AN EXTENSION OF USING JEFFREY'S EARLIER TECHNIQUE(A BAYESIAN METHOD FOR ESTIMATION OF GENERALIZED GAMMA DISTRIBUTION)

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Abstract: -This work has revealed some structural properties of the Generalized Gamma Distribution (GGD). The parameters of GGD have been calculated using the Jeffrey's & extension of Jeffrey's priors using the Bayesian technique under four different loss functions. The estimate so obtained was compared with the conventional Maximum Likelihood Estimator using MSE using simulated studies with varied sample sizes and R software. The survival function equation has been added to Jeffrey's earlier research.

Keywords: -Likelihood, Bayesian, GR Software, Jeffrey's Prior

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1. Introduction:

A flexible family is needed to model duration, and one such example is Stacy's (1962) Generalized Gamma Distribution (GGD). The GGD offers a wide range of various shape and hazard functions. (2021) Seufert Mean Percentage Score (MOS)-based assessments produce problematic and deceptive outcomes because inquiries into subjective Quality of Experience (QoE) use biased assumptions about ordinal rating scale.Economic duration analysis uses a variety of distributions, including the GGD subfamilies of exponential (Kiefer, 1984), gamma (Lancaster, 1979), and Weibull (Favero et al., 1994). The lognormal distribution, which is regarded to be a limiting distribution, was also employed by Jasggia in economics (1991). To estimate the posterior distribution of GGD, Jeffreys' prior distribution is expanded in this section of the essay. We have investigated several different loss functions to obtain a precise evaluation of the scale parameter of GGD. Research on the precise small and large eccentricity asymptotic in non-logarithmic method for linear procedures with self-governing innovations was done by Peligrad (2018). We will refer to the linear processes that we study as the "long memory case" because they are universal in nature. By using the likelihood of a single observation and its ordinates, Savsani and Ghosh (2017) devised a method for determining the posterior distribution of the Moderate Distribution. The 2017 study by

Savsani and Ghosh served as the basis for this approach. Mishra and others (2019) for the processing and classification of data, epidemiological and statistical methods are available. These approaches can be used in every single unique situation. In this post, we discussed both parametric and non-parametric approaches, their prerequisites, how to pick the best statistical measurement and analysis, and how to interpret biological data. Senators Costa and Sarmento (2019) Over the past few years, using statistical software in both professional and academic settings have grown in popularity. Everyone has used statistical software at some point in their lives, from students and professors to specialists and everyday consumers. In this paper, we provide a statistical assessment of several theoretical notions to facilitate access to these concepts.

1.1 Posterior density by using postponement of Jeffrey's previous.

Let (X_1,X_2,……, X_n) be an n-piece random model that consumes the probability density function as

$$f(x;\lambda,\beta,k)=\frac{\lambda\beta}{\Gamma k}(\lambda x)^{k\beta-1}e^{-(\lambda x)^{\beta}},\text{ for } x>0 \text{ and } \lambda,\beta,k>0.$$

Given by is the probability function.

$$L(x;\lambda,\beta,k)=\frac{\lambda^{n(k\beta)}\beta^{n}}{\Gamma^{n}(k)}\prod_{i=1}^{n} x_i^{k\beta-1}e^{-\lambda^{\beta}\sum_{i=1}^{n}x_i}.$$

We study the previousdelivery of λbeing.

$$g(\lambda)\propto[\det\,|I(\lambda)|]^{c}, c\in R^{+}$$

$$g(\lambda)=\rho\frac{1}{\lambda^{2c}} \qquad (1)$$

where ρ is constant. The following distribution of λ is known by

$$\pi_2(\lambda\mid \underline{x})\propto L(x\mid\lambda)g(\lambda) \qquad (2)$$

Using eq. (1) in eq. (2), we get

$$\pi_2(\lambda\mid \underline{x})\propto\frac{\lambda^{nk\beta-2c}\beta^{n}}{\Gamma^{n}(k)}e^{-\lambda^{\beta}\sum_{i=1}^{n}x_i^{\beta}}\prod_{i=1}^{n}(x_i)^{k\beta-1}$$

$$\pi_2(\lambda\mid \underline{x})=\rho_2\lambda^{nk\beta-2c}e^{-\lambda^{\beta}\sum_{i=1}^{n}x_i^{\beta}} \qquad (3)$$

where ρ_2 is independent of λ .

$$\rho_2^{-1}=\int_0^{\infty}\lambda^{nk\beta-2c}e^{-\lambda^{\rho}\sum_{i=1}^{n}x_i^{\beta}}d\lambda$$

On resolving the above appearance, we get

$$\rho_2 = \frac{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{nk-\frac{2c}{\beta}+1}}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}$$

calculating ρ2 in eq. (3), we get the posterior distribution given as below.

$$\pi_2(\lambda \mid \underline{x}) = \left(\frac{e^{-\lambda^{\beta}\sum_{i=1}^{n} x_i}\lambda^{nk\beta-2c}\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{nk-\frac{2c}{\beta}+1}}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\right) \quad (4)$$

1.2 Squared error loss estimate Function (SELF)

Squared error loss $l_{SI}(\hat{\lambda},\lambda) = a(\hat{\lambda}-\lambda)^2$for some constant a , the risk function is

$$R(\hat{\lambda}) = \int_0^{\infty} a(\hat{\lambda}-\lambda)^2 \pi_2(\lambda \mid \underline{x})d\lambda \quad (5)$$

By using eq. (4) in eq. (5), we take

$$R(\hat{\lambda}) = \int_0^{\infty} a(\hat{\lambda}-\lambda)^2 \left(\frac{e^{-\lambda^{\beta}\sum_{i=1}^{x_i} x_i}\lambda^{nk\beta-2c}\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{nk-\frac{2c}{\beta}+1}}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}d\lambda\right.$$

$$R(\hat{\lambda}) = \frac{a\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{nk-\frac{2c}{\beta}+1}}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\left[\hat{\lambda}^2\int_0^{\infty} e^{-\lambda^{\beta}\sum_{i=1}^{n} x_i^{\beta}}\left(\lambda^{\beta}\right)^{nk-\frac{2c}{\beta}}d\lambda + \int_0^{\infty} e^{-\lambda^{\beta}\sum_{i=1}^{n} x_i^{\beta}}\left(\lambda^{\beta}\right)^{nk+\frac{2(1-c)}{\beta}}d\lambda - 2\hat{\lambda}\int_0^{\infty} e^{-\lambda^{\beta}\sum_{i=1}^{n} x_i}\left(\lambda^{\beta}\right)^{nk+\frac{(1-2c)}{\beta}}d\lambda\right]$$

Resolving the above appearance, we have

$$R(\hat{\lambda}) = a\hat{\lambda}^2 + a\frac{\Gamma\left(nk-\frac{2c}{\beta}+\frac{2}{\beta}+1\right)}{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{\frac{2}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)} - \frac{2a\hat{\lambda}\Gamma\left(nk-\frac{2c}{\beta}+\frac{1}{\beta}+1\right)}{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{\frac{1}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}$$

Now togain Bayesian estimator, we take$\frac{\partial R(\hat{\lambda})}{\partial\hat{\lambda}} = 0$

$$\frac{\partial}{\partial\hat{\lambda}}\left[a\hat{\lambda}^2 + \frac{a\Gamma\left(nk-\frac{2c}{\beta}+\frac{2}{\beta}+1\right)}{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{\frac{2}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)} - \frac{2a\hat{\lambda}\Gamma\left(nk-\frac{2c}{\beta}+\frac{1}{\beta}+1\right)}{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{\frac{1}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\right] = 0$$

$$\hat{\lambda} = \frac{1}{\left(\sum_{i=1}^{n} x_i\right)}\left\{\frac{\Gamma\left(nk-\frac{2c}{\beta}+\frac{1}{\beta}+1\right)}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\right\} \quad (6)$$

Remark: Replacing c=1/2 in eq. (6), the same Bayes estimation is got as in eq. (2) equivalent to the Jeffrey's prior.

1.3 Estimation Al-Bayyati's loss function (ALF):

Al-Bayyati's loss function $l_{Nl}(\hat{\lambda},\lambda) = \lambda^{c_1}(\hat{\lambda}-\lambda)^2$the risk function is given as

$$R(\hat{\lambda}) = \int_0^\infty \lambda^{c_1}(\hat{\lambda} - \lambda)^2 \pi_2(\lambda \mid \underline{x}) d\lambda \qquad (7)$$

Using eq. (4) in eq. (7), we devise

$$R(\hat{\lambda}) = \frac{a\left(\sum_{i=1}^n x_i^\beta\right)^{nk-\frac{2c}{\beta}+1}}{\Gamma\left(nk - \frac{2c}{\beta} + 1\right)} \left[\hat{\lambda}^2 \int_0^\infty e^{-\lambda^\beta \sum_{i=1}^\infty x_i'} \left(\lambda^\beta\right)^{nk+\frac{(c_1-2c)}{\beta}} d\lambda + \int_0^\infty e^{-\lambda^\beta \sum_{i=1}^\infty x_j'} \left(\lambda^\beta\right)^{nk+\frac{(c_1-2c+2)}{\beta}} d\lambda - 2\hat{\lambda} \int_0^\infty e^{-\lambda^\beta \sum_{i=1}^\infty x_i} \left(\lambda^\beta\right)^{nk+\frac{(c_1-2c+1)}{\beta}} d\lambda \right]$$

Solving the above expression, we have

$$R(\hat{\lambda}) = \frac{\hat{\lambda}^2 \Gamma\left(nk + \frac{c_1}{\beta} - \frac{2c}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{9}{\beta}} \Gamma\left(nk - \frac{2c}{\beta} + 1\right)} + \frac{\Gamma\left(nk - \frac{2c}{\beta} + \frac{c_1}{\beta} + \frac{2}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{902}{\beta}} \Gamma\left(nk - \frac{2c}{\beta} + 1\right)} - \frac{2\hat{\lambda}\Gamma\left(nk - \frac{2c}{\beta} + \frac{c_1}{\beta} + \frac{1}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{9+1}{\beta}} \Gamma\left(nk - \frac{1}{\beta} + 1\right)}.$$

Now togain Bayesian estimator, we take $\frac{\partial R(\hat{\lambda})}{\partial \hat{\lambda}} = 0 \mid$

$$\frac{\partial}{\partial \hat{\lambda}} \left[\frac{\hat{\lambda}^2 \Gamma\left(nk + \frac{c_1}{\beta} - \frac{2c}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{c_1}{\beta}} \Gamma\left(nk - \frac{2c}{\beta} + 1\right)} + \frac{\Gamma\left(nk - \frac{2c}{\beta} + \frac{c_1}{\beta} + \frac{2}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{c_1+2}{\beta}} \Gamma\left(nk - \frac{2c}{\beta} + 1\right)} - \frac{2\hat{\lambda}\Gamma\left(nk - \frac{2c}{\beta} + \frac{c_1}{\beta} + \frac{1}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{c_1+1}{\beta}} \Gamma\left(nk - \frac{1}{\beta} + 1\right)} \right] = 0$$

$$\hat{\lambda} = \frac{1}{\left(\sum_{i=1}^n x_i\right)} \left\{ \frac{\Gamma\left(nk - \frac{2c}{\beta} + \frac{c_1}{\beta} + \frac{1}{\beta} + 1\right)}{\Gamma\left(nk - \frac{2c}{\beta} + \frac{c_1}{\beta} + 1\right)} \right\} \qquad (8)$$

Remark: Exchanging c=1/2 in eq. (8), the same Bayes estimation is got as in eq. (6) equivalent to the Jeffrey's previous.

1.4 Precautionary Loss Function (PLF)

By using precautionary loss function $l_{pr}(\hat{\lambda}, \lambda) = \frac{(\hat{\lambda}-\lambda)^2}{\hat{\lambda}}$ the risk function is given

$$R(\hat{\lambda}) = \int_0^\infty \frac{(\hat{\lambda}-\lambda)^2}{\hat{\lambda}} \pi_2(\lambda \mid \underline{x}) d\lambda \qquad (9)$$

Using eq. (4) in eq. (9), we take

$$R(\hat{\lambda}) = \frac{\left(\sum_{i=1}^n x_i^\beta\right)^{nk-\frac{2c}{\beta}+1}}{\hat{\lambda}\Gamma\left(nk - \frac{2c}{\beta} + 1\right)} \left[\left[\int_0^\infty e^{-\lambda^\rho \sum_{i=1}^n x_i} \left(\lambda^\beta\right)^{nk+\frac{2(1-c)}{\beta}} d\lambda + \hat{\lambda}_0^\infty e^{-\lambda^\rho \sum_{i=1}^n x_i^\beta} \left(\lambda^\beta\right)^{nk-\frac{2c}{\beta}} d\lambda \right] - 2\int_0^\infty e^{-\lambda^\beta \sum_{i=1}^\infty x_i} \left(\lambda^\beta\right)^{nk+\frac{(1-2c)}{\beta}} d\lambda \right]$$

On solving the above expression, we have

$$R(\hat{\lambda}) = \frac{\Gamma\left(nk - \frac{2c}{\beta} + \frac{2}{\beta} + 1\right)}{\hat{\lambda}\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{2}{\beta}} \Gamma\left(nk - \frac{2c}{\beta} + 1\right)} + \hat{\lambda} - \frac{2\Gamma\left(nk - \frac{2c}{\beta} + \frac{1}{\beta} + 1\right)}{\left(\sum_{i=1}^n x_i^\beta\right)^{\frac{1}{\beta}} \Gamma\left(nk - \frac{2c}{\beta} + 1\right)}.$$

Now to determine the Bayesian estimator, we have $\frac{\partial R(\hat{\lambda})}{\partial \hat{\lambda}} = 0$

$$\frac{\partial}{\partial\hat{\lambda}}\left[\frac{\Gamma\left(nk-\frac{2c}{\beta}+\frac{2}{\beta}+1\right)}{\hat{\lambda}\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{2}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}+\hat{\lambda}-\frac{2\Gamma\left(nk-\frac{2c}{\beta}+\frac{1}{\beta}+1\right)}{\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{1}{\beta}}\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\right]=0 \qquad (10)$$

Remark: Replacing c=1/2 in eq. (10), the same Bayes estimator is got as in eq. (2) equivalent to the Jeffrey's prior

1.5 Quadratic Loss Function (QLF)

By using quadratic loss function $l_{qd}(\hat{\lambda},\lambda)=\left(\frac{\hat{\lambda}-\lambda}{\lambda}\right)^2$ the risk function is given by

$$R(\hat{\lambda})=\int_0^{\infty}\left(\frac{\hat{\lambda}-\lambda}{\lambda}\right)^2\pi_2(\lambda\mid\underline{x})d\lambda \qquad (11)$$

Using eq. (4) in eq. (11), we have

$$R(\hat{\lambda})=\int_0^{\infty}\left(\frac{\hat{\lambda}-\lambda}{\lambda}\right)^2\frac{e^{-\hat{\lambda}^{\beta}\sum_{i=1}^{x_i}}\lambda^{nk\beta-2c}\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{nk-\frac{2c}{\beta}+1}}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}d\lambda$$

Solving the above formula yields.

$$R(\hat{\lambda})=\hat{\lambda}^2\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{2}{\beta}}\frac{\Gamma\left(nk-\frac{2(c+1)}{\beta}+1\right)}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}+1-2\hat{\lambda}\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{1}{\beta}}\frac{\Gamma\left(nk-\frac{(2c+1)}{\beta}+1\right)}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}.$$

Now to determine the Bayesian estimator, we have $\frac{\partial R(\hat{\lambda})}{\partial\hat{\lambda}}=0$

$$\frac{\partial}{\partial\hat{\lambda}}\left[\frac{\hat{\lambda}^2\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{2}{\beta}}\Gamma\left(nk-\frac{2(c+1)}{\beta}+1\right)}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}+1-\frac{2\hat{\lambda}\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{\frac{1}{\beta}}\Gamma\left(nk-\frac{(2c+1)}{\beta}+1\right)}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\right]=0$$

$$\hat{\lambda}=\frac{1}{\left(\sum_{i=1}^{n}x_i\right)}\left[\frac{\left\{\Gamma\left(nk-\frac{(2c+1)}{\beta}+1\right)\right\}}{\left\{\Gamma\left(nk-\frac{2(c+1)}{\beta}+1\right)\right\}}\right] \qquad (12)$$

Remark: Exchanging c=1/2 in eq. (12), the same Bayes estimator is got as in eq. (2) equivalent to Jeffrey's prior.

1.6 Estimation of Survival function (SF)

We may determine the survival function by utilizing the posterior probability density function, such that.

$$\hat{S}_2(\underline{x})=\int_0^{\infty}e^{-(\lambda x)^{\beta}}\pi_2(\lambda\mid\underline{x})d\lambda \qquad (13)$$

$$\hat{S}_1(\underline{x})=\frac{\left(\sum_{i=1}^{n}x_i^{\beta}\right)^{nk-\frac{2c}{\beta}+1}}{\Gamma\left(nk-\frac{2c}{\beta}+1\right)}\left[\int_0^{\infty}e^{-(\lambda x)^{\beta}}e^{-\lambda^{\beta}\sum_{i=1}^{n}x_i^{\beta}}\lambda^{nk\beta-2c}d\lambda\right]$$

Using eq. (4) in eq. (13), we have

$$\hat{S}_2(\underline{x}) = \left(\frac{\sum_{i=1}^{n} x_i^{\beta}}{x_i^{\beta}+\sum_{i=1}^{n} x_i^{\beta}}\right)^{nk-\frac{2c}{\beta}+1}. \qquad (14)$$

2. SIMULATION STUDY OF GENERALIZED GAMMA DISTRIBUTION (GGD)

With the aid of the R programming language, a simulation study was conducted to examine and assess the precision of the estimates for three different sample sizes (n = 25, 50, and 100), which, respectively, represented a small, medium, and massive data collection. The value of the scale parameter for the generalized gamma distribution is calculated using both conventional and Bayesian approaches to estimate. We employ Jeffrey's & an extension of Jeffrey's former inside the context of the Bayesian technique of estimation while taking several loss functions into account. We examined the values of = 1.0, 1.5, & 2.0 while figuring out the scales parameter's value. The values of c in Jeffrey's extension were 0.5, 1.0, and 1.5. The c1 loss parameter has been determined to have the values 1, -1, 2, and -2. This procedure was performed 2,000 times after the scale parameter for each approach was calculated. The tables below provide a summary of the findings.

Table 1: Mean Squared Error for $\hat{\lambda}$ under Jeffrey's prior.

N	λ	β	κ	λ_{ML}	λ_{sl}	λ_{NI}			
						C1=1	C1=-1	C1=2	C1=-2
25	1.0	0.5	0.5	1.0834	1.0244	1.2256	0.9843	1.1135	0.5734
	1.5	1.0	1.0	0.6857	0.6852	0.6632	0.7305	0.6413	0.7305
	2.0	1.5	1.0	0.6823	0.6810	0.6786	0.6863	0.6760	0.6863
50	1.0	0.5	0.5	1.1597	1.2941	1.0642	1.0851	1.1223	0.4792
	1.5	1.0	1.0	0.4263	0.4261	0.4152	0.4485	0.4044	0.4485
	2.0	1.5	1.0	0.7523	0.7523	0.7513	0.7544	0.7502	0.7544
100	1.0	0.5	0.5	1.9086	1.9086	1.9019	1.2507	0.9669	0.1199
	1.5	1.0	1.0	0.8329	0.8329	0.8275	0.8437	0.8222	0.8437
	2.0	1.5	1.0	0.7775	0.7775	0.9631	0.9647	0.7764	0.8081

ML=MaximumProbability,Sl=squared mistakeLF,Pr=defensiveLF,qd=quadraticLF, Nl=Al-Bayyati'sLF

The Al-loss Bayes estimate in most cases, especially when the loss parameter C1 is set to -2 in the just-presented table 1, Bayyati's function and Jeffrey's prior produce the minimal results. As a result, we may conclude that the Bayes estimator performs well when Al-loss is applied the standard estimator and other loss functions are contrasted with Bayyati's function.

Table 2: Mean Squared Error for $\hat{\lambda}$ under extension of Jeffrey's prior.

N	λ	β	κ	**C**	λ_{ML}	λ_{sl}	λ_{pr}	λ_{qd}	λ_{NI}			
									C1=1	C1=-1	C1=2	C1=-2
25	1.0	0.5	0.5	0.5	0.5838	0.2311	0.2950	0.1948	0.2628	0.1948	0.3089	0.0571
				1.0	0.7738	0.2578	0.1734	0.3786	0.3251	0.1366	0.1491	0.0891
				1.5	0.1787	0.1868	0.5601	0.1028	0.3462	0.1028	0.2022	0.0753
	1.5	1.0	1.0	0.5	0.8879	0.8875	0.8666	0.9304	0.8456	0.9304	0.8047	0.6241
				1.0	0.8155	0.8586	0.8367	0.9037	0.8147	0.9037	0.7717	0.4614
				1.5	0.8769	0.9630	0.9411	1.0082	0.9191	0.9883	0.8761	0.2706
	2.0	1.5	1.0	0.5	1.1271	1.2578	1.3295	1.3889	1.5483	1.4604	1.2710	1.0525
				1.0	1.4098	1.1535	1.4532	1.4886	1.6567	1.2551	1.6148	1.075
				1.5	1.3308	1.4242	1.4725	1.2556	1.4919	1.2400	1.4484	1.0278
50	1.0	0.5	0.5	0.5	0.9244	0.4822	0.1015	0.4309	0.6661	0.4309	1.0517	0.1898
				1.0	0.9288	0.2332	1.0292	0.5212	0.2195	0.5212	0.3160	0.1965
				1.5	0.6407	0.8001	0.1095	0.2509	0.1676	0.7764	0.2780	0.1382
				0.5	0.84	0.844	0.833	0.8	0.8230	0.8657	0.802	0.709

	1.5	1.0	1.0		42	1	6	657			0	7
				1.0	1.0558	1.0752	1.0655	1.0949	1.0557	1.0949	1.0361	0.9147
				1.5	0.9252	0.9670	0.9565	0.9883	0.9460	0.9883	0.9250	0.8198
	2.0	1.5	1.0	0.5	1.8551	1.8551	1.8434	1.8783	1.8320	1.8783	1.8088	1.6360
				1.0	1.6263	1.6511	1.6388	1.6761	1.6263	1.6761	1.6017	1.2201
				1.5	1.6536	1.7030	1.6907	1.7281	1.6783	1.7281	1.6535	1.2641
100	1.0	0.5	0.5	0.5	1.4077	0.5084	0.7676	0.3731	0.1842	0.2499	0.5785	0.1400
				1.0	1.1779	0.4541	0.2198	0.1993	0.5784	0.8096	0.1011	0.0982
				1.5	1.7517	0.3001	0.6765	0.7517	0.1431	0.5004	0.3364	0.1322
	1.5	1.0	1.0	0.5	0.8333	0.8332	0.8278	0.4557	0.8225	0.8440	0.8118	0.3914
				1.0	0.9041	0.9146	0.9092	0.9251	0.9041	0.9251	0.8937	0.7525
				1.5	0.8830	0.9043	0.8990	0.9150	0.8935	0.9150	0.8830	0.7436
	2.0	1.5	1.0	0.5	1.6398	1.6397	1.6336	1.6521	1.6275	1.6521	1.6153	1.1885
				1.0	1.6123	1.6248	1.6184	1.6370	1.6123	1.6370	1.5998	1.0497
				1.5	1.5403	1.5654	1.5592	1.5782	1.5527	1.5782	1.5403	1.1252

Generally, especially when loss parameter C1 is -2, Bayes' estimate using Al-loss Bayyati's function under extension of Jeffrey's previous yields the lowest values, regardless of whether the extension of Jeffrey's prior is 0.5, 1.0, or 1.5, as shown in table 2 above. Particularly when loss parameter C1 is -2, this is true. It follows that Al-loss Bayyati's function provides a Bayes estimate that is superior to other loss functions and the conventional estimator.

3. Conclusion:

Our research focused on both conventional and Bayesian estimating techniques to compute the scaling parameter of the generalized gamma distribution. The variances between the estimations are examined using the Mean Squared Error (MSE) method, and the results are shown in the tables above. Considering the results, the Bayes estimator under Al-loss Bayyati's function has the lowest MSE values for both priors (Jeffrey's and the extension of Jeffrey's prior), when compared to other loss functions and classical estimation. In most instances, this is the case. As a result, we may conclude that the Al-loss Bayyati's function-based Bayes estimator is efficient when the loss parameter C1 is set to -2.

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