# Proton-Neutron Interactions: New Perspectives on Collective Behaviour, Deformation, and Phase Transitions in Nuclei

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***Abstract* —**The reciprocal action between the final nucleons, termed $\delta V_{pn}$, was obtained utilizing the novel mass tabulation from a particular dual difference of binding energies. The findings are obtained for all even-even nuclei with N larger than 92 for their ground states; Z is between 64 and 74; and the high spin states of selected nuclei have been discussed. The outcomes show notable shell closures near magic nuclei. Overlaps of Orbits within the shell model are used to interpret $\delta V_{pn}$ values in the N>92 region. These overlaps are substantial when nucleons are in orbits that are similar and minimal when they are dissimilar. Furthermore, it is observed that there is a correlation between nucleon interactions and the rates of connection growth in terms of particle-particle (or hole-hole) and particle-hole regions. The predicted $\delta V_{pn}$ values are in perfect agreement with the observed results. We have investigated the effectiveness of Grodzin's equation ($E2_1^+ * B(E2)$), $R_{4/2}$, and $N_pN_n$ on '$\delta V_{pn}$' for the first time.

***Keywords***: Matrix element, p-n interaction, Binding energy.

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# I. INTRODUCTION

Double differences in binding energies can be used to determine the average intercommunication between the final two nucleons, or $\delta V_{pn}$, in accordance with empirical values. About fifteen years ago, this approach was first put forth [1] and put into practice [2-4] using the populace that was in existence at the time. The 2003 mass evaluation, which greatly increased the range of available masses [5], served as the impetus for further studies on these intercommunication [6–8]. These module mainly concentrated on the peculiar behavior in the large shell gap regions [6] and the alliance between the nucleons intercommunication strengths and the emergence of aggregation and distortion in the p-p, p-h, and h-h regions. Ref. [8], where numerous patterns and anomalies were detected, provides a summary of all $\delta V_{pn}$ values.

For even proton and even neutron, the term p-n intercommunication refers to the average intercommunication between the final two nucleons [9, 10, 11].

$$\delta V^{ee}_{pn}(Z, N) = 1/4[\{B(Z, N) - B(Z, N - 2)\} - \{B(Z - 2, N) - B(Z - 2, N - 2)\}] \qquad (1)$$

where B represents the nucleus' binding energy. The concept of $\delta V_{pn}$ primarily relies on removing the final nucleon’s connections with the core, as was previously indicated [9, 11], to ensure that the nuclear core is fundamentally unaffected. For a nucleus with an even number of protons and neutrons, it specifically refers to the intercommunication between the (Z-1)th and Zth valence protons and the (N-1)th and Nth neutrons.

In the scenarios of Z odd and N even, the final proton's empirical strength with the final neutron is

$$\delta V^{oe}_{pn}(Z, N) = 1/2[\{B(Z, N) - B(Z, N - 2)\} - \{B(Z - 1, N) - B(Z - 1, N - 2)\}] \qquad (2)$$

An odd-neutron nucleus follows the same rules. In the explanation that follows, we will simply refer to the interaction as $\delta V_{pn}$ and omit the superscript and parenthetical notations.

The work of Federman-Pittel [12], which was subsequently expanded to other mass regions [13], provides an explanation for the abrupt commencement of deformation in the A = 100 zone. Nazarewicz, Dobaczewski, and his colleagues [14, 15] further clarified how the orbital features affect the p-n interaction. Heyde and his research team provided the quantitative theoretical basis for the importance of the valence nucleon-neutron interaction [16, 17]. Additionally, they emphasized the significance of its monopolistic components. Additionally, they enlarged its scope beyond the emergence of low-lying invader states, which signify coexistence and the beginnings of nuclear deformation.

The $\delta V_{pn}$ values for lead (Pb) and nearby even-even nuclei that are close to 208 $_{Pb}$ are shown in Ref. [8], which amply demonstrates how much $\delta V_{pn}$ depends on the orbit [6]. The addition of more neutrons causes the $\delta V_{pn}$ values to divide into two different trends, one dropping and the other growing, starting at N = 100. Beyond N=126, the magnitude of an inferior branch's $\delta V_{pn}$ values suddenly increases until it reaches a level close to 300 keV, which is similar to the higher branch's right earlier than the shell's closing. In the sources [6, 18], this behavioral pattern is elaborated on in great detail. This trend's fundamental cause can be traced to correlations and collective effects having less relevance near doubly magical nuclei than they do in other areas. The final protons mainly inhabit low-j and high-n shell-model states for nuclei that are near the upper limits of the Z = 50–82 shell. The final neutrons likewise occupy low-j and high-n orbitals close to the top of the N=82–126 shell; a similar pattern appears. A substantial short-range residual contact develops between the valence particles as a result of the considerable spatial proximity between the nucleons orbitals. When both nucleons are located above the double magic shell closures of $^{208}Pb$, high values of $\delta V_{pn}$ can be seen. Once more, the orbits are equivalent.

## II. RESULT AND DISCUSSION

The exceptional uniformity in the mass region A=150-180 was the most notable characteristic. The relevant $\delta V_{pn}$ values are displayed in Table 1. Each element's levels of valence p-n interaction exhibit a constant, significant increase with N. The practically flawless parallel trajectories obtained by methodically adjusting the numbers to the right for each succeeding Z are an interesting finding. A falloff in $\delta V_{pn}$ at the highest neutron levels is also shown (Tab. 1) for those elements for which there is enough data available.

**Table 1- : Even-even nuclei in the A~ 150-200 mass areas shown against them are the $N_pN_n$ values with their corresponding $R_{4/2}$, $\delta V_{pn}$ in KeV and $E(2_1^+)$ * B(E2) values taken from Ref.[19,20,21].**

| Z | N | $\delta V_{pn}$ | $R_{4/2}$ | $N_pN_n$ | $E(2_1^+)$*B(E2) |
|---|---|---|---|---|---|
| 64 | 92 | 290 | 3.239 | 140 | 412.8 |
| 64 | 94 | 310 | 3.288 | 168 | 399.1 |
| 64 | 96 | 350 | 3.301 | 196 | 395.1 |
| 66 | 92 | 260 | 3.206 | 160 | 460.9 |
| 66 | 94 | 265 | 3.270 | 192 | 445.2 |
| 66 | 96 | 310 | 3.293 | 224 | 431.5 |
| 66 | 98 | 360 | 3.300 | 256 | 411.0 |
| 68 | 94 | 250 | 3.230 | 168 | 511.2 |
| 68 | 96 | 265 | 3.277 | 196 | 498.1 |
| 68 | 98 | 295 | 3.289 | 224 | 469.7 |
| 68 | 100 | 360 | 3.309 | 252 | 462.0 |
| 68 | 102 | 272 | 3.310 | 280 | 457.4 |
| 70 | 96 | 240 | 3.228 | 168 | 536.4 |
| 70 | 98 | 250 | 3.266 | 192 | 489.5 |
| 70 | 100 | 270 | 3.293 | 216 | 487.8 |
| 70 | 102 | 295 | 3.305 | 240 | 475.6 |
| 70 | 104 | 290 | 3.310 | 264 | 454.2 |
| 72 | 98 | 235 | 3.061 | 128 | 618.4 |
| 72 | 100 | 240 | 3.154 | 144 | 448.6 |
| 72 | 106 | 295 | 3.260 | 160 | 440.1 |
| 72 | 108 | 305 | 3.291 | 144 | 420.4 |
| 74 | 102 | 240 | 3.268 | 200 | 444.0 |
| 74 | 104 | 250 | 3.284 | 220 | 465.6 |
| 74 | 106 | 290 | 3.291 | 200 | 449.1 |
| 74 | 108 | 272 | 3.307 | 180 | 435.8 |

The energy of the lowest excited state in the ground band, designated as $E(2_1^+)$=E, and the excitation strength from the $0_1^+$ to $2_1^+$ state, denoted as $B(E2, 0_1^+ \rightarrow 2_1^+)$=B(E2), are the essential factors that govern the ground state structure of a certain even p-n nucleus. The vibrational and rotational properties of the nucleus, as well as the neutron number N and proton number Z, have a significant impact on these quantities. These elements work together to shape the aforementioned entities [22]. Bindra and Mittal (2018) demonstrated the reliance of Grodzin's standardized SFE product $E_{SF} * B(E2)$ and the product of rotational energy $E_{ROT} * B(E2)$ on the asymmetry constant $\gamma_0$ [23]. In Figures [1], A rapid and pronounced phase evolution occurs in the nuclei of the Gd-Hf region. This corresponds to the neutron-particle and proton-particle subregions of the shell subspaces Z = 64–74 and N = 92–108. This region has been extensively studied. In Yb nuclei, as the $\delta V_{pn}$ varies from 240–270, the Grodzins product, i.e., $E(2_1^+)*B(E2)$, decreases (tab. 1). As a result, the binding energy of atomic nuclei, which holds the protons and neutrons together, would be stronger. As a result, atomic nuclei would be more tightly bound, requiring more energy to break them apart. This increased binding energy leads to greater nuclear stability. A similar pattern is observed in other nuclei like Gd,Dy,Er, W, etc. But in the case of Hf (N = 106), the drop in $E(2_1^+)*B(E2)$ value is incredibly small as compared to the rise in $\delta V_{pn}$ value. In light of this, the Hf curve has a sharply decending character at $\delta V_{pn}$ = 290.

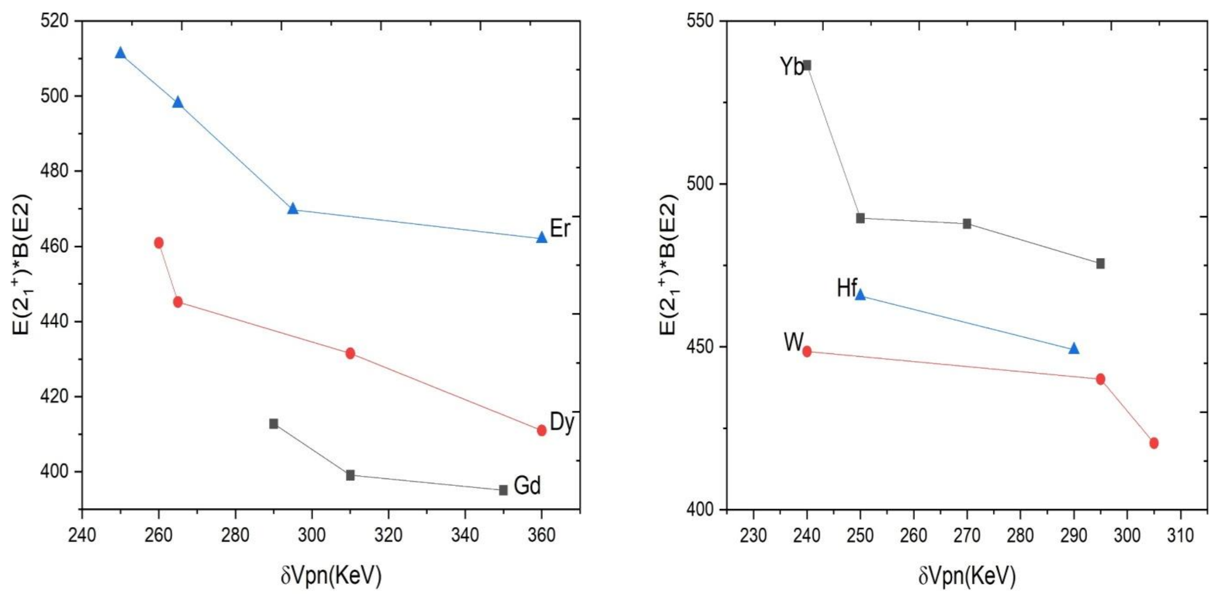


**Figure -1: $\delta V_{pn}$ values to the systematic of $E(2_1^+) * B(E2)$ in N=92-102 and N=96-108 region.**

Comparing $\delta V_{pn}$ values to the systematics of $R_{4/2}$ illustrates most dramatically the significance of high p-n interactions in causing evolving collectivity. The particularly useful plot of $\delta V_{pn}$ against $R_{4/2}$ is plotted in the lower panel of Figure 2. The calculations show an upward slope that is consistent with the empirical $\delta V_{pn}$ values for the heavier isotopes (Er–W). Even the dropoff in $\delta V_{pn}$ is reproduced in circumstances where the data go as far as sufficiently large neutron quantities. The two main differences are that Dy and Er predicted $\delta V_{pn}$ values peak a little early and that Gd, Yb, and Hf do not experience a falloff.

As soon as $R_{4/2}$ reaches 3.33, saturation is obtained. This indicates that collectivity develops more quickly in the p-p zone than in the p-h zone. Also in the figure 2, Er and Yb exhibits higher value of $R_{4/2}$, and maximum value of delta proton - neutron interaction that will results into higher growth rates of these two nuclei. Cakirli and Casten in 2006 [7] concurred with this as well. This effect should be seen in $\delta V_{pn}$ levels if the valence nucleon interaction is actually the main factor behind configuration mixing and collectivity. Greater $\delta V_{pn}$ values than in the PH region can be obtained in the PP region. Additionally, this is a logical outcome of the fractional filling concept that was just discussed: $\delta V_{pn}$ values are high when nucleons fill similar regions and low in opposed parts [19]. The actions of nuclear collectivity and the empirical intensities of valence nucleon interactions are thus, for the first time, truly connected in this image.

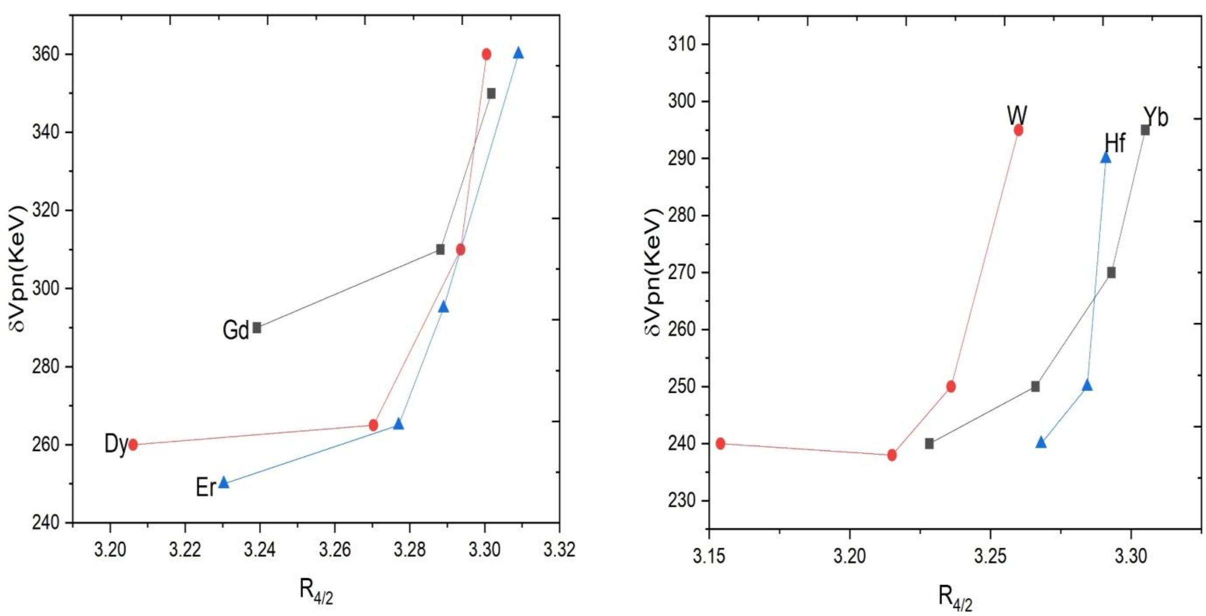


**Figure -2 : $\delta V_{pn}$ values to the systematic of $R_{4/2}$ in N=92-102 and N=96-108 region.**

In Figure 3, the evolution of $\delta V_{pn}$ over the full Z = 64–74, N =92–108 major shell is depicted and plotted against $N_pN_n$, the sum of the valence nucleons counted to the closest closed shell. The $N_pN_n$ approach essentially assumes that the orbit does not affect the p-n interaction. It's crucial to remember that the offered statement is oversimplified. The orientations of each orbit with regard to the nuclear equatorial plane determine how much the nucleon wave functions overlap, especially in a distorted field [22]. The fact that $\delta V_{pn}$ initially shows a linear relationship with $N_pN_n$ and thereafter increases at a slower rate provides empirical evidence for the $N_pN_n$ system and its subsequent improvements. With an increase in the product of valence nucleons, delta p-n interaction also increases, this will alter in the growth rates of nuclei. which will have an impact on the nuclear structure's overall stability and binding energy. Dy and Yb nuclei exhibits higher growth rates due to this sharp improvement in curve is obtained. This fact also serves as a foundation for model computations of nucleon interactions and emphasizes the implication of the monopole p-n interaction.

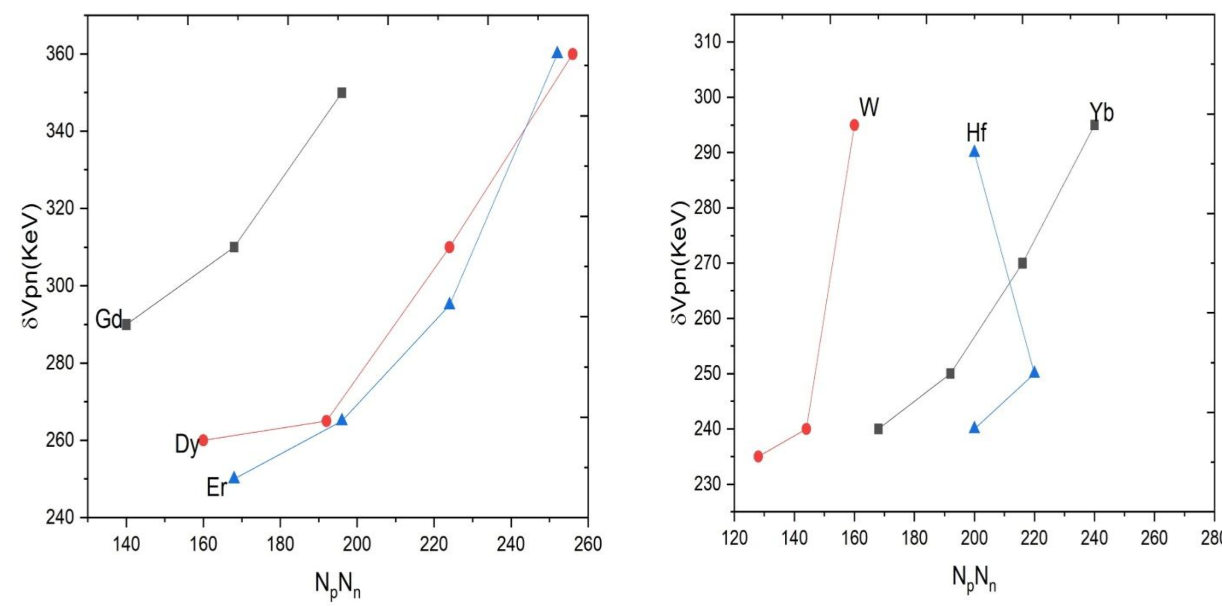


**Figure-3 : $\delta V_{pn}$ values to the systematic of $N_pN_n$ in N=92-102 and N=96-108 region.**

## III. CONCLUSION

We conducted a comprehensive investigation into the variation of the $\delta V_{pn}$ parameter, which is obtained from four masses, to estimate the average empirical strength of the nucleon interaction. Our calculations made use of a revised mass table with numerous new and enhanced experimental mass data points. In the context of symbiotic nuclei, regions with dependable shell closures, and regions with shape changes, the findings of this work are particularly noteworthy. The growth of nucleon orbital overlaps was taken into account in order to explain the establishment of a systematic split into two branches close to closed shells. The last proton and last neutron involved in empirical proton-neutron interactions have a highly distinct and recurrent pattern of parallel tracks with regard to the proton number. In order to further explore this, we computed the neucleon interaction matrix components using a zero-range interaction model and changed them into a single-particle foundation that is spherical. This research enabled us to test the hypothesis that the neutron orbits, particularly in Nilsson orbitals, show a growing amount of overlap with mid-shell proton orbits as the neutron number N rises from 92 to the mid-neutron shell. A constant deformation model also produced a fair general agreement with the observations for elements including erbium (Er), ytterbium (Yb), hafnium (Hf), and tungsten (W).

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