac{(x+\frac{3\pi}{8}+t)}{\frac{\pi}{8}} & \text{for } -\frac{3\pi}{8}-t \leq x \leq -\frac{2\pi}{8}-t \\ \frac{(x+\frac{2\pi}{8}+t)}{\frac{\pi}{8}} & \text{for } -\frac{2\pi}{8}-t \leq x \leq -\frac{\pi}{8}-t \\ \frac{(x+\frac{\pi}{8}+t)}{(\frac{\pi}{8}+t)} & \text{for } -\frac{\pi}{8}-t \leq x \leq 0 \\ 1 & \text{for } x = 0 \\ \frac{(\frac{\pi}{8}+t-x)}{(\frac{\pi}{8}+t)} & \text{for } 0 \leq x \leq \frac{\pi}{8}+t \\ \frac{(\frac{2\pi}{8}+t-x)}{(\frac{\pi}{8})} & \text{for } \frac{\pi}{8}+t \leq x \leq \frac{2\pi}{8}+t \\ \frac{(\frac{3\pi}{8}+t-x)}{(\frac{\pi}{8})} & \text{for } \frac{2\pi}{8}+t \leq x \leq \frac{3\pi}{8}+t \\ \frac{(\frac{4\pi}{8}+t-x)}{(\frac{\pi}{8})} & \text{for } \frac{3\pi}{8}+t \leq x \leq \frac{4\pi}{8}+t \\ \frac{(\frac{5\pi}{8}+t-x)}{(\frac{\pi}{8})} & \text{for } \frac{4\pi}{8}+t \leq x \leq \frac{5\pi}{8}+t \\ \frac{(\frac{6\pi}{8}+t-x)}{(\frac{\pi}{8})} & \text{for } \frac{5\pi}{8}+t \leq x \leq \frac{6\pi}{8}+t \\ \frac{(\frac{7\pi}{8}+t-x)}{(\frac{\pi}{8})} & \text{for } \frac{6\pi}{8}+t \leq x \leq \frac{7\pi}{8}+t \\ \frac{(\pi-x)}{(\frac{\pi}{8}-t)} & \text{for } \frac{7\pi}{8}+t \leq x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$$

The graphical representation of heptadecagonal fuzzy number in the interval $[-\pi,\pi]$ is given in fig. 3, that is a fuzzy integral with $[-\pi,\pi]$ for every $x,\omega \in [-\pi,\pi]$ and $\lambda \in [0,1]$. The degree set $[f_\lambda^\omega]$ of the degree of membership of $[f_\lambda^\omega]$ are referred below:

$$[f_\lambda^\omega] = [f_\lambda^-(\omega), f_\lambda^+(\omega)]$$
$$= [\pi(\lambda - \frac{36}{8}) - 7\omega, \pi(\lambda - \frac{36}{8}) + 7\omega]$$

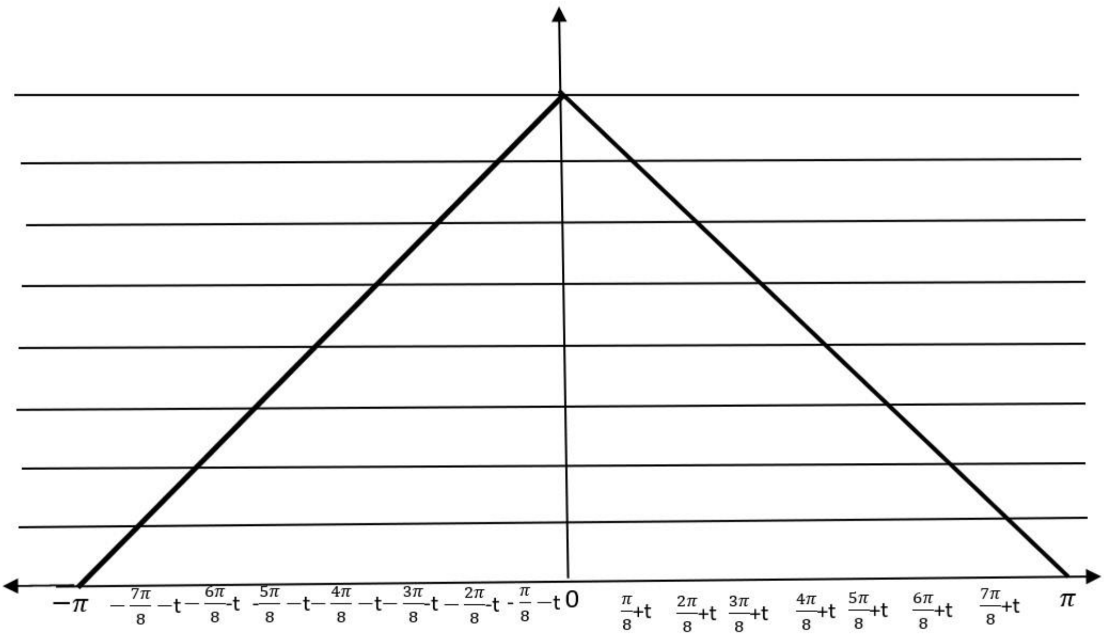

Figure 3: Graphical representation of heptadecagonal fuzzy number in the interval $[-\pi,\pi]$

The fuzzy fourier constants p_0, p_n and q_n for heptadecagonal fuzzy number is given by:

$$p_0 = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f_\lambda^-(\omega)dw, \int_{-\pi}^{\pi} f_\lambda^+(\omega)dw\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} \pi(\lambda - \frac{36}{8}) - 7\omega dw, \int_{-\pi}^{\pi} \pi(\frac{36}{8} - \lambda) + 7\omega dw\right]$$

$$= \left[2\pi(\lambda - \frac{36}{8}), 2\pi(\frac{36}{8} - \lambda)\right]$$

$$p_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f_\lambda^-(\omega)\cos(n\omega)dw, \int_{-\pi}^{\pi} f_\lambda^+(\omega)\cos(n\omega)dw\right](n \geq 0)$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} [\pi(\lambda - \frac{36}{8}) - 7\omega]\cos(n\omega)dw, \int_{-\pi}^{\pi} \pi(\frac{36}{8} - \lambda) + 7\omega)\cos(n\omega)dw\right]$$

$$= [0, 0]$$

$$q_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f_\lambda^-(\omega)\sin(n\omega)dw, \int_{-\pi}^{\pi} f_\lambda^+(\omega)\sin(n\omega)dw\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} [\pi(\lambda - \frac{36}{8}) - 7\omega]\sin(n\omega)dw, \int_{-\pi}^{\pi} \pi(\frac{36}{8} - \lambda) + 7\omega)\sin(n\omega)dw\right]$$

$$= \frac{2}{\pi}\left[\pi[\frac{92}{8} - \lambda]\frac{(-1)^n}{n} + \frac{\pi}{n}(\lambda - \frac{36}{8}), \pi[\lambda - \frac{92}{8}]\frac{(-1)^n}{n} + \frac{\pi}{n}(\frac{36}{8} - \lambda)\right]$$

The fuzzy valued function f^ω of period 2π on the set A, then the fourier series for fuzzy valued function of f^ω in the time period 2π is referred below:

$$f^\omega(t) = \frac{p_0}{2} \oplus \sum_{n=1}^{\infty}(p_n\cos(n\omega) \oplus q_n\sin(n\omega))$$

$$= \left[\pi(\lambda - \frac{36}{8}), \pi(\frac{36}{8} - \lambda)\right] \oplus \sum_{n=1}^{\infty}[0,0]\cos(n\omega) \oplus \sum_{n=1}^{\infty}\frac{2}{\pi}\left[\pi[\frac{92}{8} - \lambda]\frac{(-1)^n}{n} + \frac{\pi}{n}(\lambda - \frac{36}{8}), \pi[\lambda - \frac{92}{8}]\frac{(-1)^n}{n} + \frac{\pi}{n}(\frac{36}{8} - \lambda)\right]\sin(n\omega)$$

Hence, we obtain

$$= \Bigg[\pi(\lambda - \tfrac{36}{8}) + 4(\lambda - 8)[\sin(\omega) + \tfrac{\sin(3\omega)}{3} + \tfrac{\sin(5\omega)}{5} + ...] + 14[\tfrac{\sin(2\omega)}{2} + \tfrac{\sin(4\omega)}{4} + \tfrac{\sin(6\omega)}{6} + ...], \pi(\tfrac{36}{8} - \lambda) + 4(8 - \lambda)[\sin(\omega) + \tfrac{\sin(3\omega)}{3} + \tfrac{\sin(5\omega)}{5} + ...] - 14[\tfrac{\sin(2\omega)}{2} + \tfrac{\sin(4\omega)}{4} + \tfrac{\sin(6\omega)}{6} + ...]\Bigg]$$

Example 3.1. *Let the membership value λ be 0.5 and ω be 30°C for the heptadecagonal fuzzy number of seventeen parameters, then the fuzzy fourier series that we obtain from the fourier fuzzy valued periodic function is a symmetric periodic function.*
$[f^{30^\circ C}]_{0.5} = [-27.354889, 27.354889]$

Example 3.2. *Let the membership value λ be 0.8 and ω be 60°C for the heptadecagonal fuzzy number of seventeen parameters, then the fuzzy fourier series that we obtain from the fourier fuzzy valued periodic function is a symmetric periodic function.*
$[f^{60^\circ C}]_{0.8} = [-30.585245, 30.585245]$

4 Fuzzy fourier series for reverse order heptadecagonal fuzzy number

The level set obtained for reverse order heptadecagonal fuzzy number as shown in fig. 2 is $\mu_R\tilde{}HD(x)]_\lambda = [\mu^-_R\tilde{}HD(\lambda), \mu^+_R\tilde{}HD(\lambda)]$
$= [-(a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8)\lambda, (a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8)\lambda]$

Consider the periodic reverse order heptadecagonal fuzzy fourier function and let f_ω be 2π periodic fuzzy valued function, then the degree of membership of reverse order heptadecagonal fuzzy numer defined in interval $[-\pi, \pi]$is defined by:

$$\mu(x) = \begin{cases} 1 & \text{for } x < -\pi \\ \frac{-x}{\pi} & \text{for } -\pi \le x \le -\frac{7\pi}{8} - t \\ \frac{-x}{\frac{7\pi}{8}+t} & \text{for } -\frac{7\pi}{8} - t \le x \le -\frac{6\pi}{8} - t \\ \frac{-x}{\frac{6\pi}{8}+t} & \text{for } -\frac{6\pi}{8} - t \le x \le -\frac{5\pi}{8} - t \\ \frac{-x}{\frac{5\pi}{8}+t} & \text{for } -\frac{5\pi}{8} - t \le x \le -\frac{4\pi}{8} - t \\ \frac{-x}{\frac{4\pi}{8}+t} & \text{for } -\frac{4\pi}{8} - t \le x \le -\frac{3\pi}{8} - t \\ \frac{-x}{\frac{3\pi}{8}+t} & \text{for } -\frac{3\pi}{8} - t \le x \le -\frac{2\pi}{8} - t \\ \frac{-x}{\frac{2\pi}{8}+t} & \text{for } -\frac{2\pi}{8} - t \le x \le -\frac{\pi}{8} - t \\ \frac{-x}{\frac{\pi}{8}+t} & \text{for } -\frac{\pi}{8} - t \le x \le 0 \\ 0 & \text{for } x = 0 \\ \frac{x}{\frac{\pi}{8}+t} & \text{for } 0 \le x \le \frac{\pi}{8} + t \\ \frac{x}{\frac{2\pi}{8}+t} & \text{for } \frac{\pi}{8} + t \le x \le \frac{2\pi}{8} + t \\ \frac{x}{\frac{3\pi}{8}+t} & \text{for } \frac{2\pi}{8} + t \le x \le \frac{3\pi}{8} + t \\ \frac{x}{\frac{4\pi}{8}+t} & \text{for } \frac{3\pi}{8} + t \le x \le \frac{4\pi}{8} + t \\ \frac{x}{\frac{5\pi}{8}+t} & \text{for } \frac{4\pi}{8} + t \le x \le \frac{5\pi}{8} + t \\ \frac{x}{\frac{6\pi}{8}+t} & \text{for } \frac{5\pi}{8} + t \le x \le \frac{6\pi}{8} + t \\ \frac{x}{\frac{7\pi}{8}+t} & \text{for } \frac{6\pi}{8} + t \le x \le \frac{7\pi}{8} + t \\ \frac{x}{\pi} & \text{for } \frac{7\pi}{8} + t \le x \le \pi \\ 1 & \text{for } x > \pi \end{cases}$$

which is a fuzzy integral on $[-\pi, \pi]$ for each $x, \omega \in [-\pi, \pi]$ and $\lambda \in [0,1]$.
The level set $[f_\lambda^\omega]$ of the degree of membership of $[f_\lambda^\omega]$ are referred as,

$$[f_\lambda^\omega] = [f_\lambda^-(\omega), f_\lambda^-(\omega)]$$
$$= \Bigg[-\lambda\Bigg(\frac{36}{8}\pi + 7\omega\Bigg), \lambda\Bigg(\frac{36}{8}\pi + 7\omega\Bigg)\Bigg]$$

The reverse order heptadecagonal fuzzy fourier constants p_0, p_n and q_n are given by:

$$p_0 = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f_\lambda^-(\omega)dw, \int_{-\pi}^{\pi} f_\lambda^+(\omega)dw\right]$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} -\lambda(\frac{36\pi}{8} + 7\omega)dw, \int_{-\pi}^{\pi} \lambda(\frac{36\pi}{8} + 7\omega)dw\right]$$
$$= \left[-9\pi\lambda, 9\pi\lambda\right]$$
$$p_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f_\lambda^-(\omega)\cos(n\omega)dw, \int_{-\pi}^{\pi} f_\lambda^+(\omega)\cos(n\omega)dw\right](n \geq 0)$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} [-\lambda(\frac{36\pi}{8} + 7\omega)]\cos(n\omega)dw, \int_{-\pi}^{\pi} [\lambda(\frac{36\pi}{8} + 7\omega)]\cos(n\omega)dw\right]$$
$$= [0, 0]$$
$$q_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} f_\lambda^-(\omega)\sin(n\omega)dw, \int_{-\pi}^{\pi} f_\lambda^+(\omega)\sin(n\omega)dw\right]$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} [-\lambda(\frac{36\pi}{8} + 7\omega)]\sin(n\omega)dw, \int_{-\pi}^{\pi} [\lambda(\frac{36\pi}{8} + 7\omega)]\sin(n\omega)dw\right]$$
$$= \frac{2\lambda}{\pi}\left[[\frac{92\pi}{8}\frac{(-1)^n}{n} - \frac{36}{8}\frac{\pi}{n}], -[\frac{92\pi}{8}\frac{(-1)^n}{n} - \frac{36}{8}\frac{\pi}{n}]\right]$$

The fuzzy valued function f^ω of period 2π on the set A, then the fourier series for fuzzy valued function of f^ω in the time period 2π is referrred below:

$$f^\omega(t) = \frac{p_0}{2} \oplus \sum_{n=1}(p_n\cos(n\omega) \oplus q_n\sin(n\omega))$$
$$= \left[\frac{-9\pi\lambda}{2}, \frac{9\pi\lambda}{2}\right] \oplus \sum_{n=1}^{\infty}\left[[0,0]\cos(n\omega) \oplus \frac{2\lambda}{\pi}\left[(\frac{92\pi}{8}\frac{(-1)^n}{n} - \frac{36}{8}\frac{\pi}{n}), -(\frac{92\pi}{8}\frac{(-1)^n}{n} - \frac{36}{8}\frac{\pi}{n})\right]\sin(n\omega)\right]_{\infty}$$

Hence, we obtain

$$= \left[\frac{-9\pi\lambda}{2} - 32\lambda[\sin(\omega) + \frac{\sin(3\omega)}{3} + \frac{\sin(5\omega)}{5} + ...] + 14\lambda[\frac{\sin(2\omega)}{2} + \frac{\sin(4\omega)}{4} + \frac{\sin(6\omega)}{6} + ...], \frac{9\pi\lambda}{2} + 32\lambda[\sin(\omega) + \frac{\sin(3\omega)}{3} + \frac{\sin(5\omega)}{5} + ...] - 14\lambda[\frac{\sin(2\omega)}{2} + \frac{\sin(4\omega)}{4} + \frac{\sin(6\omega)}{6} + ...]\right]$$

Example 4.1. *Let the membership value λ be 0.4 and ω be* $45^\circ C$ *for the reverse order heptadecagonal fuzzy number of seventeen parameters, then the fuzzy fourier series that we obtain from the fourier fuzzy valued periodic function is a symmetric periodic function.*

$[f^{45^\circ C}]_{0.4} = [-2.29147, 2.29147]$

Example 4.2. *Let the membership value λ be 0.6 and ω be* $30^\circ C$ *for the reverse order heptadecagonal fuzzy number of seventeen parameters, then the fuzzy fourier series that we obtain from the fourier fuzzy valued periodic function is a symmetric periodic function.*

$[f^{30^\circ C}]_{0.6} = [-19.127986, 19.127986]$

5 Conclusion

In this research article, we introduced the fourier series with fuzziness for period 2π in interval $[-\pi,\pi]$ and also verified the similarity of fuzzy fourier coefficients using heptadecagonal fuzzy number and reverse order heptadecagonal fuzzy number. The fuzzy number space can be used for a variety of Fourier series applications in real or complex fields, such as time series analysis, image processing, signal processing, etc. Thus, the goal of this research is to extend classical analysis to fuzzy level set analysis, which deals with fuzzy valued functions across an interval $[-\pi,\pi]$.

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