Section A-Research paper

Some Results on Energies of join of Complete Graphs

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Abstract:

The energy for the adjacency matrix, degree sum matrix, degree sum adjacency matrix, and degree square sum matrix of $J_n(K_p)$ are computed in this work.

Keywords: energy, degree sum energy, degree sum adjacency energy, degree square sum energy, join of complete graphs K_p

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1. Introduction and Preliminaries

In 1978, I.Gutman was introduced the new concept energy in graph theory. Let G be a simple, undirected graph with p vertices and q edges. We adhere to the definitions provided in [9], [14], [17], and [2] for energy, degree sum energy (ie) $E_{DS}(G)$, degree sum adjacency energy $DS_AE(G)$, and degree square sum energy $E_{DSS}(G)$, respectively.

Main Results

Lemma 2.1 [5] .

Let M, N, P and Q be matrices with M invertible, then $\begin{bmatrix} M & N \\ P & Q \end{bmatrix} = |M||Q - PM^{-1}N|$

Lemma 2.2[5].

Let M, N,P and Q be matrices. Let S = $\begin{bmatrix} M & N \\ P & Q \end{bmatrix}$ if M and P commutes then

$|S| = |MQ - PN|$.

Lemma 2.3 [5].

If $A(K_p)$ is the adjacency matrix of K_p and the Spectrum of $A(K_p)$ are p-1 and $(-1)^{p-1}$ then $A^2(K_p) = (p-2)A(K_p) + (p-1)I_p$.

Definition 2.4:

Let $K_p, p \geq 2$ be complete graphs with p vertices. We take $n \geq 2$ number of complete graphs K_p and joining every first vertices together , every second vertices together and joining upto every p vertices together.Then the resulting graph is called join of complete regular graphs K_p with order $pn, \frac{p^2n-2pn+pn^2}{2}$ edges, and regular of degree $d = p + n - 2$. It is denoted by $J_n(K_p)$.

Theorem 2.5

If $J_n(K_p)$ is the join of $n \geq 2$ number of complete graphs K_p. Then $S_p\left(J_n(K_p)\right) = \begin{pmatrix} p+n-2 & n-2 & p-2 & -2 \\ 1 & p-1 & n-1 & np-n-p+1 \end{pmatrix}$ and $E\left(J_n(K_p)\right) = 4np - 4p - 4n + 4$.

Proof:

Let $J_n(K_p)$ be the join of $n \geq 2$ number of complete graphs.

Then the adjacency matrix of $J_n(K_p)$ is

$$A\left(J_n(K_p)\right) = \begin{pmatrix} A(K_p) & I_p & \cdots & I_p \\ I_p & A(K_p) & \cdots & I_p \\ \vdots & \vdots & \ddots & \vdots \\ I_p & I_p & \cdots & A(K_p) \end{pmatrix}$$

and the characteristic polynomial of $A\left(J_n(K_p)\right)$ is

$$\left|\lambda I_{pn} - A\left(J_n(K_p)\right)\right| = \begin{vmatrix} \lambda I_p - A(K_p) & -I_p & \cdots & -I_p \\ -I_p & \lambda I_p - A(K_p) & \cdots & -I_p \\ \vdots & \vdots & \ddots & \vdots \\ -I_p & -I_p & \cdots & \lambda I_p - A(K_p) \end{vmatrix}$$

By using elementary transformations $C_1 \rightarrow C_1 + C_2 + \cdots + C_n$ and $R_i \rightarrow R_i - R_1$, $i = 2,3, \dots, n$. We get

$$\left|\lambda I_{pn} - A\left(J_n(K_p)\right)\right|$$

$$= \begin{vmatrix} (\lambda-(n-1))I_p - A(K_p) & -I_p & \cdots & -I_p \\ 0 & (\lambda+1)I_p - A(K_p) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (\lambda+1)I_p - A(K_p) \end{vmatrix}$$

$$= (\left(\lambda-(n-1))I_p - A(K_p)\right)((\lambda+1)I_p - A(K_p))^{n-1}$$

By using lemma 2.3, $S_p\left(J_n(K_p)\right) = \begin{pmatrix} p+n-2 & n-2 & p-2 & -2 \\ 1 & p-1 & n-1 & np-n-p+1 \end{pmatrix}$

and $E\left(J_n(K_p)\right) = 4np - 4p - 4n + 4$.

Theorem 2.6

The adjacency matrix of complete graphs of order p is multiplied twice by its own degree to obtain the degree sum matrix if G is a regular graph with p vertices.

Proof:

Let G be a $d-$ regular graphs with p vertices then degree sum matrix of G is sum of two degrees v_i and v_j if $i \neq j$ otherwise zero.

Therefore $DS(G) = \begin{pmatrix} 0 & 2d & \cdots & 2d \\ 2d & 0 & \cdots & 2d \\ \vdots & \vdots & \ddots & \vdots \\ 2d & 2d & \dots & 0 \end{pmatrix}$

We know that the main diagonal of the adjacency matrix of K_p is 0 and every other entry is 1.As a result ,the aforementioned matrix is equal to a multiple of $2d$ by the corresponding adjacency matrix of complete graphs, where d is that matrix's degree. Hence proved.

Theorem 2.7

If $J_n(K_p)$ is join of complete regular graphs then $S_p\left(DS\left(J_n(K_p)\right)\right) =$

$\begin{bmatrix} 2d(pn-1) & -2d \\ 1 & pn-1 \end{bmatrix}$ and $E_{DS}\left(J_n(K_p)\right) = 4d(pn-1)$.

Proof:

By theorem 2.6 , $DS\left(J_n(K_p)\right) = 2dA(K_p)$.

Then the characteristic polynomial $DS\left(J_n(K_p)\right)$ is

$\left|\mu I_{pn} - DS\left(J_n(K_p)\right)\right| = |\mu I_{pn} - 2dA(K_{pn})|$.

By using lemma 2.3, $\left|\mu I_{pn} - DS\left(J_n(K_p)\right)\right| = (\mu - 2d(pn-1))(\mu + 2d)^{pn-1}$

Therefore $S_p\left(DS\left(J_n(K_p)\right)\right) = \begin{bmatrix} 2d(pn-1) & -2d \\ 1 & pn-1 \end{bmatrix}$ and $E_{DS}\left(J_n(K_p)\right) = 4d(pn-1)$.

Theorem 2.8

If G is a regular graph with p vertices then the degree sum adjacency matrix is the adjacency matrix of that graph of order p multiplied by twice the degree of that matrix.

Proof:

Let G be a $d-$ regular graphs with p vertices then degree sum matrix of G is sum of two degrees v_i and v_j if v_i is adjacent to v_j and 0 otherwise.

$$\text{Therefore } DS_A(G) = \begin{pmatrix} 0 & 2da_{12} & \cdots & 2da_{1p} \\ 2da_{21} & 0 & \cdots & 2da_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ 2da_{p1} & 2da_{p2} & \ldots & 0 \end{pmatrix}$$

$$= 2d\begin{pmatrix} 0 & a_{12} & \cdots & a_{1p} \\ a_{21} & 0 & \ldots & a_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ a_{p1} & a_{p2} & \ldots & 0 \end{pmatrix}$$

$DS_A(G) = 2dA(G)$, where $A(G)$ is the adjacency matrix of G.

Hence result.

Theorem 2.9

Consider the graph $J_n(K_p)$ then

$S_p(DS_A(J_n(K_p))) = \begin{pmatrix} 2d^2 & 2d(n-2) & 2d(p-2) & -4d \\ 1 & p-1 & n-1 & np-n-p+1 \end{pmatrix}$ and

$DS_AE\left(J_n(K_p)\right) = 2d^2 + 8pdn - 10dn + 8d - 6dp - 4p + 4.$

Proof:

Let $J_n(K_p)$ be join of complete graphs and $\beta = \{\beta_1, \beta_2, \ldots \beta_{pn}\}$ be the eigen values of degree sum adjacency matrix of $J_n(K_p)$.

By using theorem 2.8, $DS_A\left(J_n(K_p)\right) = 2dA\left(J_n(K_p)\right)$

By using theorem 2.5 , $\left|\beta I_{pn} - DS_A\left(J_n(K_p)\right)\right| = (\beta - 2d^2)(\beta - 2d(n-$
2)p−1(β−2dp−2)n−1(β+4d)pn−p−n+1.

Therefore

$S_p(DS_A(J_n(K_p))) = \begin{pmatrix} 2d^2 & 2d(n-2) & 2d(p-2) & -4d \\ 1 & p-1 & n-1 & np-n-p+1 \end{pmatrix}$ and

$DS_AE\left(J_n(K_p)\right) = 2d^2 + 8pdn - 10dn + 8d - 6dp - 4p + 4.$

Theorem 2.10

Let G be a regular graph with p vertices then $DSS(G) = 2d^2A(K_p)$.

Proof:

If $i \neq j$ then the degree square sum matrix of G is equal to the sum of the squares of the degrees corresponding to v_i and v_j and 0 otherwise.

Therefore $DSS(G) = \begin{bmatrix} 0 & 2d^2 & \cdots & 2d^2 \\ 2d^2 & 0 & \cdots & 2d^2 \\ \vdots & \vdots & \ddots & \vdots \\ 2d^2 & 2d^2 & \cdots & 0 \end{bmatrix} = 2d^2A(K_p).$

Hence $DSS(G) = 2d^2A(K_p)$.

Theorem 2.11

If $J_n(K_p)$ is join of complete regular graphs then $S_p\left(DSS\left(J_n(K_p)\right)\right) =$

$\begin{bmatrix} 2d^2(pn-1) & -2d^2 \\ 1 & pn-1 \end{bmatrix}$ and $E_{DSS}\left(J_n(K_p)\right) = 4d^2(pn-1).$

Proof:

By using theorem 2.10, $DSS\left(J_n(K_p)\right) = 2d^2 A(K_p)$.

Then the characteristic polynomial $DSS\left(J_n(K_p)\right)$ is

$\left|\alpha I_p - DSS\left(J_n(K_p)\right)\right| = \left|\alpha I_p - 2d^2 A(K_p)\right|$.

By using lemma 2.3, $S_p\left(DSS\left(J_n(K_p)\right)\right) = \begin{bmatrix} 2d^2(pn-1) & -2d^2 \\ 1 & pn-1 \end{bmatrix}$

Therefore $E_{DSS}\left(J_n(K_p)\right) = 4d^2(pn-1)$.

Observations:

The following table gives the details of join of 3 number of complete graphs with 4 vertices.

S.No	Matrix	Eigen value	Energy
1	Adjacency Matrix	5,-2,-2,-2,-2,-2, -2,2,2,1,1,1	24
2	Degree Sum Matrix	110,-10,-10, -10,-10,-10, -10,-10, -10,-10, -10,-10	220
3	Degree Sum Adjacency Matrix	50,-20,-20,-20, -20,-20,-20, -20,-20,-20, 10,10,10	240

4	Degree Square Sum Matrix	550,-50,-50, -50,-50,-50,-50,-50,-50, -50,-50,-50	1100

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