



BINARY GENERALIZED STAR SEMI CLOSED SET IN BINARY TOPOLOGICAL SPACES

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Article History: Received: 19.05.2023

Revised: 28.06.2023

Accepted: 24.07.2023

Abstract

In this paper focuss on binary g^* s-closed in binary topological spaces and certain properties of these investigated. Further, we have given an appropriate examples to understand the abstract concepts clearly.

Keywords: binary g -closed set, binary g^* -closed set and binary g^* s-closed set

Subjclass: [2010]54C05, 54C08, 54C10.

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DOI: 10.31838/ecb/2023.12.s2.399

1. Introduction and Preliminaries

In 1970 Levine [5] gives the concept and properties of generalized closed (briefly g -closed) sets and the complement of g -closed set is said to be g -open set. Njasted [14] introduced and studied the concept of α -sets. Later these sets are called as α -open sets in 1983. Mashhours et.al [8] introduced and studied the concept of α -closed sets, α -closure of set, α -continuous functions, α -open functions and α -closed functions in topological spaces. Maki et.al [6, 7] introduced and studied generalized

α -closed sets and α -generalized closed sets. In 2011, S.Nithyanantha Jothi and P.Thangavelu [9] introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from X to Y which is defined to be the ordered pairs (A, B) where $A \subseteq X$ and $B \subseteq Y$. In this paper focus on binary g^* -closed in binary topological spaces and certain properties of these investigated. Further, we have given an appropriate examples to understand the abstract concepts clearly.

Throughout this paper, (X, Y) denote binary topological spaces (X, Y, \mathcal{M}) .

Let X and Y be any two nonempty sets. A binary topology [9] from X to Y is a binary structure $\mathcal{M} \subseteq \mathbb{P}(X) \times \mathbb{P}(Y)$ that satisfies the axioms namely

1. (ϕ, ϕ) and $(X, Y) \in \mathcal{M}$,
2. $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$, and
3. If $\{(A_\alpha, B_\alpha) : \alpha \in \delta\}$ is a family of members of \mathcal{M} , then $(\bigcup_{\alpha \in \delta} A_\alpha, \bigcup_{\alpha \in \delta} B_\alpha) \in \mathcal{M}$.

If \mathcal{M} is a binary topology from X to Y then the triplet (X, Y, \mathcal{M}) is called a binary topological space and the members of \mathcal{M} are called the binary open subsets of the binary topological space (X, Y, \mathcal{M}) . The elements of $X \times Y$ are called the binary points of the binary topological space (X, Y, \mathcal{M}) . If $Y = X$ then \mathcal{M} is called a binary topology on X in which case we write (X, \mathcal{M}) as a binary topological space.

Definition 1.1 [9] Let X and Y be any two nonempty sets and let (A, B) and $(C, D) \in \mathbb{P}(X) \times \mathbb{P}(Y)$. We say that $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.

Definition 1.2 [9] Let (X, Y, \mathcal{M}) be a binary topological space and $A \subseteq X$, $B \subseteq Y$. Then (A, B) is called binary closed in (X, Y, \mathcal{M}) if $(X \setminus A, Y \setminus B) \in \mathcal{M}$.

Proposition 1.3 [9] Let (X, Y, \mathcal{M}) be a binary topological space and $(A, B) \subseteq (X, Y)$.

Let $(A, B)^{1*} = \bigcap \{A_\alpha : (A_\alpha, B_\alpha) \text{ is binary closed and } (A, B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \bigcap \{B_\alpha : (A_\alpha, B_\alpha) \text{ is binary closed and } (A, B) \subseteq (A_\alpha, B_\alpha)\}$. Then $((A, B)^{1*}, (A, B)^{2*})$ is binary closed and $(A, B) \subseteq ((A, B)^{1*}, (A, B)^{2*})$.

Proposition 1.4 [9] Let (X, Y, \mathcal{M}) be a binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B)^{1*} = \bigcup \{A_\alpha : (A_\alpha, B_\alpha) \text{ is binary open and } (A_\alpha, B_\alpha) \subseteq (A, B)\}$ and $(A, B)^{2*} = \bigcup \{B_\alpha : (A_\alpha, B_\alpha) \text{ is binary open and } (A_\alpha, B_\alpha) \subseteq (A, B)\}$.

Definition 1.5 [9] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ is called the binary closure of (A, B) , denoted by $b\text{-cl}(A, B)$ in the binary space (X, Y, \mathcal{M}) where $(A, B) \subseteq (X, Y)$.

Definition 1.6 [9] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ defined in proposition 1.4 is called

the binary interior of (A, B) , denoted by $b\text{-int}(A, B)$. Here $((A, B)^{1*}, (A, B)^{2*})$ is binary open and $((A, B)^{1*}, (A, B)^{2*}) \subseteq (A, B)$.

Definition 1.7 [9] Let (X, Y, \mathcal{M}) be a binary topological space and let $(x, y) \subseteq (X, Y)$. The binary open set (A, B) is said to be a binary neighbourhood of (x, y) if $x \in A$ and $y \in B$.

Proposition 1.8 [9] Let $(A, B) \subseteq (C, D) \subseteq (X, Y)$ and (X, Y, \mathcal{M}) be a binary topological space. Then, the following statements hold:

1. $b\text{-int}(A, B) \subseteq (A, B)$.
2. If (A, B) is binary open, then $b\text{-int}(A, B) = (A, B)$.
3. $b\text{-int}(A, B) \subseteq b\text{-int}(C, D)$.
4. $b\text{-int}(b\text{-int}(A, B)) = b\text{-int}(A, B)$.
5. $(A, B) \subseteq b\text{-cl}(A, B)$.
6. If (A, B) is binary closed, then $b\text{-cl}(A, B) = (A, B)$.
7. $b\text{-cl}(A, B) \subseteq b\text{-cl}(C, D)$.
8. $b\text{-cl}(b\text{-cl}(A, B)) = b\text{-cl}(A, B)$.

Definition 1.9 A subset (A, B) of a binary topological space (X, Y, \mathcal{M}) is called

1. a binary semi open set [13] if $(A, B) \subseteq b\text{-cl}(b\text{-int}(A, B))$.
2. a binary pre open set [3] if $(A, B) \subseteq b\text{-int}(b\text{-cl}(A, B))$,
3. a binary regular open set [12] if $(A, B) = b\text{-int}(b\text{-cl}(A, B))$.

Definition 1.10 A subset (A, B) of a binary topological space (X, Y, \mathcal{M}) is called

1. a binary g -closed set [10] if $b\text{-cl}(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary open.
2. a binary gs -closed set [15] if $b\text{-scl}(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary open.
3. a binary sg -closed set [15] if $b\text{-scl}(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary semi open.

Definition 1.11 [2] Let (A, B) be a subset of a binary topological space (X, Y) . Then (A, B) is called a binary g^* -closed set if $b\text{-cl}(A, B) \subseteq (P, Q)$ whenever $(A, B) \subseteq (P, Q)$ and (P, Q) is binary g -open in (X, Y) .

2 Binary g^* s-closed sets

Definition 2.1 A subset (A, B) of a binary topological space (X, Y, \mathcal{M}) is called binary generalized star semi closed sets (briefly binary g^* s-closed) if $bs\text{-cl}(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary g -open in (X, Y) .

Theorem 2.2 Every binary closed set is binary g^* s-closed set.

Proof. Let (A, B) be a binary closed set of (X, Y) and $(A, B) \subseteq (S, T)$, (S, T) is binary g -open in (X, Y) . Since (A, B) is binary closed, $b\text{-cl}(A, B) = (A, B)$. So $(A, B) \subseteq (S, T)$ implies $bs\text{-cl}(A, B) \subseteq (S, T)$. But $bs\text{-cl}(A, B) \subseteq b\text{-cl}(A, B)$ implies $bs\text{-cl}(A, B) \subseteq (S, T)$, $(A, B) \subseteq (S, T)$, (S, T) is binary g -open in (X, Y) .

Therefore (A, B) is binary g^* s-closed set.

Example 2.3 Let $X = \{a, b\}$, $Y = \{1, 2\}$ and $\mathcal{M} = \{(\emptyset, \emptyset), (\emptyset, \{1\}), (\{a\}, \{1\}), (X, Y)\}$. Then the set $(\{a\}, \{2\})$ is binary g^* s-closed set but not binary closed.

Theorem 2.4 Every binary g^* -closed set is binary g^* s-closed set.

Proof. Let (A, B) be a binary g^* -closed set of (X, Y) and $(A, B) \subseteq (S, T)$, (S, T) is binary g -open in (X, Y) . Since (A, B) is binary g^* -closed, $b\text{-cl}(A, B) = (A, B)$. So $(A, B) \subseteq (S, T)$ implies $b\text{-cl}(A, B) \subseteq (X, Y)$. But $bs\text{-cl}(A, B) \subseteq b\text{-cl}(A, B)$ implies $bs\text{-cl}(A, B) \subseteq (S, T)$, $(A, B) \subseteq (S, T)$, (S, T) is binary g -open in (X, Y) . Therefore (A, B) is binary g^* s-closed set.

Example 2.5 Let $X = \{a, b\}$, $Y = \{1, 2\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{1\}), (\{a\}, \{1\}), (\{b\}, \{1\}), (X, \{1\}), (X, Y)\}$. Then the set $(\{b\}, \phi)$ is binary g^* s-closed set but not binary g^* -closed set.

Theorem 2.6 Every binary g^* s-closed set is binary gs -closed set.

Proof. Let (A, B) be a binary g^* s-closed set of (X, Y) and let (S, T) be binary open set such that $(A, B) \subseteq (S, T)$. Since every binary open set is binary g -open, and (A, B) is binary g^* s-closed, we have $bs\text{-cl}(A, B) \subseteq (S, T)$. Therefore (A, B) is binary gs -closed set.

Example 2.7 Let $X = \{a, b, c\}$, $Y = \{1, 2\}$ and $\mathcal{M} = \{(\phi, \phi), (\{a\}, \{1\}), (\{b\}, \phi), (\{b\}, \{2\}), (\{a, b\}, \{1\}), (\{a, b\}, Y), (X, Y)\}$. Then the set $(\{a\}, \phi)$ is binary gs -closed set but not in binary g^* s-closed.

Remark 2.8 The notion binary g^* s-closed sets and binary g -closed sets are independent.

Example 2.9 In Example 2.3, the set $(\{a\}, \phi)$ is binary g^* s-closed set but not binary g -closed set. Then the set $(\{b\}, \{1\})$ is binary g -closed set but not binary g^* s-closed set.

Theorem 2.10 The union of two binary g^* s-closed sets in (X, Y, \mathcal{M}) is also a binary g^* s-closed sets in (X, Y, \mathcal{M}) .

Proof. Let (A, B) and (C, D) be two binary g^* s-closed sets in (X, Y, \mathcal{M}) . Let (S, T) be a binary g -open set in (X, Y) such that $(A, B) \subseteq (S, T)$ and $(C, D) \subseteq (S, T)$. Then we have $(A, B) \cup (C, D) \subseteq (S, T)$. As (A, B) and (C, D) are binary g^* s-closed sets in (X, Y, \mathcal{M}) . $bs\text{-cl}(A, B) \subseteq (S, T)$ and $bs\text{-cl}(C, D) \subseteq (S, T)$. Now $bs\text{-cl}((A, B) \cup (C, D)) = bs\text{-cl}(A, B) \cup bs\text{-cl}(C, D) \subseteq (S, T)$. Thus we have $bs\text{-cl}((A, B) \cup (C, D)) \subseteq (S, T)$ whenever $((A, B) \cup (C, D)) \subseteq (S, T)$, (S, T) is binary g -open set in (X, Y, \mathcal{M}) which implies $(A, B) \cup (C, D)$ is a binary g^* s-closed set in (X, Y, \mathcal{M}) .

Theorem 2.11 Let (A, B) be a binary g^* s-closed subset of (X, Y, \mathcal{M}) . If $(A, B) \subseteq (C, D) \subseteq bs\text{-cl}(A, B)$, then (C, D) is also binary g^* s-closed subset of (X, Y, \mathcal{M}) .

Proof. Let $(C, D) \subseteq (S, T)$ where (S, T) is binary g -open in (X, Y) . Then $(A, B) \subseteq (C, D)$ implies $(A, B) \subseteq (S, T)$. Since (A, B) is binary g^* s-closed, $bs\text{-cl}(A, B) \subseteq (S, T)$. Also $(C, D) \subseteq bs\text{-cl}(A, B)$ implies $bs\text{-cl}(C, D) \subseteq bs\text{-cl}(A, B)$. Thus $bs\text{-cl}(C, D) \subseteq (S, T)$ and so (C, D) is binary g^* s-closed.

Theorem 2.12 Let (A, B) be binary g^* s-closed in (X, Y, \mathcal{M}) . Then $bs\text{-cl}(A, B) - (A, B)$ has no non-empty binary g -closed set.

Proof. Let (A, B) be binary g^* s-closed sets in (X, Y, \mathcal{M}) , and (E, F) be binary g -closed subset of $bs\text{-cl}(A, B) - (A, B)$. That is $(E, F) \subseteq bs\text{-cl}(A, B) - (A, B)$, which implies that $(E, F) \subseteq bs\text{-cl}(A, B) - (A, B)$. That is $(E, F) \subseteq bs\text{-cl}(A, B)$ and $(E, F) \subseteq (A, B)^c$. $(E, F) \subseteq (A, B)^c$ implies that $(a, b) \subseteq (E, F)^c$ where $(E, F)^c$ is a binary g -open set. Since (A, B) is binary g^* s-closed, $bs\text{-cl}(A, B) \subseteq (E, F)^c$. That is $(E, F) \subseteq (bs\text{-cl}(A, B))^c$. Thus $(E, F) \subseteq bs\text{-cl}(A, B) \cap (bs\text{-cl}(A, B))^c$. Therefore $(E, F) = (\phi, \phi)$.

Theorem 2.13 The intersection of any two binary g^* s-closed sets in (X, Y, \mathcal{M}) is also a binary g^* s-closed sets in (X, Y, \mathcal{M}) .

Proof. Let (A, B) and (C, D) be two binary g^* s-closed sets in (X, Y, \mathcal{M}) . Let (S, T) be a binary g -open set in (X, Y) such that $(A, B) \subseteq (S, T)$ and $(C, D) \subseteq (S, T)$. Then $bs-cl(A, B) \subseteq (S, T)$ and $bs-cl(C, D) \subseteq (S, T)$. Therefore $bs-cl((A, B) \cap (C, D)) \subseteq (S, T)$, (S, T) is binary g -open set in (X, Y, \mathcal{M}) . Since (A, B) and (C, D) are binary g^* s-closed sets. Hence $(A, B) \cap (C, D)$ is a binary g^* s-closed set in (X, Y, \mathcal{M}) .

Example 2.14 Let $X = \{a, b\}$, $Y = \{1, 2\}$ and $\mathcal{M} = \{\emptyset, \emptyset, (\emptyset, \{1\}), (\{a\}, \{1\}), (X, Y)\}$. Then the binary g^* s-closed sets are $\{(\emptyset, \emptyset), (\emptyset, \{2\}), (\{a\}, \emptyset), (\{a\}, \{2\}), (\{b\}, \emptyset), (\{b\}, \{2\}), (\{b\}, Y), (X, \emptyset), (X, \{2\}), (X, Y)\}$. Let $A = (\{a\}, \{2\})$ and $B = (\{b\}, \{2\})$ and $A \cap B = (\emptyset, \{2\})$ is also binary g^* s-closed set.

Theorem 2.15 Every binary g^* s-closed set is binary sg -closed set.

Proof. Let (A, B) be a binary g^* s-closed set of (X, Y) and let (S, T) be binary open set such that $(A, B) \subseteq (S, T)$. Since every binary semi-open set such that $(A, B) \subseteq (S, T)$. Since every binary semi-open set is binary g -open set and (A, B) is binary g^* s-closed, we have $bs-cl(A, B) \subseteq (S, T)$. Therefore (A, B) is binary sg -closed set.

Example 2.16 In Example 2.7, the set $(\emptyset, \{1\})$ is binary sg -closed set but not binary g^* s-closed set.

Remark 2.17 From the above discussions we have the following implications.

binary-closed \rightarrow binary g^* -closed \rightarrow binary g^* s-closed \rightarrow binary sg -closed \rightarrow binary gs -closed

3 Binary g^* s-open sets

Definition 3.1 A subset (A, B) of a binary topological space (X, Y, \mathcal{M}) is called binary generalized star semi-open (briefly, binary g^* s-open), if $(A, B)^c$ is also binary g^* s-closed.

Theorem 3.2

1. Every binary open set is binary g^* s-open.
2. Every binary g^* -open set is binary g^* s-open.

Proof. Proof follows from the Theorem 2.4 and 2.6.

Remark 3.3 For a subset a of a binary topological space (X, Y, \mathcal{M}) .

1. $\cup bg^*s-int(A, B) = bg^*s-cl((X, Y) - (A, B))$
2. $\cap bg^*s-int(A, B) = bg^*s-cl((X, Y) - (A, B))$.

Theorem 3.4 A subset $(A, B) \subseteq (X, Y)$ is binary g^* s-open iff $(E, F) \subseteq bs-int(A, B)$ whenever (E, F) is binary g -closed set and $(E, F) \subseteq (A, B)$.

Proof. Let (A, B) be binary g^* s-open set and suppose $(E, F) \subseteq (A, B)$ where (E, F) is binary g -closed. Then $(X, Y) - (A, B)$ is binary g^* s-closed set contained in the binary g -open set $(X, Y) - (E, F)$. Hence $bs-cl((X, Y) - (A, B)) \subseteq ((X, Y) - (E, F))$ and $(X, Y) - bs-int(A, B) \subseteq ((X, Y) - (E, F))$. Thus $(E, F) \subseteq bs-int(A, B)$.

Conversely, if (E, F) is binary g -closed set with $(E, F) \subseteq bs-int(A, B)$ and $(E, F) \subseteq (A, B)$. Then $(X, Y) - bs-int(A, B) \subseteq ((X, Y) - (E, F))$. Thus $bs-cl((X, Y) - (A, B)) \subseteq ((X, Y) - (E, F))$. Hence $((X, Y) - (A, B))$ is binary g^* s-closed set and (A, B) is binary

g^* s-open set.

Theorem 3.5 If $bs\text{-int}(A, B) \subseteq (C, D) \subseteq (A, B)$ and if (A, B) is binary g^* s-open, then (C, D) is binary g^* s-open.

Proof. Let $bs\text{-int}(A, B) \subseteq (C, D) \subseteq (A, B)$, then $(A, B)^c \subseteq (C, D)^c \subseteq bs\text{-cl}(A, B)^c$, where $(A, B)^c$ is binary g^* s-closed and hence $(C, D)^c$ is also binary g^* s-closed by Theorem 2.11. Therefore, (C, D) is binary g^* s-open.

Remark 3.6 If (A, B) and (C, D) are binary g^* s-open subsets of a binary topological space (X, Y) , then $(A, B) \cup (C, D)$ is also binary g^* s-open in (X, Y) , as seen from the following example.

Example 3.7 Let $X = \{a, b\}$, $Y = \{1, 2\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{2\}), (\{a\}, \{1\}), (\{a\}, Y), (X, \{1\}), (X, Y)\}$. Then the binary g^* s-open sets are $\{(\phi, \phi), (\phi, \{1\}), (\phi, \{2\}), (\phi, Y), (\{a\}, \phi), (\{a\}, \{1\}), (\{a\}, \{2\}), (\{a\}, Y), (X, \{1\}), (X, Y)\}$. Let $A = (\phi, \{1\})$ and $B = (\{a\}, \{2\})$ are binary g^* s-open sets, then $A \cup B = (\phi, \{1\}) \cup (\{a\}, \{2\}) = (\{a\}, Y)$ is also binary g^* s-open sets.

Theorem 3.8 If (A, B) is binary g^* s-closed, then $b\text{-cl}(A, B) - (A, B)$ is binary g^* s-open.

Proof. Let (A, B) be binary g^* s-closed. Let (E, F) be binary g -closed such that $(E, F) \subseteq bs\text{-cl}(A, B) - (A, B)$. Then $(E, F) = (\phi, \phi)$. Since $bs\text{-cl}(A, B) - (A, B)$ cannot have any non-empty binary g -closed set. Therefore, $(E, F) \subseteq bs\text{-int}(bs\text{-cl}(A, B) - (A, B))$. Hence $bs\text{-cl}(A, B) - (A, B)$ is binary g^* s-open.

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