Section A-Research paper
ISSN 2063-5346

Odd Hexagonal Graceful Labeling on Some Graphs

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Abstract:

In this paper we have developed Hexagonal graceful labeling by including new conditions and it is called Odd Hexagonal Graceful Labeling. Let G be a graph and let its vertex set and edge set be v(G) and E(G) respectively. Let $f: V(G) \rightarrow \{0,1,2,\ldots,H_{2q-1}\}$ be an injective function, where q is the number of edges of G and H_n be the n[th] hexagonal number given by $H_n = \frac{2n(2n-1)}{2}$. If the function f induces the function f^* on E(G) such that $f^*(uv) = |f(u) - f(v)|$ for all the edges uv ∈E(G) with $f^*(E(G)) = \{H_1, H_3, H_5, \ldots, H_{2q-1}\}$ we say f is odd hexagonal graceful labeling and the graph which admits such a labeling is called an odd hexagonal graceful graph.

Keywords: Graceful labeling, Hexagonal graceful number, Hexagonal graceful labeling, odd Hexagonal graceful labeling, odd Hexagonal graceful graphs

AMS Subject Classification (2020):05C78

1.Introduction

In 1967 Rosa [1] introduced Graceful Labeling, and later he called it as β-labeling. The term "graceful" was due to Solomon.W.Golomb [1]. A Graceful labeling is a special type of graph labeling of a graph on edges in which nodes are labeled with a subset of distinct non-negative integers from 0 and the edges are labeled with the absolute difference between node values. A. Rama Lakshmi and M.P. Syed Ali Nisaya introduced polygonal graceful labeling [2] of some simple graphs. S.Selestin Lina and S.Asha [3] introduced even triangular graceful labeling on simple graphs. Inspired by their works the study undertaken in this paper involves odd hexagonal graceful labeling on some simple and special graphs. We review several key definitions that are relevant to the current study.

Definition:1.1

A function f is said to be graceful [1] of a graph G with q-edges if f is 1-1 from V → {0, 1, 2, …, q} such that for each edge uv assigned the label | f (u) – f (v) |, the resulting edge labels are distinct numbers {1, 2, …, q}.

Definition:1.2

A polygonal graceful labeling [2] of a graph G is an injective function η: $V(G) \rightarrow Z+$, where $Z+$ is a set of all non-negative integers that induces a bijection η^*: $E(G) \rightarrow \{Ps(1), Ps(2), \ldots, Ps(q)\}$, where $Ps(q)$ is the qth polygonal number such that $\eta^*(uv) = |\eta(u) - \eta(v)|$ for every edge $e = uv \in E(G)$. A graph which admits such labeling is called a polygonal graceful graph. For $s = 3$, the above labeling gives triangular graceful labeling. For $s = 4$, the above labeling gives tetragonal graceful labeling and so on.

Definition:1.3

Consider the graph $G = (V(G), E(G))$. Assume that $e = uv$ is an edge of G and that of W is not a vertex of G. The edge e is subdivided when it is replaced by the edges $e' = uw$ and $e'' = wv$. In other words, baycentric subdivision is a graph formed by inserting a vertex of degree 2 into each edge of the original graph. $S(G)$ represents the baycentric subdivision [4] of any graph G.

Definition:1.4

The graph generated by taking one copy of G_1(which has p vertices) and p_1 copies of G_2 and then linking the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_1 is known as the corona $G_1 \odot G_2$[5] of two graphs G_1 and G_2

Definition:1.5

Let P_m and P_n be two paths. Let $u_1, u_2, u_3, \ldots, u_i$ be the vertices of m copies of P_m and vertices of m copies of P_n be v_{ij}, 1≤ i ≤ m and 1≤ j ≤ n. The graph obtained by adjoining the end vertex v_{i1}, 1≤ i ≤ m of P_n to each u_i , 1≤ i ≤m is denoted by$P_m * P_n$[3]. It has mn vertices and mn-1 edges.

Definition:1.6

A family of stars that resembles a banana tree has a new vertex next to each star's end vertex.
Let T be a banana tree represents the $\{K_{1,n_1}, K_{1,n_2}, \ldots, K_{1,n_t}\}, t \geq 1$ [6] family of stars. Let v_i stand for the star's central vertex and u_{ij}, $j = 1,2, \ldots, n_i$ for its end vertices where $i = 1,2, \ldots, t$. Let w stands for the new vertex that connects the vertex $u_{i,1}$ of each star with $i = 1,2, \ldots, t$.

2.Methodology

2.1 Definition`

Let G be a graph with p vertices and q edges. Let the set of its vertex and edge be V(G) and E(G) respectively. Let $f: V(G) \to \{0,1,2,\dots,H_{2q-1}\}$ be an injective function, with n^th hexagonal number $H_n = \frac{2n(2n-1)}{2}$ and q being the number of edges in G. If the function f induces the function f^* on E(G) such that $f^*(uv) = |f(u) - f(v)|$ for all the edges uv ∈E(G) with $f^*(E(G)) = \{H_1, H_3, H_5, \dots, H_{2q-1}\}$ we say that the function f is odd hexagonal graceful labeling and the graph that admits such a labeling is called an odd hexagonal graceful graph.
Here, we demonstrate how to label a graph's vertices using a mathematical formulation and demonstrate that the labeling is an odd hexagonal graceful labeling.

3.Results and Discussions

3.1 Theorem

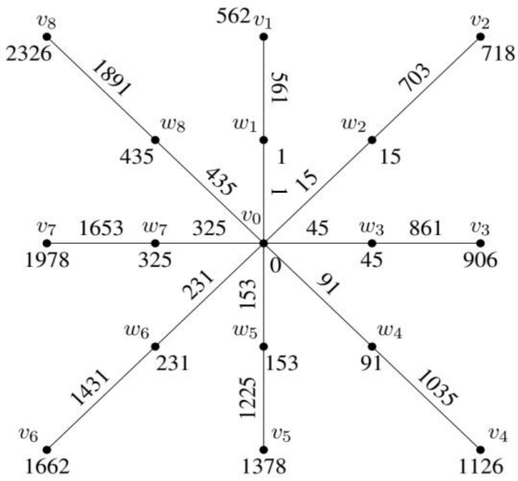
The baycentric subdivision $S(K_{1,n})$ of the star $K_{1,n}$[4] admits odd hexagonal graceful labeling.

Proof:

Let $K_{1,n}$ be the star with apex vertex v_0 and pendant vertices $v_1, v_2, v_3, \dots, v_n$.
Let $e_i = v_0 v_i$ for $i = 1,2,\dots,n$.
To obtain baycentric subdivision $G = S(K_{1,n})$ of the star $K_{1,n}$ subdivide each edge of the star $K_{1,n}$ by the vertices $w_1, w_2, w_3, \dots, w_n$ where each w_i is added between v_0 & v_i, $i = 1,2,\dots,n$.
The fact that $|V(G)| = 2n + 1$ & $|E(G)| = 2n$ is noted.
Create a function $f: V(G) \to \{0,1,2,\dots,H_{2q-1}\}$ as shown:
$f(v_0) = 0$,
$f(w_i) = H_{2i-1}$, where $1 \leq i \leq n$
In general, $f(w_n) = H_{2n-1}$
$f(v_i) = f(w_i) + H_{2n+(2i-1)}$, $1 \leq i \leq n$
In general, $f(v_n) = f(w_n) + H_{4n-1}$
It follows that f is injective and f induces the function f^*on E(G) such that $f^*(uv) = |f(u) - f(v)|$ for all $uv \in E(G) = \{H_1, H_2, \dots, H_{2q-1}\}$.
f is an odd hexagonal graceful labeling.
Thus, the odd hexagonal graceful labeling of baycentric subdivision $S(K_{1,n})$ of the star $K_{1,n}$ is formed.

3.2 Example

The figure depicts an Odd hexagonal graceful labeling of the graph $S(K_{1,8})$[4].



$S(K_{1,8})$

3.3 Theorem

The Corona $P_3 \odot \overline{K}_n$ [5] is an odd hexagonal graceful graph.

Proof:

Let p & q stands for the number of vertices and edges in this graph $P_3 \odot \overline{K}_n$ respectively.

Let $|V(G) = 3n + 3$ & $|E(G) = 3n + 2$

Define a function $f: V(G) \rightarrow \{0,1,2, \dots, H_{2q-1}\}$ as follows:

Let $f(v_1) = H_1, f(v_2) = 0, f(v_3) = H_{2q-1}$

For $i = 1$ & $1 \le j \le n$, $f(v_{ij}) = f(v_i) + H_{2j+1}$

In general, $f(v_{1n}) = f(v_1) + H_{2n+1}$

For $i = 2$ & $1 \le j \le n$, $f(v_{ij}) = f(v_i) + H_{(2n+1)+2j}$

In general, $f(v_{2n}) = f(v_2) + H_{(2n+1)+2n}$

$ie)$ $f(v_{2n}) = f(v_2) + H_{(4n+1)}$

For $i = 3$ & $1 \le j \le n$,

$f(v_{ij}) = f(v_i) + H_{(4n+1)+2j}$

In general, $f(v_{3n}) = f(v_3) + H_{(4n+1)+2n}$

$ie.$, $f(v_{3n}) = f(v_3) + H_{(6n+1)}$

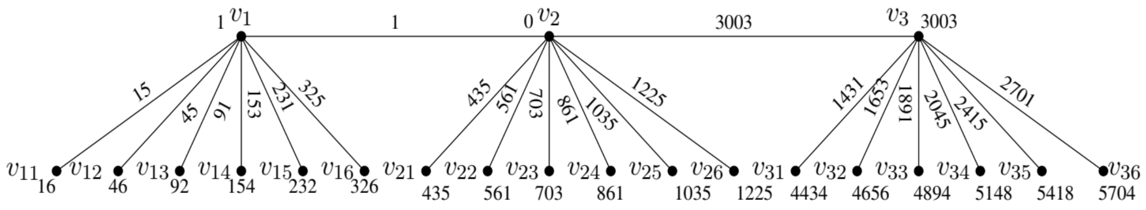
Since f is injective, f induces the function f^*on E(G) such that $f^*(uv) = |f(u) - f(v)|$ for every uv $where$ $E(G) = \{H_1, H_2, \dots, H_{2q-1}\}$.

f is an odd hexagonal graceful labeling.

The Corona graph $P_3 \odot \overline{K}_n$ is an odd hexagonal graceful graph.

3.4 Example

The Corona graph $P_3 \odot \overline{K}_n$ [5] admits odd hexagonal graceful labeling.



$P_3 \odot \bar{K}_6$

3.5 Theorem

$P_m * P_n$ [3] is an odd hexagonal graceful graph where (m$> n$).

Proof:

Let $u_1, u_2, u_3, \ldots, u_m$ be the vertices of path P_m and vertices m copies of P_n be v_{ij}, $1 \leq i \leq m \; and \; 1 \leq j \leq n$ are identified with the corresponding u_i, $1 \leq i \leq m$.

Let $G = P_m * P_n (m > n)$

Then G has mn vertices and $mn - 1$ edges.

Let us define a function $f: V(G) \to \{0,1,2, \ldots, H_{2q-1}\}$ as follows.

Let $f(v_{11}) = 0$

For $K = 1, 2 \leq j \leq m$ and $1 \leq i \leq n$, $f(v_{jk}) = f(v_{ik}) + H_{2i-1}$

In general, $f(v_{m1}) = f(v_{n1}) + H_{2n-1}$

For $K = 2$, $1 \leq j \leq m$ and $n + 1 \leq i \leq 2n + 1$, $f(v_{jk}) = f(v_{ji}) + H_{2i-1}$

In general, $f(v_{m2}) = f(v_{m1}) + H_{2(2n+1)-1}$

$ie)$ $f(v_{m2}) = f(v_{m1}) + H_{4n+1}$

For $K = 3$, $1 \leq j \leq m$ and $2n + 2 \leq i \leq 3n + 2$, $f(v_{jk}) = f(v_{j2}) + H_{2i-1}$

In general, $f(v_{m3}) = f(v_{m2}) + H_{2(3n+2)-1}$

$ie)$ $f(v_{m3}) = f(v_{m2}) + H_{6n+4-1}$

$ie)$ $f(v_{m3}) = f(v_{m2}) + H_{6n+3}$

For $K = 4$, $1 \leq j \leq m$ and $3n + 3 \leq i \leq 4n + 3$, $f(v_{jk}) = f(v_{j3}) + H_{2i-1}$

In general, $f(v_{m4}) = f(v_{m3}) + H_{2(4n+3)-1}$

$ie)$ $f(v_{m4}) = f(v_{m3}) + H_{8n+5}$

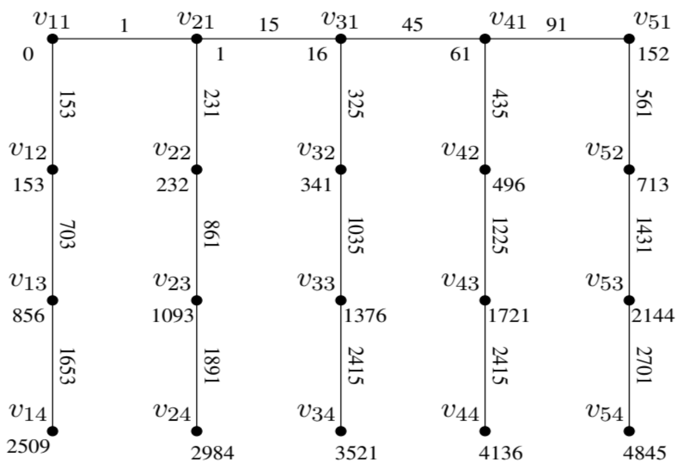
Therefore, f is injective and f induces the function f^*on E(G) such that $f^*(uv) = |f(u) - f(v)|$ for all $uv \in E(G) = \{H_1, H_2, \ldots, H_{2q-1}\}$.

f is an odd hexagonal graceful labeling.

Hence the graph $P_m * P_n$ where (m$> n$) is an odd hexagonal graceful graph.

3.6 Example

The figure depicts an odd hexagonal graceful graph of the graph $P_m * P_n$ where (m$> n$) [3].

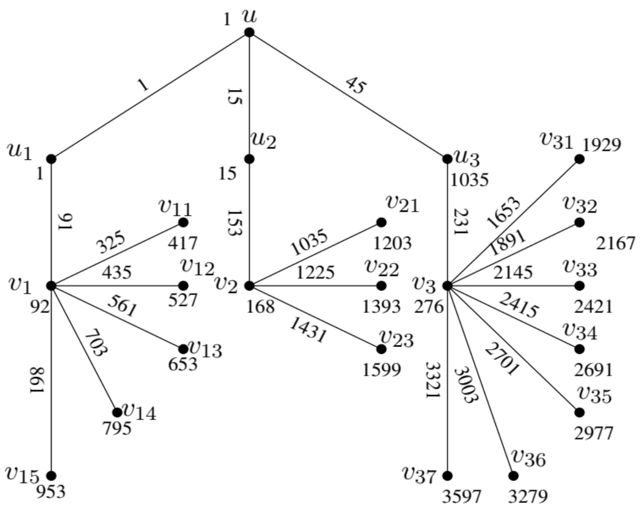


$P_5 * P_4$

3.7 Theorem
If T be a banana tree that represents the $\{K_{1,m_1}, K_{1,m_2}, \dots, K_{1,m_n}\}$ family of stars for all m, n [6] then G is an odd hexagonal graceful graph.
Proof:
Let T be a banana tree that resembles the $\{K_{1,m_1}, K_{1,m_2}, \dots, K_{1,m_n}\}$ family of stars for all m, n.
Let v_i denotes the central vertex and for $j = 1,2, \dots, n_i$ let v_{ij} denotes the end vertices of the i^{th} star K_{1,n_i} where $i = 1,2, \dots, n$.
Let u denotes the new vertex joining the vertex u_i which connects v_i for $1 \leq i \leq n$.
The banana tree graph has $2n + m + 1$ vertices and $2n + m$ edges.
Let us define a function $f: V(G) \to \{0,1,2, \dots, H_{2q-1}\}$ as follows.
Let $f(u) = 0$
For $1 \leq i \leq n$, let $f(u_i) = f(u) + H_{2i-1}$
$f(u_n) = f(u) + H_{2n-1}$,
For $1 \leq i \leq n$ & $n + 1 \leq j \leq 2n$, $f(v_i) = f(u_i) + H_{2j-1}$
In general, $f(v_n) = f(u_n) + H_{4n-1}$
For $2n + 1 \leq i \leq 2n + m_1, j = 1$ & $1 \leq k \leq 2n + 1$, $f(v_{jk}) = f(v_j) + H_{2i-1}$
In general, $f(v_{1m_1}) = f(v_1) + H_{4n+2m_1-1}$
For $(2n + 1) + m_1 \leq i \leq (2n + 5) + m_2, j = 2$ & $1 \leq k \leq m_2$, $f(v_{jk}) = f(v_j) + H_{2i-1}$
In general, $f(v_{2m_2}) = f(v_2) + H_{4n+2m_2+9}$
For $(2n + 6) + m_2 \leq i \leq (2n + 8) + m_3, j = 3$ & $1 \leq k \leq m_3$, $f(v_{jk}) = f(v_j) + H_{2i-1}$
In general, $f(v_{3m_3}) = f(v_3) + H_{4n+2m_2+15}$
Since f is injective, f induces the function f^*on E(G) such that $f^*(uv) = |f(u) - f(v)|$ for every uv $where$ $E(G) = \{H_1, H_2, \dots, H_{2q-1}\}$.
f is an odd hexagonal graceful labeling.
Hence a banana tree that represents the $\{K_{1,m_1}, K_{1,m_2}, \dots, K_{1,m_n}\}$ family of stars for all m, n is an odd hexagonal graceful graph.

3.8 Example
A banana tree that represents the $\{K_{1,m_1}, K_{1,m_2}, \dots, K_{1,m_n}\}$ family of stars for all m, n[6]

admits odd hexagonal graceful labeling.



The Banana tree T corresponding to the family of stars $\{K_{1,5}, K_{1,3}, K_{1,7}\}$

4.Conclusion

Here we established the fact that certain classes of star and path related graphs are odd hexagonal graceful graphs. We also extend this work to find other graphs which admits odd hexagonal graceful labeling and also the graceful labeling of graphs with polygonal numbers. Other researches can find more results using graceful labelings of graphs with polygonal numbers.

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